Creating a Powerful Mathematics Learning Community: Strategies Within and Beyond the Classroom

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#reimaginingmath
Session Goals

• Strengthen your VISION of what a powerful mathematics learning community looks and feels like
• Know more about the elements of powerful mathematics learning communities
• Reflect on your own mathematics learning community
Characteristics of Classrooms that Promote Meaningful Mathematics Learning for All

1. **Teach toward the understanding of powerful mathematics**
   - Focus is on making sense of deep, meaningful mathematics concepts and relationships
   - Assessment serves to inform students’ continued growth in mathematics

2. **View students as sense makers with valuable, important ideas**
   - Students’ thinking and reasoning are valued
   - Tasks connect to and build on students’ varied interests and experiences

3. **Nurture a mathematics community of learners**
   - Norms promote inclusion and positive sense of identity toward math
   - Routines foster collaboration and communication
   - Families and communities are used as resources
Math Talk Agreements

• Mental math problem
• Everyone quiet during think time
• Silent signals
  – Start with fist against your upper chest
  – Turn thumb clockwise to indicate a strategy but not yet a solution
  – Quietly raise thumb when have a solution
  – Try to find another strategy, holding up index finger and so on for other strategies
Today’s Math Talk

2.8 x 15
## What Talk Moves Were Used?

<table>
<thead>
<tr>
<th>Move/habit</th>
<th>Explanation and use</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Revoicing</strong></td>
<td>• Repeat some or all of what the student said; then ask the student to respond and verify whether the revoicing is correct.</td>
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<tr>
<td>“So, you’re saying . . .”</td>
<td>• Revoicing can be used to clarify or highlight an idea.</td>
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<tr>
<td><strong>Repeating</strong></td>
<td>• Ask a student to repeat or rephrase another student’s idea.</td>
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<tr>
<td>“Can you repeat what he said in your own words?”</td>
<td>• Repeating encourages students to listen closely to others and allows more time to consider important ideas.</td>
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<tr>
<td>“Can someone say that in a different way?”</td>
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<tr>
<td><strong>Reasoning</strong></td>
<td>• After students have time to process another student’s idea, ask students to analyze someone else’s claim.</td>
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<tr>
<td>“Do you agree or disagree—and why?”</td>
<td>• Refrain from supporting one position or another.</td>
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<td></td>
<td>• Ask students to explain “why” to encourage them to apply their thinking to someone else’s contribution.</td>
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<td>• Reasoning allows students to engage in each other’s ideas.</td>
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</table>

Source: Adapted from Chapin, O’Connor, and Anderson, *Classroom Discussions: Using Math Talk to Help Students Learn*, 2009
What Talk Moves Were Used?

| Building on                  | • Prompt students to expand on the idea already stated.  
|                             | • “Building on” validates student contributions and allows more perspectives to be considered. |
| "Does someone want to say something more about that?" | |
| "Does someone want to raise any questions about the point made?" | |
| Wait time                   | • Wait at least 10 seconds after asking a question before calling on someone for an answer.  
| "Take your time... we'll wait." | • Wait at least the same amount of time after calling on a student to give the student time to process and organize his or her thoughts.  

Source: Adapted from Chapin, O'Connor, and Anderson, *Classroom Discussions: Using Math Talk to Help Students Learn*, 2009
Reflect and Discuss

1. What was the role of the teacher during the Math Talk?
2. What was the role of students during the Math Talk?
3. How was this different from “traditional” 20th century math lessons?
## Levels of Classroom Discourse

