OPENMP PARALLELIZATION OF BURGERS’ EQUATION MODEL OF TURBULENT DUCT FLOW

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Abstract. In this work we present a multi-scale LES-like turbulence modeling procedure in the context of Burgers’ equation with exact solution and investigate parallelization beyond simply employing the standard OpenMP parallel DO loops.

1 INTRODUCTION

The well-known Burgers’ equation (see, e.g., [1]) has been widely used in various preliminary CFD studies because of its close resemblance to a 1-D incompressible Navier–Stokes (N–S) equations. It differs only in a term $p_x$, which in the N–S case is a pressure gradient with $p$ being determined from the divergence-free condition on the velocity field, while in the present case $p_x$ is prescribed as part of the problem in such a way as to provide an exact solution. Here, we do this so as to mimic turbulent flow (including both spatial and temporal fluctuations) in a duct. We solve this problem with a synthetic-velocity form of implicit LES constructed to permit simultaneous calculation of large- and subgrid-scale (SGS) velocities, thus obtaining an additional level of parallelization.

2 ANALYSIS/RESULTS

We begin by noting that a two-scale solution to the N–S equations can be represented as $u(x, t) = \tilde{u}(x, t) + u^*(x, t)$, where ‘$\tilde{}$’ denotes a large-scale, filtered part corresponding to the first “few,” say several thousand, terms of a Fourier representation; and ‘$^*$’ denotes the series remainder—approximated with a SGS model. In the Burgers’ equation case the vector velocity field is simply the scalar $u(x, t) = \tilde{u}(x, t) + u^*(x, t)$.

There are numerous formulations leading to multi-scale treatments—see, e.g., [2] for turbulent N–S contexts—and here we employ

$\tilde{u}_t + (\tilde{u}^2)_x = -p_x + \nu \tilde{u}_{xx}$,

$u^*_t + (u^*^2)_x = \nu u^*_{xx}$,

with $\nu$ being kinematic viscosity ($= 1/Re$ in Burgers’ equation). Note that the complete forcing term, $p_x = \tilde{p}_x + \tilde{p}_x^*$, is used in Eq. (1a) rather than splitting it between the two equations. This first equation is solved by standard techniques (2nd-order centered differencing in space, trapezoidal integration in time) with use of a Shuman filter to control aliasing as needed.

A Galerkin procedure, restricted to a single mode, leading to a logistic map [3] (a “poor-man’s” Burgers’ equation)—analogous to the “poor-man’s” N-S (PMNS) equations of [4]—is employed for the Eq. (1b). This leads to SGS chaotic behaviors analogous to physical turbulence. Following each large-scale time step these results are combined to begin the new time step.

An advantage of a relatively simple 1-D problem is the ability to easily formulate exact solutions. We have done this in the present case and produced a result that is very similar
to physical turbulent duct flow. This and stationary-state simulations for three different grid spacings are presented in part (a) of the figure below. We see from this that as grid spacing is refined, the synthetic-velocity model approaches the exact solution which has been evaluated on a grid of 10001 points. We also show, in part (b), time series (on a very restricted subinterval to improve visualization) for both $\tilde{u}$ and $u^*$ (501 grid-point case) to demonstrate relative amplitudes of the two different scales—in basic agreement with physical results for $Re = 20000$ employed here. Notice that the red, nearly straight, line is the filtered large-scale solution with amplitude $\sim 0.934$. The chaotic black curve is the SGS model result with peak-to-peak amplitude of $\sim 0.04$ at a point $\sim 0.2$ units out from the wall, shown in part (a).

With regard to parallelization, we find that executing large and SGS scales in parallel leads to reducing run times by nearly a factor of two using two threads within the OMP SECTIONS framework. Since the current model contains only two scales, this is all that can be done except for OpenMP DO-loop parallelization, which is in progress. But these results suggest a way to significantly improve run times for many-scale problems.

3 SUMMARY/CONCLUSIONS

In this abstract we have presented a 1-D synthetic-velocity turbulence model based on the logistic map and tested it against an exact duct-flow solution to Burgers’ equation to demonstrate validity of such eddy-viscosity-free approaches. We have additionally argued that such techniques introduce a further level of parallelism, and we have shown this to be easily exploited via OMP SECTIONS constructs.

REFERENCES


