DOMAIN DECOMPOSITION BASED EXPONENTIAL TIME DIFFERENCING METHOD FOR FLUID DYNAMICS

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Abstract. In this paper, the application of an exponential time differencing method in conjunction with WENO spacial scheme to fluid dynamic problems in a parallel computing environment is presented. In[1], second order time marching schemes for semi-discrete Euler equations are derived. Following their work, the application of PCEXP to WENO discretization in space is discussed, in which case the exact Jacobian matrix is obtained with the chain rule analytically. The computation is parallelized on the basis of domain decomposition method. Numerical tests against several one- and two-dimensional problems demonstrate that the present method is accurate and efficient.

1 INTRODUCTION

The semi-discrete Euler equations, obtained after a spacial discretization can be written in the following ODE form:

\[
\frac{du}{dt} = R(u)
\]  

A second order exponential time integration scheme of Eq.(1) has been derived by Li et.al[1]

\[
u_\ast = u_n + \int_0^{\Delta t} e^{(\Delta t-\tau)J_n} \, d\tau R(u_n)
\]

\[
u_{n+1} = u_\ast + \frac{1}{2} \int_0^{\Delta t} e^{(\Delta t-\tau)J_n} \, d\tau [N(u_\ast) - N(u_n)]
\]

where \( J_n \) denotes the Jacobian of \( R(u) \) at time \( t_n \), and \( N \) the remaining nonlinear term. \( R \) is related with spatial discretization schemes.

In our application, WENO discretization[2] in space is discussed, in which case the exact Jacobian matrix is obtained with the chain rule, and the required matrix functions to vectors are computed by Krylov subspace approximation[3].

The computation is parallelized on the basis of non-overlapping domain decomposition method shown in Figure 1 and 2.
2 TEST CASES

A benchmark case of isentropic vortex transportation from[4] is considered. Comparison of density at center line between the analytical solution, TVDRK3 scheme and ETD2 scheme is done. The results are in good agreement as shown in Figure 3.

![Domain decomposition](image1)

![Auxiliary grids](image2)

![2D isentropic vortex propagating on a uniform grid](image3)

Figure 1: Domain decomposition
Figure 2: Auxiliary grids
Figure 3: 2D isentropic vortex propagating on a uniform grid

3 CONCLUSIONS

In the final paper we will provide detailed application of the second order exponential integration scheme in conjunction with WENO reconstruction, and its parallelized methodology, especially on how to prevent numerical oscillations. Results about parallel speedup will also be presented.

REFERENCES


