Let’s Communicate!

Foster math learning one conversational interaction at a time.

NWMC
Whistler, BC
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Levels of Discourse in a Mathematics Classroom  
Moving Responsibility for Understanding to Students

Participating in a mathematical community through discourse is as much a part of learning mathematics as the conceptual understanding of the mathematics itself. As students learn to make and test conjectures, question, and agree or disagree about problems, they are learning the essence of what it means to do mathematics (Catherine C. Stein).

<table>
<thead>
<tr>
<th>Level 0: Teacher Directed Dependent on Teacher Direction</th>
<th>Level 1: Teacher Directed Dependent on Teacher Modeling</th>
<th>Level 2: Student Centered Dependent on Teacher Facilitation</th>
<th>Level 3: Student Centered Dependent on Student Participation as a Member of a Learning Community</th>
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<tbody>
<tr>
<td>• Students learn about their roles and responsibilities from the teacher in order to work productively in whole group, small group and independent settings.</td>
<td>• Teacher models expected roles and responsibilities for whole group, small group and independent settings.</td>
<td>• Students apply knowledge of roles and responsibilities as they work in whole group, small group and independent settings. Teacher provides feedback as students apply knowledge of roles and responsibilities in their work with others.</td>
<td>• Students hold themselves and other students accountable for whole/small group and independent roles and responsibilities.</td>
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<tr>
<td>• The teacher asks questions and affirms the accuracy of answers. Students give short answers to the teacher’s questions.</td>
<td>• The teacher asks students direct questions about their thinking while other students listen.</td>
<td>• The teacher asks open-ended questions to solicit student thinking and asks students to comment on one another’s work. Students answer the questions posed to them.</td>
<td>• Students ask questions of one another to clarify ideas.</td>
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<tr>
<td>• The teacher introduces and explains mathematical ideas. Students listen.</td>
<td>• The teacher explains student strategies, filling in any gaps before continuing to present mathematical ideas. The teacher may ask one student to help another by showing how to do a problem.</td>
<td>• Students voluntarily provide additional information about their thinking. The teacher provides confirmation and redirection based on student explanations.</td>
<td>• Student ideas build upon one another as they thoroughly explain and justify their thinking and listen to the explanation of others. The teacher asks questions about ideas, solutions, strategies and thinking in order to extend understanding.</td>
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Springfield Public Schools  
August, 2009
# Productive Talk Moves

Each talk move listed below is a suggested action that is found to be effective for making progress toward supporting mathematical thinking and learning. Each move serves various purposes. These are not the only talk moves that can be used to support productive mathematical talk. However, these are a solid base for which to begin to encourage conversations in a math classroom.

<table>
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<tr>
<th>Talk Move</th>
<th>Definition/Ideas</th>
<th>Looks like/Sounds like</th>
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| Revoicing     | - This talk move allows teachers to interact with a student who is unclear.  
- The teacher tries to repeat some or all of what the student has said  
- Then the teacher asks the student to respond and verify whether or not the revoicing is correct.  
- Few students will improve if teachers only call on the clear students.  
- This move can also be effective when the teacher understands what a student has said but is not sure that the other students in the class understand. | In a whole group discussion a third grade class discusses if 24 is an even or odd number  
1. Philip: Well, if we could use three, then it could go into that, but three is odd. So then if it was...but...three is even. I mean odd. So if it’s odd, then it’s not even.  
2. Teacher: OK, let me see if I understand. So you’re saying that 24 is an odd number?  
3. Philip: Yeah. Because three goes into it, because twenty-four divided by three is eight.  
The teacher is confused about the student’s comments and asks the question back of the student to see if her understanding is correct. This gives the student a chance to clarify. This also gives the teacher a chance to see if the student has a misconception or an understanding of the concept.  
4. Teacher: Can anyone repeat what Philip just said in his or her own words? Miranda?  
5. Miranda: Um, I think I can. I think he said that 24 is odd because it can be divided by three.  
6. Teacher: Is that right Philip? Is that what you said?  
7. Philip: Yes.  
If other students could not or did not hear what a speaker said, they cannot easily participate in further exchanges. Over time, as students come to realize that people are listening closely to what they say, they increasingly make efforts to make their contributions comprehensible. |
| Teacher revoices. | “So you’re saying that it’s an odd number”                                                                                                                                                                                                 |                                                                                                                                                                                                                      |
| “Can you repeat what he just said in your own words?” | - This talk move extends the responsibility to the students in the classroom.  
- By asking one student to repeat or rephrase what another student has said, it requires the class to listen to each other and make sense of ideas shared.  
- After the student has restated, go immediately back to the original student for follow up.  
- This move also allows the rest of the class to hear the idea shared one more time so they have more time to process what was shared. |                                                                                                                                                                                                                      |
# Productive Talk Moves

