The *Comprehensive Mathematics Instruction (CMI) Framework:*
A new lens for examining teaching and learning in the mathematics classroom

Scott Hendrickson, Department of Mathematics Education,
Brigham Young University
Sterling C. Hilton, Department of Educational Leadership and Foundations,
Brigham Young University
Damon Bahr, Department of Teacher Education,
Brigham Young University

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Along with previously released NCTM *Standards* (NCTM 1989, 1991, 1995), the NCTM publication *Principles and Standards for School Mathematics* identifies "key issues in contemporary mathematics education" and "sets out a carefully developed and ambitious but attainable set of expectations for school mathematics" (NCTM, 2000, p. 379), including a vision of mathematics instruction in which "students confidently engage in complex mathematical tasks chosen carefully by teachers, [as] teachers help students make, refine, and explore conjectures on the basis of evidence and use a variety of reasoning and proof techniques to confirm or disprove those conjectures." (p. 3)

To what extent is this vision and set of expectations being realized in mathematics classes throughout the United States? Observations based on the TIMSS 1995 and 1999 Video Studies (Jacobs et al., 2006) led the TIMSS researchers to conclude "the nature of classroom mathematics teaching observed in the videotapes reflects the kind of traditional teaching that has been documented during most of the past century, . . . the nature of mathematical thinking and reasoning, and the conceptual mathematical work, remain unaligned with the intent of *Principles and Standards*." (pp. 28, 30)

Although the most recent of the TIMSS Video Studies is over nine year old, experience suggests that not much has changed from the classroom episodes observed in 1999. Results from the video study suggested the following:

- Teachers often feel that their lessons are in accord with current ideas about the teaching and learning of mathematics if they involve externally observable practices such as the use of technology, inclusion of real-world problems, or collaborative group work.
- While U. S. eighth grade mathematics teachers devote approximately one-third of their lesson time to "private interaction" (time when students work on their own or in small groups), over 75% of this time consisted of working on "repeating procedures that had been demonstrated earlier in the lesson or learned in previous lessons", rather than "engaging in complex mathematical tasks, working productively and reflectively [to] make, refine and explore conjectures on the basis of evidence" as envisioned by the NCTM *Standards*.
- While more eighth grade teachers in 1999 incorporated such lesson features as sharing alternative strategies and examining solution methods than in 1995, the actual number of opportunities for students to do so was

as sharing alternative strategies and examining solution methods than in 1995, the actual number of opportunities for students to do so was extremely limited. Only 5% of the problems discussed publicly in the 1999 eighth grade mathematics lessons involved the sharing of alternative solution methods, and only 2% of shared problems were actively discussed, examined, critiqued and compared.

- None of the U. S. lessons in the 1999 Video Study showed evidence of students "developing a rationale, making generalizations, or using counterexamples" as methods of mathematical reasoning.

One might wonder, what has contributed to this lack of understanding and limited implementation of the NCTM *Standards* and other recommendations for mathematics education reform? In 1999 Chazan and Ball expressed frustration with "current math education discourse about the teacher's role in discussion-intensive teaching." They argue that educators are often left "with no framework for the kinds of specific, constructive pedagogical moves that teachers might make." (p. 2) While the NCTM *Standards* have promoted a vision and set of expectations for mathematics instruction, the conscientious lack of a prescriptive pedagogy often leaves teachers without a clear sense of direction.

The *Comprehensive Mathematics Instruction* (CMI) Framework was designed to provide access to reform-based pedagogical strategies for K-12 mathematics instructors. It was also designed to bridge the gap between the good pedagogical strategies of traditional instruction and the recommendations of reform-based instruction. The CMI Framework was developed over several years of collaborative efforts between professors from four departments (Educational Leadership, Mathematics, Mathematics Education, and Teacher Education) at Brigham Young University and representatives from five surrounding school districts representing one-third of the students in Utah. The CMI Framework consists of three major components: a *Teaching Cycle*, a *Learning Cycle*, and a *Continuum of Mathematical Understanding*.

