TIME DISCRETIZATION STRATEGIES FOR A 3D LID-DRIVEN CAVITY BENCHMARK WITH PETSC

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Abstract. We present PETSc parallel implementations of three time discretization strategies to solve the 3D incompressible Navier-Stokes equations in velocity-vorticity formulation applied to the lid-driven cavity with aspect ratio 3:1 benchmark with second order finite differences discretization in space. Different features of the PETSc software are investigated allowing fast prototyping of parallel numerical methods adapted to the proposed strategies. Comparisons on parallel efficiency, numerical accuracy, and flow behavior will also be given.

1 INTRODUCTION

The flow in a 3D lid-driven cavity with aspect ratio 3.1 is modelled by the incompressible Navier-Stokes equations in the velocity-vorticity ($\vec{V} - \vec{\omega}$) formulation that follows:

$$\frac{\partial \vec{\omega}}{\partial t} - \vec{\nabla} \times (\vec{V} \times \vec{\omega}) = \frac{1}{Re} \Delta \vec{\omega} + B.C. + I.C \quad (1)$$

$$\Delta \vec{V} = -\vec{\nabla} \times \vec{\omega} + B.C. + I.C. \quad (2)$$

No slip boundary conditions are imposed for $t \geq 0$: $\vec{V} = (V_x = 0, V_y = 0, V_z = 0)$ for $(x = \pm\frac{3}{2}, y = \pm\frac{1}{2}, z = -\frac{1}{2})$, $\vec{V} = (0, 1, 0)$ for $(z = +\frac{1}{2})$ while $\vec{\omega} \triangleq \vec{\nabla} \times \vec{V}$ is used for $\vec{\omega}$. Initial conditions are $\vec{V} = \vec{\omega} = \vec{0}$ for $t \leq 0$, the start of training is therefore impulsive.

This problem has been implemented in a research parallel code, requiring 3 years man of development, and using ADI for time marching and multigrid accelerated by domain decomposition method for velocity [1]. It has been efficient numerically and in elapsed time in a benchmarking codes comparison [2].

2 TIME DISCRETIZATION STRATEGIES WITH PETSC

We investigate different features of the PETSc [3] parallel software to implement three time discretizing strategies for (1)-(2) that take more and more terms implicitly in the equations and boundary conditions. This benchmark constitutes the core of a lecture to teach PETSc software to engineer students in applied mathematics in few hours.
The first strategy solves (1) with taking the convection term explicitly and then solves (2) with the curl of the computed vorticity as right hand side:

\[
\begin{align*}
(\mathbb{I} - \frac{\Delta t}{Re} \Delta) \mathbf{\omega}^{n+1} & = \mathbf{\omega}^n + \Delta t \mathbf{\nabla} \times (\mathbf{V}^n \times \mathbf{\omega}^n) \\
\Delta \mathbf{V}^{n+1} & = -\mathbf{\nabla} \times \mathbf{\omega}^{n+1}, \text{ after solving (3)}
\end{align*}
\]

Strategy 2 solves simultaneously (1)-(2) with taking the convection term semi-implicitly:

\[
\begin{pmatrix}
\mathbb{I} - \frac{\Delta t}{Re} \Delta, & \Delta t \mathbf{\nabla} \times (\mathbf{\nabla} \times .) \\
\mathbf{\nabla} \times ., & \Delta.
\end{pmatrix}
\begin{pmatrix}
\mathbf{\omega}^{n+1} \\
\mathbf{V}^{n+1}
\end{pmatrix}
= 
\begin{pmatrix}
\mathbf{\omega}^n \\
0
\end{pmatrix}
\]

The PETSc implementation of strategy 1 (resp. 2) involves distributed array (DA) with one degree (resp. two) of freedom (DOF) with two (resp. one) Krylov solvers (KSP) with time invariant (resp. updating a part of the matrix at each time step) matrices (Mat).

The third strategy solves (1)-(2) totally implicitly:

\[
\begin{pmatrix}
(Id - \frac{\Delta t}{Re} \Delta) \mathbf{\omega}^{n+1} - \Delta t \mathbf{\nabla} \times (\mathbf{\nabla} \times \mathbf{\omega}^{n+1}) - \mathbf{\omega}^n \\
\Delta \mathbf{V}^{n+1} + \mathbf{\nabla} \times \mathbf{\omega}^{n+1}
\end{pmatrix}
= 
\mathbf{0}
\]

Its PETSc implementation involves a DA with two DOF with one nonlinear solver (SNES).

3 RESULTS

Results will focus on the points that follow.

- The PETSc parallel implementation details, the advantages of the software and some limitation we faced in terms of numerical developments.
- The comparison of the three strategies with respect to the choices of KSP and SNES solvers in terms of numerical accuracy and parallel efficiency.
- The three strategies effects on the CFL number and on the flow behavior.

REFERENCES

