ON THE DEVELOPMENT OF A GPU-BASED MIXED LAGRANGIAN-EULERIAN MODEL FOR SOLVING THE INCOMPRESSIBLE NAVIER-STOKES EQUATIONS

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Abstract. In this research, the mixed Lagrangian-Eulerian method (MLE) is proposed to overcome disadvantage of mesh and meshless methods which is convective instability due to false diffusion errors and low order accuracy of spatial discretization schemes, respectively. It is evident that the proposed MLE method shows better results than those of the conventional particle method.

1 INTRODUCTION

In this work, the mixed Lagrangian-Eulerian method (MLE) is proposed to solve the incompressible Navier-Stokes equations. Within the framework of MLE, the momentum equations will be solved on the moving particle (Lagrangian). On the other hand, the continuity equation will be solved on the Cartesian grid (Eulerian) in order to retain the nature of ellipticity of pressure. Meanwhile, the total derivative of velocity terms shown in momentum equations is estimated by simply advecting the moving particles in Lagrangian sense, thereby eliminating the convection instability difficulty and improving the solution accuracy in the absence of false diffusion error. Moreover, the massively parallel property of particle method makes us to exploit the computing power of GPU to accelerate all the calculations. The proposed MLE method can be applied to simulate the fluid flow problem from low to high Reynolds or Rayleigh number. The particle code can be also verified and validated by solving several benchmark problems with success since the predicted numerical results compare well with benchmark solutions.

2 NUMERICAL METHODS

Projection method is adopted to solve the incompressible Navier-Stokes equations.
1. Intermediate velocities are calculated after computing velocity diffusion term by using the sixth order accurate combined compact difference (CCD6) \[1\] scheme.

2. Intermediate positions are calculated by advecting the particles with the intermediate velocities.

3. Intermediate velocities are interpolated back to the uniform grids by using the third-order accurate moving least-squares (MLS3) scheme.

4. The pressure Poisson equation (PPE) is solved by using the second-order accurate central difference scheme.

5. Velocities are updated by correcting intermediate velocities with pressure gradient term taken into account.

3 RESULTS - Taylor-Green flow at \(Re = 100\)

![Numerical results](image)

(a) Spatial rates of convergence  (b) Decay of maximum velocity

Figure 1: Numerical results

4 CONCLUSIONS

Due to the potential of parallelism, the parallel MLE method is successfully developed to solve the incompressible Navier-Stokes equations. From the numerical results, it is evident that the proposed method shows more accurate results than those of the conventional particle method.

REFERENCES