The Teaching Cycle

Successful inquiry-based teaching moves through phases of a *Teaching Cycle* (Figure 1) that begins by engaging students in a worthwhile mathematical task (*Launch*), allows students time to grapple with the mathematics of the task (*Explore*), and concludes with a class discussion in which student thinking is examined and exploited for its potential learning opportunities (*Discuss*).

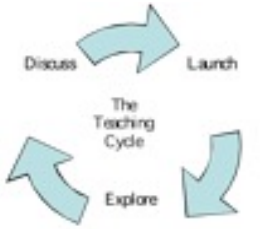


Figure 1: The Teaching Cycle

The *Teaching Cycle* is not a new idea being proposed by the CMI Framework, although the phases of the cycle might be referred to by different names in the literature, such as the Launch-Explore-Summarize instructional model of the *Connected Mathematics Project* (Lappan, Fey, Fitzgerald, Friel, & Phillips, 2006; Schroyer & Fitzgerald, 1986). The CMI Framework contains a careful articulation of the purposes for each phase of the *Teaching Cycle* and a description of teacher and student roles during these phases of guided-inquiry. What is new, however, is the CMI Framework explicitly recognizes that the phases of the *Teaching Cycle* look and function differently depending on the purpose of the lesson, as determined by the *Learning Cycle*.

The Learning Cycle

Student learning progresses through a *Learning Cycle* (Figure 2) that first surfaces students' thinking relative to a selected mathematical purpose (*Develop Understanding*), then extends and solidifies correct and relevant thinking (*Solidify Understanding*), and finally refines thinking in order to acquire fluency consistent with the mathematical community of practice both inside and outside the classroom (*Practice Understanding*).

The *Learning Cycle* is unique to the CMI Framework and suggests both how understanding develops and how it can be guided through different types of lessons. It is particularly important to realize that the phases of the *Learning* Cycle influence and

understanding develops and how it can be guided through different types of lessons. It is particularly important to realize that the phases of the *Learning* Cycle influence and modify the *Launch, Explore* and *Discuss* phases of the *Teaching Cycle*. For example, the *Launch* during a *Develop Understanding* lesson may consist of engaging students in an open-ended task designed to elicit a variety of alternative solution strategies; whereas, the *Launch* of a *Solidify Understanding* lesson may consist of a string of related problems designed to elicit and examine a particular strategy.