| Agree/Disagree | 8. Teacher: Miranda, do you agree or disagree with what Philip said?  
9. Miranda: Well, I sort of...like, I disagree?  
10. Teacher: Can you tell us why you disagree with what he said? What's your reasoning?  
11. Miranda: Because I thought that we said yesterday that you could divide even numbers by two. And I think you can divide twenty-four by two. And it's twelve. So isn't that even? |
|---|---|
| "Do you agree or disagree and why?" | - This move encourages students to apply their own reasoning to someone else’s reasoning.  
- The teacher should not support one position or another but should use the talk move to elicit other ideas.  
- It is critical that students support their decision by explaining their reasoning as it supports students’ mathematical learning. |
| Add on | Continuing the conversation but begins by revoicing:  
12. Teacher: So we have two different ideas here about the number twenty-four. Philip, you are saying that twenty-four is odd because you can divide it by three?  
14. Teacher: And Miranda, you are saying that it’s even because you can divide it by two? Is that correct?  
15. Miranda: Yes.  
16. Teacher: OK, so what about other people? Who would like to add to this discussion? |
| "Who can add an idea to this discussion?" | - This move increases participation by asking other students to either agree or disagree or to add other comments.  
- This prompting for more input on previous statements will, over time, result in students showing more willingness to weigh in on what the group is considering. |
| Wait time | After asking the question in line 16, the teacher waits about 5 seconds and a few hands might go up. The students know the teacher will wait for more responses so that it is not always the same super fast two or three students who will answer all the questions. After 15-20 seconds slowly other hands go up. After 45 seconds the teacher calls on Eduardo who takes about ten seconds before responding:  
17. Eduardo: Yes, I agree with Miranda’s idea, because the only way we learned to find out if something is even is to divide by two. And if we divide 24 by three, we can also divide it by 4. And we can divide it by 6 too. So I think we should stick with two only. |
| Wait time 1 | - This move is not about talking at all but instead about silence.  
- Wait time 1: After a teacher has asked a question, students should be given about 5-10 seconds to think about the ideas. |
| Wait time 2 | - Wait time 2: Wait time also should come into play after a student has been called on. After calling on a particular student, that student should be given at least the same amount of time to organize his/her thoughts.  
- The amount of time to wait should be determined based on what the students must think about. |

# Checklist

**Goals for Productive Discussions and Nine Talk Moves**

<table>
<thead>
<tr>
<th><strong>Goal One</strong></th>
<th>Help Individual Students Share, Expand and Clarify Their Own Thinking</th>
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</table>
| □ 1. **Time to Think** | - Partner Talk  
- Writing as Think Time  
- Wait Time |
| □ 2. **Say More:** | “Can you say more about that?”  
“What do you mean by that?”  
“Can you give an example?” |
| □ 3. **So, Are You Saying...?:** | “So, let me see if I’ve got what you’re saying. Are you saying...?”  
(always leaving space for the original student to agree or disagree and say more) |

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<tr>
<th><strong>Goal Two</strong></th>
<th>Help Students Listen Carefully to One Another</th>
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| □ 4. **Who Can Rephrase or Repeat?** | “Who can repeat what Javon just said or put it into their own words?”  
(After a partner talk) “What did your partner say?” |

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<thead>
<tr>
<th><strong>Goal Three</strong></th>
<th>Help Students Deepen Their Reasoning</th>
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</thead>
</table>
| □ 5. **Asking for Evidence or Reasoning** | “Why do you think that?”  
“What’s your evidence?”  
“How did you arrive at that conclusion?” |
| □ 6. **Challenge or Counterexample** | “Does it always work that way?”  
“How does that idea square with Sonia’s example?”  
“What if it had been a copper cube instead?” |

| **Goal Four** | Help Students Think With Others |
|----------------|--------------------------------|---|
| □ 7. **Agree/Disagree and Why?** | “Do you agree/disagree? (And why?)”  
“What do people think about what lan said?”  
“Does anyone want to respond to that idea?” |
| □ 8. **Add On:** | “Who can add onto the idea that Jamal is building?”  
“Can anyone take that suggestion and push it a little further?” |
| □ 9. **Explaining What Someone Else Means** | “Who can explain what Aisha means when she says that?”  
“Who thinks they could explain why Simon came up with that answer?”  
“Why do you think he said that?” |
Which figure does not belong? Explain.

