Division of Fractions Progression
There are two types of division:
1. Determining the number of groups when the size of the group is known, and
2. Determining the size of one group (per 1) when the number of groups is known.

In the NC revised standards, determining the number of groups is 6th grade, and determining the size of one group is 7th grade. The common denominator approach is used in 6th grade. Students get to the standard algorithm in 7th grade.

Common Denominator Approach
Sets A, B, and C determine the number of groups when the size of the group is known.

A. Dividing a whole number by a unit fraction
A bow uses $\frac{1}{2}$ yard of ribbon. How many bows can be made from 3 yards of ribbon? Write a number sentence.

Number Sentence: $3 \div \frac{1}{2} = _____$

Teacher Notes: The question to be answered: How many $\frac{1}{2}$s are in 3?

Move from the model to the symbolic representation:

$3 \div \frac{1}{2} = 6$ 
$3 \cdot 2 = 6$ 
or 
$6 \div \frac{1}{2} = 6$

Continue with similar problems until students see a pattern. Ask students to model the problems to justify their work. Possible problems are:
1. $3 \div \frac{1}{4}$
2. $3 \div \frac{1}{3}$
3. $4 \div \frac{1}{3}$
4. $5 \div \frac{1}{4}$

Note: Students may notice that dividing the numerators and dividing the denominators will also produce 6. Does this work all the time? Why?
Think about the following problem: $12 \div 3 = 4$. This is true because $4 \cdot 3 = 12$. Following this reason, then $\frac{6}{2} \div \frac{1}{2} = 6$ means that $6 \cdot \frac{1}{2} = \frac{6}{2} = 3$. 

B. Dividing a whole number by a non-unit fraction (moves to work with mixed numbers).

*The purpose of this sequence is to create the need for a common denominator. Answers are whole numbers. Remainders will be addressed in Part C.*

Another bow uses \( \frac{2}{3} \) yard of ribbon. How many bows can be made from 4 yards of ribbon? Write a number sentence.

**Number Sentence:** \( 4 \div \frac{2}{3} = \_\_\_ \)

**Teacher Notes:** The question to be answered: How many \( \frac{2}{3} \)'s are in 4?

Move from the model to the representation:

\[
4 \div \frac{2}{3} = 6 \\
\frac{12}{3} \div \frac{2}{3} = 6
\]

*NOTE: Some students may recognize that 4 was multiplied by 3 to find the number of thirds and then divide by 2 to group the thirds into groups of \( \frac{2}{3} \). This could be written as \( 4 \cdot 3 \div 2 \) or \( 4 \cdot \frac{3}{2} \).*

*Continue with similar problems until students see a pattern. In this first set of problems the answers are whole numbers. Possible problems are:*

1. \( 6 \div \frac{3}{5} \)
2. \( 3 \div \frac{3}{4} \)
3. \( 4 \div \frac{2}{5} \)
4. \( 4 \div 1\frac{1}{3} \)

*For problem 4, the question being answered is: How many groups of \( \frac{4}{3} \) are in 4?*
C. Dividing mixed numbers by mixed numbers to find the number of groups. Need to get a common denominator.

Now try:
5. \( \frac{4}{5} \div \frac{2}{3} \)
6. \( \frac{3}{8} \div 1\frac{1}{4} \)

Finally move to problems without common denominators.
7. \( \frac{4}{2} \div \frac{3}{4} \)
8. \( \frac{2}{3} \div 1\frac{1}{6} \)

Word Problems:
a. A ribbon is 4 m long. Yu needs a piece \( \frac{2}{3} \) m long for each gift. How many gifts can she wrap?
b. Sam lives 10 miles from school. Nina lives \( \frac{2}{5} \) miles from school. How many times farther from school does Sam live than Nina?
c. Ravi lives 12 miles from school and \( \frac{4}{3} \) miles from the library. How many times closer is he to the library than to school?
D. Naming the remainder when dividing fractions.
Note: Learning to name the remainder helps students recognize why a problem can show division by thirds and end up with halves as the answer. 
Begin by presenting the following problem to students.

You have 3 pounds of fish. You are planning on serving each person \( \frac{2}{3} \) pound of fish. How many full servings are possible?
How can you name the amount left over?
Number Sentence: \( 3 \div \frac{2}{3} = _____ \)

There are 4 full servings. Since each serving (the divisor) is 2 thirds and there is only 1 third remaining, the amount remaining is 1 part out of 2 parts or \( \frac{1}{2} \) of a serving.

So, \( 3 \div \frac{2}{3} = 4 \frac{1}{2} \)
This means that there are 4 full servings and \( \frac{1}{2} \) of another serving.

The following examples focus on naming the remainder. (taken from JUMP Math). Use these to help students name remainders.

