Mathematically Productive Teaching Routine
Structuring Student Math-Talk

**STRUCTURES FOR THE MATHEMATICIANS DYAD, TRIAD, AND QUAD**

When students work in a Mathematicians Dyad, Triad, or Quad, the math-talk:
- **Always** begins with “Mathematicians Think Time” (i.e., time to think privately) about the task.
- **Always** focuses on each group member’s mathematical reasoning, sense making, representations, justifications, and/or generalizations.
- **Always** ends with a discussion of ways their ideas are mathematically the same and/or different.
- **Always** follows a prescribed structure that provides students “practice” with status-free, and mathematically productive student-to-student interaction.

| LISTEN & COMPARE | A. Partner #1 explains her/his ideas while the other partner(s) silently listen to understand Partner #1’s thinking.
| B. When the teacher announces, “Finish your thought and switch roles,” repeat step A for Partner #2. The teacher sets the timing according to the complexity of the question/task and student backgrounds.
| C. (for triads and quads) Repeat until all partners have reported. |
| REVOICE & COMPARE | A. Partner #1 speaks while the other partner(s) silently listen to understand Partner #1’s mathematical thinking.
| B. When the teacher announces, “Finish your thought and Partner #X revoice,” Partner #X carefully revoices Partner #1’s ideas without judging, adapting, or commenting about the correctness or sensibility of the ideas.
| C. Partner #1 clarifies as needed.
| D. When the teacher announces, “Rotate Partners,” Partner #2 speaks while the other partner(s) silently listen to understand.
| E. When the teacher announces, “Finish your thought and Partner #Y revoice,” Partner #Y carefully revoices Partner #2’s ideas.
| F. Partner #2 clarifies as needed.
| G. (for triads and quads) Repeat until all partners have revoiced and reported. |
| INTERPRET & COMPARE | A. Two partners exchange their written work for a task. During Private Think Time, the partners study each other's work and, without any discussion, try to understand each other’s reasoning.
| B. Partner #1 reports her interpretation of Partner #2’s reasoning.
| C. Partner #2 clarifies.
| D. Partner #2 reports his interpretation of Partner #1’s reasoning.
| E. Partner #1 clarifies. |
| COLLABORATION ROLE PLAY | A. The teacher arranges the students in teams (dyads, triads, or quads) and distributes a role card to each student (all teams use the same roles). Student role assignments may be purposeful or random.
| B. Team members read their roles and sample role-talk to their partners.
| C. The teacher poses a task and monitors while teams role play and complete the task.
| D. The teacher periodically calls for “Role Huddles” in a corner of the room where the Mathematical Habits of Interaction are posted. During a Role Huddle (e.g., all Facilitators), the teacher gives additional directions related to the specific role and one or more of the Mathematical Habits of Interaction and/or Mathematical Habits of Mind. The Role Players return to their teams with information from the huddle.
| E. Upon completion of the task and any required team reporting/presentations, the teams debrief their effectiveness, challenges, and next steps as collaborative mathematicians. |
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Purposes
- Support the development of student-to-student interaction that is consistently equitable, status-free, and mathematically productive. (For these purposes, mathematically productive means that student interactions consistently include students’ mathematical reasoning, sense making, representations, justifications, and generalizations.)
- Provide formative assessment information that drives instructional decisions

Student Outcomes
- Equitable, status-free, and mathematically productive student-to-student interaction
- Increased metacognitive skills
- Increased capacity to articulate and clarify their math thinking
- Productive socio-mathematical norms
- Increased math content knowledge
- Improved Mathematical Habits-of-Mind/Interaction
- Increased accountability and engagement
- Increased self-efficacy as mathematicians

Groupings: Affordances and Drawbacks

<table>
<thead>
<tr>
<th>Mathematicians Dyad (2)</th>
<th>Mathematicians Triad (3)</th>
<th>Mathematicians Quad (4)</th>
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<tbody>
<tr>
<td>+ Accountability for engagement</td>
<td>+ More strategies/modeling</td>
<td>+ Can flex between Dyads &amp; Quads</td>
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<tr>
<td>+ One-to-one modeling</td>
<td>+ Fewer groups to monitor</td>
<td>+ Fewer groups to monitor</td>
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<tr>
<td>+ Takes less time</td>
<td>- Potential status issues for the &quot;third wheel&quot;</td>
<td>+ More strategies/modeling</td>
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<tr>
<td>+/- Fewer strategies to compare</td>
<td>- Discussion takes longer</td>
<td>- Discussion takes longer</td>
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<tr>
<td>+/- Less public</td>
<td>- Nonproductive if one disengages</td>
<td>- Can be easier for individuals to &quot;ride&quot; on others’ ideas</td>
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<tr>
<td>- Nonproductive if one disengages</td>
<td>- Comparing strategies could be complicated</td>
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Logistics
- **Time** 4 – 15 minutes plus the ending plenary discussion; partners are always allocated equal “air-time.”
- **Who** All students either randomly mixed or purposely combined for differentiation, modeling, status treatment, management, or specific content purposes. Partners may change regularly or be assigned for a set duration.
- **Set-up** Referencing groupings by a name (e.g., Mathematicians Dyad/Triad/Quad) alerts students to move quickly into a known structure. When structuring student talk, it is helpful to demonstrate how partners should be positioned (e.g., pairs in a knees-to-knees position, how to make eye contact, etc.). Movement provided by an occasional Stand-up Mathematicians Dyad, Triad, or Quad in which all partners stand or a specified member of each team stands to report, can boost energy, accountability, and attention.
- **Task** The math task should: (1) always be central to the core mathematical purpose(s) of the lesson, and (2) always require that the student mathematicians explain their math reasoning, sense making, representations, justifications, and/or generalizations

**Grouping and Structure** Determine whether the most mathematically productive grouping will be a Mathematicians Dyad, Triad, or Quad and whether the most productive routine will be Listen & Compare, Revoice & Compare, or Interpret & Compare.

Protocol
- **Introduce the task and implement the structure.** Provide students the task – carefully worded to assure emphasis on math reasoning, sense making, representations, justifications, and/or generalizations. Include step-by-step-visual support for the structure process and deep math thinking.
- **Monitor the math-talk.** Listen for trends in student thinking. Select and sequence ideas for a plenary.
- **Facilitate a plenary discussion of student ideas.** Invite selected students to report about their thinking, their partners’ thinking, and/or their combined ideas. Sequence student reporting, interject information, and question to focus, scaffold, and/or advance student thinking about key concepts and connections.
- **Reflect about the process.** Ask the student mathematicians, “How was today’s Mathematicians Dyad/Triad/Quad process helpful for you?” A process debrief is not always necessary, but is important periodically. It fosters student ownership and increases engagement and accountability.