Which expression does not belong? Explain.

<table>
<thead>
<tr>
<th>$x^2 - 4$</th>
<th>$-x^2 + 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x + 3$</td>
<td>$x^2 - 6x + 9$</td>
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</table>
Create Your Own Task
The figure below is composed of eight circles, seven small circles and one large circle containing them all. Neighboring circles only share one point, and two regions between the smaller circles have been shaded. Each small circle has a radius of 5 cm.

Calculate:

a. The area of the large circle.

b. The area of the shaded part of the figure.
ASKING QUESTIONS

Asking the right question is an art to be cultivated by all educators. Low-level quizzes that ask for recall or simple computation are a dime a dozen but a good high-level open-ended question that gives students a chance to think is a treasure! These questions might be used as teaching or "leading" questions as well as for assessment purposes. Both questions and responses may be oral, written, or demonstrated by actions taken. The questions and their responses will contribute to a climate of thoughtful reflectiveness.

Some suggestions about assessment questioning:

- Prepare a list of possible questions ahead of time, but unless the assessment is very formal, be flexible. You may learn more by asking additional or different questions.
- Use plenty of wait time; allow students to give thoughtful answers.
- For formal assessment, leading questions and feedback are not generally used, although some assessment techniques include teaching during the examination.
- Make a written record of your observations. A checklist may or may not be appropriate.

*This is a starter list. You will want to build a collection of your own good questions.*

<table>
<thead>
<tr>
<th>Problem Comprehension</th>
<th>What is this problem about? What can you tell me about it?</th>
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<tbody>
<tr>
<td></td>
<td>How would you interpret that?</td>
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<tr>
<td></td>
<td>Would you please explain that in your own words?</td>
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<tr>
<td></td>
<td>What do you know about this part?</td>
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<tr>
<td></td>
<td>Do you need to define or set limits for the problem?</td>
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<td></td>
<td>Is there something that can be eliminated or that is &quot;missing&quot;?</td>
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<tr>
<td></td>
<td>What assumptions do you have to make?</td>
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</table>

<table>
<thead>
<tr>
<th>Approaches and Strategies</th>
<th>Where could you find the needed information?</th>
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<tbody>
<tr>
<td></td>
<td>What have you tried? What steps did you take?</td>
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<tr>
<td></td>
<td>What did not work?</td>
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<tr>
<td></td>
<td>How did you organize the information? Do you have a record?</td>
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<tr>
<td></td>
<td>Did you have &quot;a system? A strategy? A design?</td>
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<tr>
<td></td>
<td>Have you tried (tables, trees, lists, diagrams...)?</td>
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<tr>
<td></td>
<td>Would it help to draw a diagram or make a sketch?</td>
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<tr>
<td></td>
<td>How would it look if you used these materials?</td>
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<tr>
<td></td>
<td>How would you research that?</td>
</tr>
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<table>
<thead>
<tr>
<th>Relationships</th>
<th>What is the relationship of this to that?</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>What is the same? What is different?</td>
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<tr>
<td></td>
<td>Is there a pattern?</td>
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<td></td>
<td>Let's see if we can break it down. What would the parts be?</td>
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<tr>
<td></td>
<td>What if you moved this part?</td>
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<tr>
<td></td>
<td>Can you &quot;write another problem related to this one?</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Flexibility</th>
<th>Have you tried making a guess?</th>
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<tbody>
<tr>
<td></td>
<td>Would another recording method work as well or better?</td>
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<tr>
<td></td>
<td>What else have you tried?</td>
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<tr>
<td></td>
<td>Give me another related problem. Is there an easier problem?</td>
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<tr>
<td></td>
<td>Is there another way to (draw, explain, say...) that?</td>
</tr>
</tbody>
</table>

This is a page from *Assessment Alternatives in Mathematics*, a booklet from the California Mathematics Council and EQUALS.
ASKING QUESTIONS (continued)

Communication
Can students describe or depict the strategies they are using? Do they articulate their thought processes? Can they display or demonstrate the problem situation?

- Would you please reword that in simpler terms?
- Could you explain what you think you know right now?
- How would you explain this process to a younger child?
- Could you write an explanation for next year's students (or some other audience) of how to do this?
- Which words were most important? Why?

Curiosity and Hypotheses
Is there evidence of conjecturing, thinking ahead, checking back?

- Can you predict what will happen?
- What was your estimate or prediction?
- How do you feel about your answer?
- What do you think comes next?
- What else would you like to know?