1. For each of the following problems, write a number sentence including the quotient.
Answers:

Diagram A:

\[ 1 \div \frac{3}{8} = 2\frac{5}{3} \]

Diagram B:

\[ 1 \div \frac{2}{5} = 2\frac{1}{2} \]

Diagram C:

\[ 1 \div \frac{4}{9} = 2\frac{1}{4} \]

Write and solve a number sentence to match each diagram:

Diagram A:

\[ 1 \div \frac{3}{8} = 2\frac{5}{3} \]

Diagram B:

\[ 1 \div \frac{2}{5} = 2\frac{1}{2} \]

Diagram C:

\[ 1 \div \frac{4}{9} = 2\frac{1}{4} \]

Additional Word Problems
(from The Rational Number Project)

1. You have 2\(\frac{1}{2}\) pounds of candy. Packages contain \(\frac{3}{4}\) of a pound. How many full packages can be filled? How can we describe the amount left over?
2. A scoop holds \(\frac{3}{4}\) cup. How many scoops of birdseed are needed to fill a bird feeder that holds 3\(\frac{1}{2}\) cups of birdseed?
3. You bought 2\(\frac{1}{6}\) pints of ice cream for your party. You plan on serving each friend about \(\frac{1}{3}\) of a pint. How many full servings can you serve. What part of a serving is left?
**Standard Algorithm**

The remaining sets determine the size of one group (per 1) when the number of groups is known.

E. **Dividing a unit fraction by a whole number.**

There is $\frac{1}{2}$ of a rectangular cake remaining. If it is divided among 3 people, how much will each person get?

![Diagram](image)

Each person receives 1 of the pieces. Each piece is now $\frac{1}{6}$ of the original cake.

Number Sentence: $\frac{1}{2} \div 3$

**Teacher Notes:** The question to be answered: How much does one person get?

Move from the model to the representation:

\[
\frac{1}{2} \div 3 = \frac{1}{6} \\
\frac{1}{3} \cdot \frac{1}{3} = \frac{1}{6}
\]

**Note:** Ask students: What is the relationship between $3, \frac{1}{3}$, and finding the “per 1”?

Continue with similar problems until students see a pattern. Ask students to model the problems to justify their work. Possible problems are:

1. $\frac{1}{2} \div 4$
2. $\frac{1}{2} \div 2$
3. $\frac{1}{3} \div 3$
4. $\frac{1}{4} \div 2$
F. Dividing non-unit fractions by whole numbers
At the end of the night a pizza place had \( \frac{3}{4} \) of a pizza left over. The 3 employees each took home the same amount of leftover pizza. How much pizza did each employee take home?
\[
\frac{3}{4} \div 3
\]

\[
\frac{3}{4} \div 3 = \frac{3}{4} \cdot \frac{1}{3} = \frac{3}{12}
\]

Each person gets 3 of the pieces. Each piece represents \( \frac{1}{12} \), so each person gets \( \frac{3}{12} \).

Continue with similar problems until students see a pattern. Possible problems are:
1. \( \frac{2}{5} \div 4 \)
2. A land developer wants to divide up \( \frac{4}{5} \) of an acre of land into equally-sized lots for houses. He wants to build 2 houses. How big can he make each lot?
3. Mrs. Morton had \( \frac{2}{3} \) of a cherry pie left over. She split the leftover pie evenly between her 2 children. What fraction of a pie did each child get?

G. Dividing Fractions by Fractions
1. Sam can walk \( \frac{1}{8} \) of a mile in \( \frac{1}{4} \) hour. How far can he walk in 1 hour?

Number sentence:
\[
\frac{1}{8} \div \frac{1}{4} = \frac{1}{8} \cdot \frac{4}{1} = \frac{1}{2}
\]

Since the question is finding “per one hour” the \( \frac{1}{4} \) hour is the divisor.

Sam can walk \( \frac{1}{2} \) mile in 1 hour
2. If 4 servings of potatoes fill $\frac{2}{3}$ of a container. How many servings of potatoes will fill 1 container?

First the 4 servings of potatoes is divided between the 2 thirds.

\[
4 \div \frac{2}{3} \rightarrow 4 \div 2
\]

If each part of the container contains 2 servings, then one container would have 6 servings of potatoes.

\[
4 \div \frac{2}{3} \rightarrow 4 \div 2 \times 3
\]

Number Sentence: \[4 \div \frac{2}{3} = _____
\[4 \cdot \frac{3}{2} = 6
\]

3. Isaac read $\frac{3}{4}$ of a book in $\frac{2}{3}$ of an hour. How much of the book can he read in 1 hour?

Each color represents $\frac{2}{3}$ of an hour, which also represents $\frac{3}{4}$ of a book.

There are 3 groups of $\frac{3}{4}$ to get to a whole amount; however, we need to find the amount of the book read in 1 hour, so divide by 2.

Number Sentence: \[\frac{3}{4} \div \frac{2}{3} = _____
\[\frac{3}{4} \cdot \frac{3}{2} = \frac{9}{8} \text{ or } 1\frac{1}{8} \text{ hours}
\]

The model also shows that in one hour there is $\frac{3}{4}$ and half of another $\frac{3}{8}$ or $\frac{3}{8} + \frac{3}{8}$ is also $\frac{9}{8}$.

Display each of the division number sentences and the corresponding multiplication sentence.

Introduce the concept of a reciprocal to get to the standard algorithm.

The context determines whether you are solving for the number of groups (common denominator) or the number in 1 group (multiply by the reciprocal).