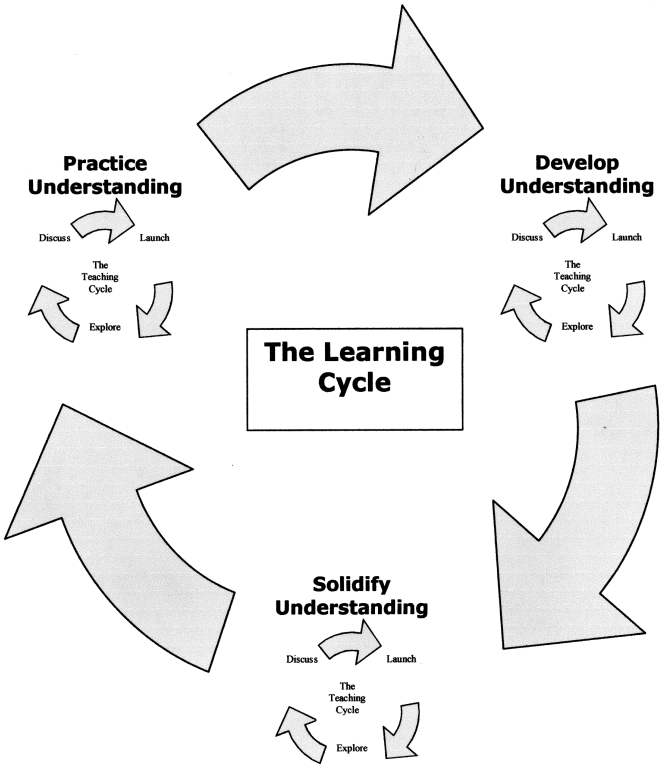


Figure 2: The Learning Cycle

Continuum of Mathematical Understanding

Mathematical understanding encompasses at least three connected but distinct domains as represented by the horizontal lines of a *Continuum of Mathematical Understanding* (Figure 3): conceptualizing mathematics, doing mathematics, and representing mathematics. Mathematical understanding progresses continually along the continuum, but it is useful to note three sets of distinct landmarks of progression along the continuum that are associated with each of the three phases of the *Learning Cycle*.

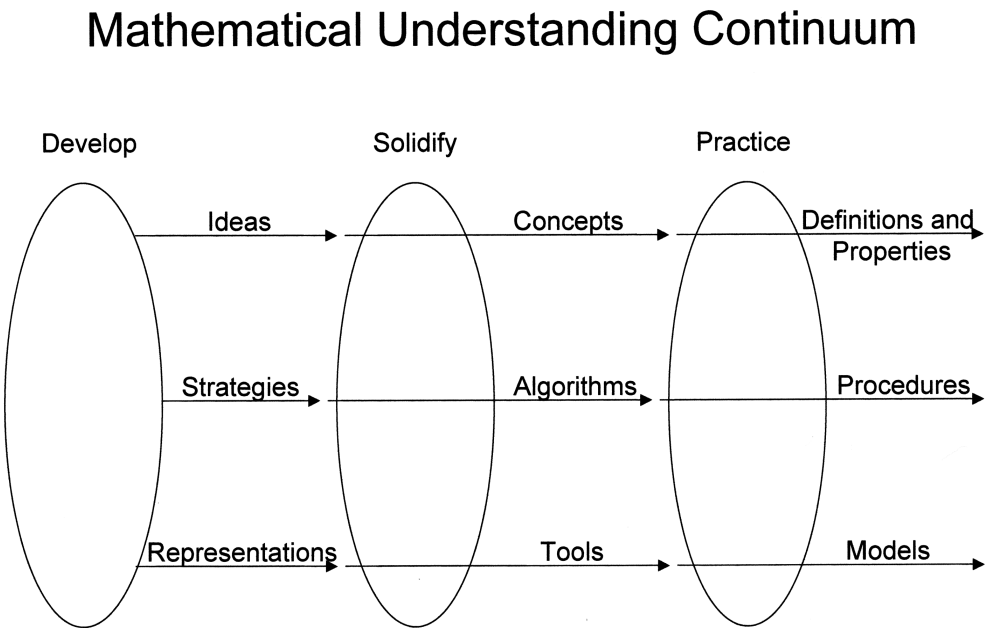


Figure 3: The Continuum of Mathematical Understanding

Emerging mental images are fragile as they are surfaced during students' initial experiences with tasks designed to elicit those images (*Develop Understanding*). We call these fragile images: ideas, strategies, and representations. These ideas, strategies and representations need to be examined for accuracy and completeness, as well as extended and connected through multiple exposures and experiences until they become more tangible, solid and useful (*Solidify Understanding*). In the CMI Framework, ideas that

and connected through multiple exposures and experiences until they become more tangible, solid and useful (*Solidify Understanding*). In the CMI Framework, ideas that become more solid and firm are called concepts; solid strategies become algorithms; and useful representations become tools. Although understanding has been developed and solidified, it needs further refinement to become fluent (*Practice Understanding*). In the CMI Framework concepts that are refined become definitions or properties; algorithms that become fluent are called procedures; and refined tools become models. These definitions and properties, procedures, and models must be consistent with the broader mathematical "community of practice".