Equality and Equity
Do all students participate to the same degree? Is the quality of participation opportunities the same?

- Did you work together? In what way?
- Have you discussed this with your group? with others?
- Where would you go for help?
- How could you help another student without telling the answer?
- Did everybody get a fair chance to talk?

Solutions
Do students reach a result? Do they consider other possibilities?

- Is that the only possible answer?
- How would you check the steps you have taken, or your answer?
- Other than retracing your steps, how can you determine if your answers are appropriate?
- Is there anything you have overlooked?
- Is the solution reasonable, considering the context? How did you know you were done?

Examining Results
Can students generally, prove their answers? Do they connect the ideas to other similar problems or to the real world?

- What made you think that was what you should do?
- Is there a real-life situation where this could be used?
- Where else would this strategy be useful?
- What other problem does this seem to lead to?
- Is there a general rule?
- How were you sure your answer was right?
- How would your method work with other problems?
- What questions does this raise for you?

Mathematical Learning
Did students use or learn some mathematics from the activity? Are there indications of a comprehensive curriculum?

- What were the mathematical ideas in this problem?
- What was one thing you learned (or 2 or more)?
- What are the variables in this problem? What stays constant?
- How many kinds of mathematics were used in this investigation?
- What is different about the mathematics in these two situations?
- Where would this problem fit on our mathematics chart?

Self-Assessment
Do students evaluate their own processing, actions, and progress?

- What do you need to do next?
- What are your strengths and weaknesses?
- What have you accomplished?
- Was your own group participation appropriate and helpful?
- What kinds of problems are still difficult for you?

This is a page from Assessment Alternatives in Mathematics, a booklet from the California Mathematics Council and EQUALS.
Taxi Service

Desmond is organizing a trip to the airport for a party of 75 people. He can use two types of taxis. A small taxi costs $44 for the trip and holds up to 4 passengers. A large taxi costs $70 for the trip and holds up to 7 passengers.

1a. If Desmond orders 6 large taxis, how many small taxis will he need?

1b. How much will the total cost be?

Desmond can organize the trip more cheaply than this!

2. How many taxis of each type should Desmond order to keep the total cost as low as possible? Explain.
THE ART OF QUESTIONING IN MATHEMATICS
From the NCTM Professional Teaching Standards

HELP STUDENTS WORK TOGETHER TO MAKE SENSE OF MATHEMATICS:
• What do you think about what ______ said?
• Do you agree? Disagree? Why or why not?
• Does anyone have the same answer but a different way to explain it?
• Would you ask the rest of the class that question?
• Do you understand what they are saying?
• Can you convince the rest of us that that makes sense?

HELP STUDENTS TO RELY MORE ON THEMSELVES TO DETERMINE WHETHER SOMETHING IS MATHEMATICALLY CORRECT:
• Why do you think that?
• Why is that true?
• How did you reach that conclusion?
• Does that make sense?
• Can you make a model and show that?

HELP STUDENTS TO LEARN TO REASON MATHEMATICALLY:
• Does that always work? Why or why not?
• Is that true for all cases? Explain.
• Can you think of a counter example?
• How could you prove that?
• What assumptions are you making?

HELP STUDENTS TO CONJECTURE, INVENT, AND SOLVE PROBLEMS:
• What would happen if ______? What if not?
• Do you see a pattern? Explain.
• What are some possibilities here?
• How did you predict the next one? What about the last one?
• How did you think about the problem?
• What decision do you think he/she should make?
• What is a like and what is different about your method of solution and his/hers?

HELP STUDENTS TO CONNECT MATHEMATICS, ITS IDEAS, AND ITS APPLICATIONS:
• How does this relate to ______?
• What ideas that we have learned before were useful in solving this problem?
• Have we ever solved a problem like this one before?
• What uses of mathematics did you find in the newspaper last night?
• Can you give me an example of ______?
Additional Task to consider...

A small and regular fruit salad consists of blueberries, raspberries, grapes, and cherries. The small fruit salad has a total of 280 pieces of fruit and the regular fruit salad has a total of 525 pieces of fruit. There are twice as many raspberries as blueberries, three times as many grapes as cherries, and four times as many cherries as raspberries.

How many each fruit are there in the small and regular fruit salad? Using the same ratio of fruit, how many cherries do you think there should be in a large fruit salad? Explain.

Think about the following to help you:

What is the most common fruit in the salad? What is the least common fruit in the salad? What are the ratios of each piece of fruit to the others?