*(Principles to Action, p. 32)*

<table>
<thead>
<tr>
<th>Level</th>
<th>Teacher role</th>
<th>Questioning</th>
<th>Explaining mathematical thinking</th>
<th>Mathematical representations</th>
<th>Building student responsibility within the community</th>
</tr>
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<tbody>
<tr>
<td>0</td>
<td>Teacher is at the front of the room and dominates conversation.</td>
<td>Teacher is only questioner. Questions serve to keep students listening to teacher. Students give short answers and respond to teacher only.</td>
<td>Teacher questions focus on correctness. Students provide short answer-focused responses. Teacher may give answers.</td>
<td>Representations are missing, or teacher shows them to students.</td>
<td>Culture supports students keeping ideas to themselves or just providing answers when asked.</td>
</tr>
<tr>
<td>1</td>
<td>Teacher encourages the sharing of math ideas and directs speaker to talk to the class, not to the teacher only.</td>
<td>Teacher questions begin to focus on student thinking and less on answers. Only teacher asks questions.</td>
<td>Teacher probes student thinking somewhat. One or two strategies may be elicited. Teacher may fill in an explanation. Students provide brief descriptions of their thinking in response to teacher probing.</td>
<td>Students learn to create math drawings to depict their mathematical thinking.</td>
<td>Students believe that their ideas are accepted by the classroom community. They begin to listen to one another supportively and to restate in their own words what another student has said.</td>
</tr>
<tr>
<td>2</td>
<td>Teacher facilitates conversation between students, and encourages students to ask questions of one another.</td>
<td>Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.</td>
<td>Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.</td>
<td>Students label their math drawings so that others are able to follow their mathematical thinking.</td>
<td>Students believe that they are math learners and that their ideas and the ideas of their classmates are important. They listen actively so that they can contribute significantly.</td>
</tr>
<tr>
<td>3</td>
<td>Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.</td>
<td>Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse.</td>
<td>Teacher follows student explanations closely. Teacher asks students to contrast strategies. Students defend and justify their answers with little prompting from the teacher.</td>
<td>Students follow and help shape the descriptions of others’ math thinking through math drawings and may suggest edits in others’ math drawings.</td>
<td>Students believe that they are math leaders and can help shape the thinking of others. They help shape others’ math thinking in supportive, collegial ways and accept the same support from others.</td>
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# Levels of Classroom Discourse

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<td><strong>Level 2</strong></td>
<td>Teacher asks probing questions and facilitates some student-to-student talk. Students ask questions of one another with prompting from teacher.</td>
<td>Teacher probes more deeply to learn about student thinking. Teacher elicits multiple strategies. Students respond to teacher probing and volunteer their thinking. Students begin to defend their answers.</td>
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<td>Students carry the conversation themselves. Teacher only guides from the periphery of the conversation. Teacher waits for students to clarify thinking of others.</td>
<td>Student-to-student talk is student initiated. Students ask questions and listen to responses. Many questions ask “why” and call for justification. Teacher questions may still guide discourse.</td>
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<td><strong>Level 3</strong></td>
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20th Century Instructional Routine

I do.  We do.  You do.
Instruction that Supports Learning with Meaning

• Allow students to use their own informal problem-solving strategies, at least initially, and then guide their mathematical thinking toward more effective strategies and advanced understandings.

• Encourage math talk so that students can clarify their strategies to themselves and others, and compare the benefits and limitations of alternate approaches.

• Design intentional activities/tasks that effectively bridge commonly held preconceptions and targeted mathematical understandings.

https://www.nap.edu/catalog/11101/how-students-learn-mathematics-in-the-classroom
21st Century Routine that Supports Learning with Meaning

THINK

SHARE

COMPARE

To learn more, see this paper: Fostering Student Engagement in the Mathematical Practices
How Might We Support Students to Engage in a Discourse-Rich Community?

Individual

Pairs

Small Group

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Participation Structures to Support a Discourse-Rich Math Community

Individual Think Time
Participation Structures to Support a Discourse-Rich Math Community

Individual Think Time

Focused Pair Share
Tools to Support a Discourse-Rich Math Community

Conversational Prompts

Expressing an Opinion
I think that...
It seems to me that...
In my opinion...

Asking for Clarification
What do you mean?
Will you explain ________ again?
I have a question about that.

Disagreeing
I disagree with you because...
I got a different answer than you.
I see it another way.
Interactive Math Walls
Teachers play a major role in establishing and distributing (and disrupting) power relations in the classroom.
Digital Discourse Tools: Voicethread

38 + 61
Digital tools for sharing and collaboration include:

- Screencast-O-Matic (tutorial)
- SeeSaw (teacher tutorial and teacher ideas)
- FlipGrid (ideas from teacher blog)
- and many more!!
Promoting a Math Discourse Community

1. **What type of spaces** are available for students to engage in mathematical thinking and discourse...
   - Independently?
   - In pairs?
   - In small groups?