We have tried to capture this progression of conceptualizing, doing, and representing mathematics through the *Continuum of Mathematical Understanding*. While the words used on the *Continuum* mean different things to different people, we have adopted these words to try to capture the progression of the often stated goal of mathematical instruction: deepening mathematical understanding.

Potential Uses of the CMI Framework

The CMI Framework was designed to help classroom teachers strengthen their instructional practices in order to deepen students' mathematical understanding. To this end, the CMI Framework can be used by the classroom teacher as a pedagogical tool before, during, and after teaching. Prior to teaching, the Framework provides a planning model for designing lessons to meet intended purposes and desired learning outcomes. Teachers are expected to anticipate student thinking and to anticipate how they will orchestrate a discussion from this thinking in order to develop and solidify correct understanding.

Teaching episodes in which students are allowed to explore mathematical ideas can appear chaotic and seem unmanageable for teachers who are accustomed to presenting ideas and taking on the role of sole mathematical authority in the classroom. The CMI Framework provides a method for teachers to make sense of student work that is taking place in the classroom and to decide what to do with the ideas that are emerging.

The CMI Framework also provides language and structure for a classroom teacher (and other observers of the lesson) to reflect upon what took place during the lesson. Being able to name instructional strategies and pedagogical moves allows them to become objects of reflection and refinement; therefore, teacher intent, student understanding, and the transactions that occurred between and among students and the classroom teacher can be discussed using the language and structure of the CMI Framework. Finally, the CMI Framework focuses teachers on future work—suggesting where they can go next with student thinking and suggesting possible paths for how to get there.

Implications of the CMI Framework

The CMI Framework encompasses both the procedural goals of traditional mathematics instruction and the conceptual goals of reform-based mathematics instruction; however, it does not advocate an equal balance between these two approaches to instruction. For example, traditional mathematics instruction usually *begins and ends* at the far right of the *Continuum of Mathematical Understanding* by presenting students with definitions and properties, procedures, and models without providing students with opportunities to explore, examine or refine the conceptual underpinnings of these learning outcomes. On the other hand, implementation of reform-based instruction often *begins and ends* at the far left of the *Continuum of Mathematical Understanding* by only helping students surface ideas, strategies and representations without providing students with opportunities to solidify and practice them. The CMI Framework provides a structure and model for teachers to guide students' individually-constructed ideas towards a community of shared mathematical definitions and properties, procedures, and models. By using the *Teaching Cycle,* teachers guide students through the *Learning Cycle* in order to help them progress along the *Continuum of Mathematical Understanding*.

An instructional model that focuses on the conceptual, procedural, and representational understanding of mathematics demands more of teachers than just turning to the next page of the math textbook. It requires a "pedagogical content knowledge" (Shulman, 1986) different from the mathematical content and pedagogical knowledge most teachers receive in their preservice education or develop through

turning to the next page of the math textbook. It requires a "pedagogical content knowledge" (Shulman, 1986) different from the mathematical content and pedagogical knowledge most teachers receive in their preservice education or develop through teaching experience. Therefore, it is necessary for teachers who want to use the CMI Framework to have the opportunity through in-depth professional development to deepen both their mathematical content knowledge and their mathematical pedagogical knowledge.

The CMI Framework was designed to help both preservice and classroom teachers implement instructional practices that will lead to student mathematical understanding. We have discovered, however, that it can also be used to unify the discourse of mathematics education researchers who are trying to shift instructional practices to accommodate deeper student thinking and conceptual understanding.

Of necessity, this article has only introduced the major components of the CMI Framework; the Framework itself includes specific language describing purposes, particular instructional practices, and teacher and student roles within both the *Teaching* and *Learning Cycles*. We recognize, of course, that good mathematics instruction cannot be reduced to a recipe. The CMI Framework is intended to be a "framework"—a structured set of basic principles and productive practices that can lead students to deeper mathematical understanding. The authors encourage inquiries and feedback as we continue to build a community of practice worthy of the mathematical education of our students.

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