LATERAL TORSIONAL BUCKLING OF WOOD BEAMS:
FEA-MODELLING AND SENSITIVITY ANALYSIS

Qiuwu Xiao¹, Ghasan Doudak², Magdi Mohareb³

ABSTRACT: A finite element model was developed for glue-laminated wood beams modelled as an orthotropic material and comparisons with the classical solution as well as experimental results were made. The model was able to capture the buckling response and capacity of such cases and was extended to assess the influence of orthotropic constitutive properties on the lateral torsional buckling capacity of wooden beams.

KEYWORDS: Lateral torsional buckling, timber beam, orthotropic material, finite element

1 INTRODUCTION

Lateral torsional buckling (LTB) is a failure mode that occurs when the member is bent about the major axis of the cross-section where simultaneous lateral displacement and twist take place suddenly. For large span unsupported members, the resistance based on LTB may be less than that based on material failure. Theoretically, the lateral buckling resistance of a beam with a rectangular cross-section is given by

\[ M_{ltb} = C_b \frac{\pi}{L_u} \sqrt{EI/GJ} \]  

where \( C_b \) is equivalent moment gradient factor, \( L_u \) is unbraced length, \( E \) is modulus of elasticity, \( I_y \) moment of inertia about weak axis, \( G \) is shear modulus, \( J \) is torsion constant. The current study aims to obtain critical moment for glue-laminated beams through experimental testing and finite element modelling.

2 MATERIAL DESCRIPTION

Wood can be considered as an orthotropic material. For the purpose of modelling, mechanical property along the grains and perpendicular to the grains are obtained through testing following the ASTM D198-09 Standard [1].

3 MODEL DESCRIPTION AND VALIDATION AGAINST CLASSICAL LTB SOLUTION

A finite element model was developed to investigate the lateral torsional buckling capacity of Pine lodgepole glue-laminated beams, the beams cross-sections were 80mm in width, by 600mm in depth and 5000mm in span. The dimensions were selected specifically to ensure that elastic lateral torsional buckling takes place and are thus independent of the strength properties of the beam material. The beams were assumed to be simply supported, restrained laterally and torsionally at both ends without intermediate lateral bracing along the beam span. Linear elastic Eigen value finite element analysis were conducted for various scenarios of loading, including; a) concentrated load applied at mid-span, b) equal end moments inducing uniform moments c) uniformly distributed load. The C3D8 eight-noded brick element was used from the Abaqus library of elements to model the problem with three degrees of freedom were used [2]. The element dimensions were 10mm in width and depth and 20mm in length; therefore, there were 8 elements along the width, 60 elements along the depth and 250 element along the span of the beam.

A comparison between the results of the FE model and the classical lateral torsional buckling solution is presented in Table 1. Model input for the finite element model were based on published values taken from CSA Standards O86-09 Engineering design in wood [3] and the Wood Handbook [4].

<table>
<thead>
<tr>
<th>Table 1: Comparison of FE model and Classical Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant moment</td>
</tr>
<tr>
<td>Classical LTB solution</td>
</tr>
<tr>
<td>FE model</td>
</tr>
<tr>
<td>FEA/Cl assical</td>
</tr>
</tbody>
</table>

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The FE model was able to predict the capacity of the beams as determined by the well accepted classical solution. Figure 1 shows the buckled configuration for a beam subjected to uniformly distributed load as predicted by the FE model.

**Figure 1: ABAQUS model of LTB**

### 4 COMPARAISON OF FE MODEL WITH LTB EXPERIMENTAL RESULTS

To verify the accuracy of linear elastic Eigen value finite element analysis, a comparison was made between experimental programs and the numerical analysis. Shown here is an example from a testing program by Hooly and Madsen’s test [5]. It can be seen that the model is able to capture the beam behaviour under different loading configurations.

**Table 2: Comparison of FE model and Experiment results**

<table>
<thead>
<tr>
<th>Test number</th>
<th>Test results (kNm)</th>
<th>FE model results(kNm)</th>
<th>FE model/test</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.542</td>
<td>0.559</td>
<td>1.031</td>
</tr>
<tr>
<td>2</td>
<td>0.514</td>
<td>0.551</td>
<td>1.072</td>
</tr>
<tr>
<td>3</td>
<td>0.466</td>
<td>0.443</td>
<td>0.951</td>
</tr>
<tr>
<td>4</td>
<td>0.508</td>
<td>0.457</td>
<td>0.900</td>
</tr>
</tbody>
</table>

Comparison between the model and results from an experimental program conducted at the University of Ottawa’s structural lab will also be presented in the full length paper.

### 5 SENSITIVITY ANALYSIS RESULTS AND CONCLUSION

The effect of the various mechanical properties on the critical moment was systematically investigated by varying the magnitude of the constitutive parameters of the orthotropic material model.

Presented here, as an example, is the case for uniform moment. Based on the reference parameters (Young modulus in the longitudinal direction $E_L=10300\text{MPa}$, shear modulus in the transverse direction $G_T=473.8\text{MPa}$) for Pine lodgepole glue-laminated beam, the critical moment was 67.91kNm. The magnitude of the input constitutive parameters were changed by a factor of 1.5 and 0.5 while keeping other parameters unchanged. The ratio between the resulting critical moment and that based on the reference case are shown in Table 3. The results show that the critical moments are affected by the modulus of elasticity $E_L$ along the longitudinal direction, and the shear modulus $G_T$ along transverse. In contrast, the modulus of elasticity $E_T$, Poisson’s ratio $\nu_T$ along transverse, Poisson’s ratio $\nu_{RT}$ and shear modulus $G_{RT}$ about radial and tangential axes have a negligible effect on critical moment.

**Table 3: Proportions of critical moments based on the reference case and those based on changing the input value of the constitutive parameter**

<table>
<thead>
<tr>
<th>Parameter varied</th>
<th>Mcr by 1.5 times of parameter/(Mcr reference)</th>
<th>Mcr by 0.5 times of parameter/(Mcr reference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_L$</td>
<td>1.2430</td>
<td>0.6964</td>
</tr>
<tr>
<td>$G_T$</td>
<td>1.2022</td>
<td>0.7343</td>
</tr>
<tr>
<td>$E_T$</td>
<td>1.0012</td>
<td>0.9980</td>
</tr>
<tr>
<td>$\nu_T$</td>
<td>1.0004</td>
<td>0.9998</td>
</tr>
<tr>
<td>$\nu_{RT}$</td>
<td>1.0003</td>
<td>0.9999</td>
</tr>
<tr>
<td>$G_{RT}$</td>
<td>1.0053</td>
<td>0.9884</td>
</tr>
</tbody>
</table>

### 6 CONCLUSIONS

A finite element model was developed for glue-laminated wood beams modelled as orthotropic material. Comparisons were made with the classical solution for lateral torsional buckling as well as experimental results. The model was able to capture the buckling response and capacity of such cases and was extended to assess the influence of orthotropic constitutive properties on the lateral torsional buckling capacity of wooden beams.

**REFERENCES**