EFFICIENT $p$-MULTIGRID METHOD BASED ON EXPONENTIAL TIME DISCRETIZATION FOR COMPRESSIBLE STEADY FLOWS

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Abstract. A $p$-multigrid framework is developed for solving steady-state compressible flows discretized with a $p$-order spatial discontinuous Galerkin method. The algorithm that based on a global coupling, exponential scheme provides strong damping effects to accelerate the convergence to steady-state solutions, while high-frequency error modes are smoothed out with a $s$-stage preconditioned Runge-Kutta method. The performance of the developed method is investigated in $p$-independent convergence studies. Numerical results of 2D and 3D problems demonstrate the effectiveness of the developed framework for rapid computations of compressible steady flows.

1 INTRODUCTION

An important requirement for computational fluid dynamics applications is the capability to predict steady flow such as the case of flow past a complex geometry, so that key performance parameters e.g. the lift and drag coefficients can be estimated. While the classical second-order methods are still being used extensively, high-order spatial discretizations attract more and more attentions. Most of the spatial discretizations have rested on the use of traditional time marching schemes combining with various acceleration methods. Among these methods, the $p$-multigrid acceleration is very natural in the context of modal discontinuous Galerkin methods. In this paper, we consider a $p$-multigrid framework consisting of an exponential time marching method and a $s$-stage preconditioned Runge-Kutta method as an effective way to increase the feasibility of arbitrarily $p$-order discontinuous Galerkin methods (DG) for steady-state flow computations.

2 NUMERICAL RESULTS

Two sample compressible flow cases are shown for investigating the effectiveness of the developed method: Mach number $M = 0.3$ flows past a circle in quasi-2D and a sphere in 3D. The $p$-multigrid method is used for driving the spatial DG solutions to their steady states with $p = 1 \sim 3$ orders of accuracy, and the velocity contours of the $p = 3$ solutions are given in Fig. 1. Convergence histories of $p$-multigrid (pMG) iterations versus the
$L_2$ norm of density residual are given in Fig. 2, which exhibit $p$-independent convergence rates.

Figure 1: $M = 0.3$ flows past a circle (left) and a sphere (right) with DG $p = 3$

Figure 2: $M = 0.3$ flows past a circle (left) and a sphere (right) with DG $p = 3$

REFERENCES