2. **What supports** do students have within these spaces in terms of...
   - Access to tools (manipulatives, whiteboards, other supplies)?
   - Norms for productive engagement with the mathematics and with one another?
   - A variety of ways of to express and share their thinking (e.g., paper to write on; laminated sentence starters; digital tools for recording and sharing)?
Extending the Classroom: Families and Communities
Family Involvement vs. Engagement

“A school striving for parent engagement... tends to lead with its ears—listening to what parents think, dream, and worry about.”

(Ferlazzo, 2011, p. 10)
Home and School Partnerships

Elementary School

- Mallory's family is going to buy oranges. The Grand Marke sells oranges at $0.99 for 100 pounds. How much does 1 pound of oranges cost at Grand Marke?

- It will cost $9.9 to buy 1 pound of oranges.

- What I thought of First

- What happened was doing it.
Connecting to the Community

- A photo can serve as a powerful tool to investigate the mathematics in students’ daily lives.
- What’s the mathematics in this picture?

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Mrs. Rodriquez bought a large cup of mangos and a small cup of watermelon. The small cup costs $1.75 and the larger cup costs twice as much as the smaller cup. If Mrs. Rodriquez paid $10.00, how much money should she get back?

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Parents Sharing Mathematics

See this website for examples:
http://tacib.weebly.com/parents--stem.html
Parents Sharing Mathematics

Empaquetado de Pelotas

- La pelota de baloncesto del jugador Lew Alcindor viene en una caja cúbica cuyas aristas miden 1 pie de largo. Estas cajas se mandan a las tiendas en una caja más grande que contiene 24 pelotas. **EJERCICIO: ¿Cuántos arreglos se pueden hacer con 24 pelotas?** Usa los cubos para producir los arreglos y la tabla a continuación para organizar tus datos.

<table>
<thead>
<tr>
<th>Largo</th>
<th>Ancho</th>
<th>Alto</th>
<th>Volumen</th>
<th>Área de Superficie</th>
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1. ¿Cómo puedes garantizar que tienes todos los arreglos posibles?
2. ¿Cuál es la mejor forma de arreglar las pelotas? ¿Y por qué?
3. ¿Por qué crees que la compañía no manda las pelotas en paquetes con 26 pelotas?

Packaging Basketballs

- The Lew Alcindor basketball comes in a cube-shaped box that measures 1 foot along each edge. These smaller boxes are shipped in one large box that contains 24 basketballs.
- **TASK: in how many possible arrangements you can package the 24 boxes?** Use the set of cubes provided and a table like the one below to organize your data.

<table>
<thead>
<tr>
<th>Length</th>
<th>Width</th>
<th>Height</th>
<th>Volume</th>
<th>Surface Area</th>
<th>Sketch</th>
</tr>
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</table>

1. How can you be certain that you've found all possible arrangements?
2. What is **best** way to arrange these and why?
3. Why doesn't the company send these in sets of 26 basketballs?
The Quinceañera Problem

Parent Interview: Another parent who plans parties shared how she uses math to calculate numbers of tables, chairs, etc.

This became a problem to investigate two-digit multiplication.

What are the possible seating arrangements?

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The Quinceañera Problem

Martín is helping plan his sister’s quinceañera. The space where it will be held measures 30 feet by 50 feet.

- Based on the number of guests attending, how many of each size table should Martín recommend?
- Show how the tables will be arranged and how you calculated the total number of chairs.
1. How do you elicit, learn about, and build on students’ cultural and community funds of knowledge in mathematics?

2. How do you and your school encourage and value bi-directional exchanges of information with parents to support students’ mathematics learning?
Promoting Meaningful Mathematics Learning for All

1. Teach toward the understanding of powerful mathematics
2. View students as sense makers with valuable, important ideas
3. Nurture a mathematics community of learners

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Thank you!!

Questions?

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