NC Math 3 Standards

**Explain volume formulas and use them to solve problems.**
G-GMD.3 Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

**Visualize relationships between two-dimensional and three-dimensional objects**
G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

**Apply geometric concepts in modeling situations.**
G-MG.1 Apply geometric concepts in modeling situations
- Use geometric and algebraic concepts to solve problems in modeling situations.
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.

Prior knowledge:
- Students can identify shapes by their appropriate names.
- Students can identify the measurements necessary to calculate the volume.
- Students understand the relationship between radius and diameter of a circle.
- Students understand volume is measured in unit cubes.
- Students can calculate the area of squares, rectangles, triangles and circles.
- Students can calculate the perimeter of circles and polygons.
**Explain volume formulas and use them to solve problems.**

G-GMD.3 Use the volume formulas for prisms, cylinders, pyramids, cones, and spheres to solve problems.

| Level 1 | Students can calculate the volume of a three dimensional shape given  
|         | - a three dimensional shape (prism, cylinder, pyramid, cone and/or sphere)  
|         | - measurements labeled or enough information to determine the measurements (ex: the length is twice the height)  
|         | - volume formula sheet  
|         | Find the volume of each shape: |
|         | ![Volume of an 14 ft cone](image.png)  
|         | ![Volume of a KC Apple Juice](image.png)  
| Level 2 | Students can adjust volume calculations in order to solve problems given  
|         | - partial or composite three dimensional figures  
|         | - measurements labeled or enough information to determine the measurements needed (ex: diameter given and radius needed)  
|         | - volume formula sheet  
|         | Students can solve for a measurement of a three dimensional figure given  
|         | - drawing or description of a three dimensional figure  
|         | - the volume of the figure  
|         | - all measurements except one missing measurement  
|         | Find the volume of each figure  
|         | ![Volume of an ice cream](image.png)  
|         | ![Volume of a cube](image.png)  
|         | The volume of a square pyramid is 180 ft³ and has a height of 10 ft. Find the length of the side of the base.  
| Level 3 | Students can solve multi-step volume problems given  
|         | - drawing or description of three dimensional figure(s)  
|         | - measurements of the figure  
|         | - at least one additional aspect of the problem  
|         | A toy manufacturer has designed a new piece for use in building models. It is a cube with side length 7 mm and it has a 3 mm diameter circular hole cut through the middle. The manufacturer wants 1,000,000 prototypes. If the plastic used to create the piece costs $270 per cubic meter, how much will the prototypes cost?  
|         | ![Multi-step volume problem](image.png)  

A golden-colored cube is handed to you. The person wants you to buy it for $100, saying that is a gold nugget. You know gold should weight 19.3 g per cubic centimeter. You measure the cube and find that it is 2 cm on each side, and weighs 40 g. Should you buy the cube? Why or why not?

A cylinder has a radius of 4 in and a height of 9 in. What would be the height of a cone with the same radius and the same volume?

### Level 4

Students can use the proportional relationships between figures to compare
- the volume of cones, cylinders and spheres with the same radius
- the volumes of pyramids and prisms with congruent bases and same height

If the volume of a rectangular pyramid is $93\frac{1}{3}$ ft$^3$, what is the volume of a prism with a congruent base and the same height?

If the volume of a sphere is $240\frac{2}{3}$ ft$^3$, what is the volume of the cylinder with the same radius and double the height?

#### Visualize relationships between two-dimensional and three-dimensional objects

G-GMD.4 Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

### Level 1

Given a
- drawing of a three dimensional object intersected by a plane or
- physical three dimensional object with ability to slice it with a plane

Students can
- draw the two dimensional cross section
- identify the two dimensional cross section
- define the cross section as the intersection of a plane and a three dimensional object

A plane can intersect a cylinder in different ways. Draw and describe the cross section formed for each of the cylinders on the right.
<table>
<thead>
<tr>
<th>Level 2</th>
<th>Given</th>
<th>Consider the intersection of a plane and a cone. If the plane were parallel to the base of the cone, what would be the shape of the cross section?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>● a three dimensional object and ● instructions on how to slice it by a plane (e.g. parallel to the base, through opposite edges, etc)</td>
<td>Draw a vertical and a horizontal cross section of the rectangular prism and identify each.</td>
</tr>
<tr>
<td>Students can</td>
<td>● draw the plane within the three dimensional figure ● identify the three dimensional cross section</td>
<td></td>
</tr>
</tbody>
</table>

**Visualize relationships between two-dimensional and three-dimensional objects**

G-GMD.4 Identify the shapes of two-dimensional cross sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

<table>
<thead>
<tr>
<th>Level 1</th>
<th>Given a ● simple two dimensional shape and an axis of rotation ● set of possible three dimensional objects</th>
<th>Which of the following is generated by rotating the two dimensional shape around the given axis?</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students can ● identify the three dimensional object generated</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Level 2</th>
<th>Given a ● two dimensional shape and various axes of rotation ● set of possible three dimensional objects</th>
<th>Select all of the shapes that can be generated by rotating the two dimensional shape around any of the axes given.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Students can ● identify all the possible three dimensional object generated</td>
<td></td>
</tr>
</tbody>
</table>


<table>
<thead>
<tr>
<th>Level 3</th>
<th>Given a three dimensional object</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students can</td>
<td>describe or draw the two dimensional shape that generated the three dimensional object</td>
</tr>
</tbody>
</table>

Draw the two dimensional shape and indicate the axis of rotation that generates the three dimensional object.

**Apply geometric concepts in modeling situations.**

G-MG.1 Apply geometric concepts in modeling situations
- Use geometric and algebraic concepts to solve problems in modeling situations.
- Use geometric shapes, their measures, and their properties, to model real-life objects.
- Use geometric formulas and algebraic functions to model relationships.
- Apply concepts of density based on area and volume.
- Apply geometric concepts to solve design and optimization problems.
| Level 1 | Students can identify shapes and formulas relevant to the context.  
Students show at least one attempt to investigate or solve the task. |
|--------|---------------------------------------------------------------------|
| Level 2 | Students can represent with the situation with a correct model.  
Students can analyze and perform operations with their model. The operations and analysis lack attention to detail. |
| Level 3 | Students can analyze and perform operations accurately with their model to draw conclusions. |

3. Derive the formula to calculate the volume of unused space given the figure on the right. Let \( r \) be the radius of the sphere.

\[
V = \pi r^2 h
\]

\[
V = \frac{1}{3} \pi r^3
\]

\[
\pi r^2 h - \frac{1}{3} \pi r^3
\]

- Student understands that the volume of a cylinder and volume of a sphere are required. Makes an attempt to write the formula. Does not attend to the model having 3 spheres or that the formula should be in terms of \( r \).  
- Student has a correct model using subtraction to calculate the empty space and recognizes there are 3 spheres. Lack of detail that the height of the cylinder is \( 6r \) and not \( 3r \). Doesn't recognize that the answer does not make sense (i.e. cannot have negative space).  
- Student has a correct answer and solution process is accurate.
| Level 4 | Students can interpret their results. Students validate their conclusions by comparing them to the situation and either improve the model (when needed) or report their conclusions. |

|  | 3. Derive the formula to calculate the volume of unused space given the figure on the right. Let $r$ be the radius of the sphere. $V = \pi r^2(b r)$  
$V = 3(\frac{1}{3}\pi r^3) - \frac{2}{3}\pi r^3$  
The empty space is $\frac{1}{3}$ of the volume of the cylinder. |

|  | Student relates the solution back to original situation and makes a comparison between empty space and volume of cylinder. This extension of knowledge may require or become evident during class discussion and/or extension questions. |