From Modeling to Algorithms: Supporting Teachers in Building Students’ Understanding of Operations with Rational Numbers, 4-8
Dr. Katie Mawhinney, Appalachian State
Dr. Kay Pitchford, UNC Pembroke

Who’s in the Room?
- K-5 classroom teachers?
- K-5 school-based leaders?
- K-5 district leaders?
- Teachers of other grade levels?
- Other district leaders?
- University students?
- IHE instructors?
- Others?

Agenda
- Overview of Tools Project
- Models for Arithmetic with Rational Numbers in Grades 1 – 5
- Models for Arithmetic with Rational Numbers in Grades 6 - 8
- Questions
Resources Available

- **Instructional Resources:**
  - Instructional Frameworks, Lessons, Games, Formative Assessment Tasks, Number Talk Resources, Content Briefs,
- **Parent/Family Resources:**
  - Letters, Games, Tips & Video Clips
- **Professional Development Resources**
  - PLC and/or school-level professional development

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**From Modeling to Algorithms**

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**Time for Math!**

The 5th graders want to raise money for their overnight camping trip by selling cornbread during the school district Chili Cook-Off contest. All the pans of cornbread are squares. A full pan of cornbread costs $12. The customers can buy any fractional part of a pan of cornbread and pay that fraction of $12.

Mr. Farmer walks up to buy cornbread and the pan is ½ full. He wants to buy 3/4 of the remaining cornbread. How much will he spend on the cornbread? Show your thinking with words, pictures, and/or numbers.

Adapted from Illustrative Mathematics
Time for Math!

What do you have to know to solve the problem?
What do you have to understand to solve the problem?
What do you have to be able to do to solve the problem?

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Fraction Models in Grades 1-5
Establishing Community: Cluster 1

Locate the Standards

- Go to www.tools4ncteachers.com
- Scroll to the District Leaders section
- Click on Documents
- Click on K-8 Mathematics Standards

Establishing Community: Cluster 1

Locate an Unpacking Document

- Go to www.tools4ncteachers.com
- Click on a grade level
- Scroll down
- Click on Unpacking Document

Establishing Community: Cluster 1

Connect your work to the standards:

**Fraction Models in 1st and 2nd Grades**

Connect your work to the standards:
Fraction Models in 3rd Grade

Connect your work to the standards:

NC.3.M.2: Interpret fractions with denominators of 2, 3, 4, 6, and 8 using area and length models.
- Using an area model, explain that the numerator of a fraction represents the number of equal parts of the whole fraction.
- Using a number line, explain that the numerator of a fraction represents the number of lengths of the unit fraction.

NC.3.M.3: Represent equivalent fractions with area and length models by
- Extending and decomposing fractions into equivalent fractions using related fractions, halves, fourths, and eighths.
- Explaining that a fraction with the same numerator and denominator equals one whole.
- Extending whole numbers to fractions, and recognizing fractions that are equivalent to whole numbers.

Tools for Teachers
Establishing Community: Cluster 1

Connect your work to the standards:

Fraction Models in 4th and 5th Grades

Connect your work to the standards:

This problem will take more than one step. It is being used as an illustration only!
Establishing Community: Cluster 1

Connect your work to the standards:

**Fraction Models in 4th and 5th Grades**

Connect your work to the standards:

- **NS.4.F.1**: Explain why a fraction is equivalent to another fraction by using area and length fraction models, with attention to how the numerator and denominator differ even though the two fractions are the same size.
- **NS.4.F.3**: Use fraction models and some division problems.
  - Interpret a fraction as an equal-sharing context, where a quantity is divided into equal parts.
  - Model and interpret fraction as the inverse of the operation by the denominator.
- **NS.4.P.7**: Some one-step word problems involving division of whole numbers by two whole numbers and division of whole numbers by unit fractions using area and length models, and equations to represent the problem.

![Fraction Models Example](image)

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![Fraction Models Example](image)
Fraction Models in 4th and 5th Grades

Connect your work to the standards:

- **4.MP.1**: Explanations of a solution should include reasoning by using area and length fraction models, to which students show how the fraction and ratio of the whole part of the problem are in the same way.
- **4.NF.3**: Add and subtract fractions with equal numerators and denominators (e.g., 2/5 + 2/5, 4/3 - 1/3). This understanding is used to solve problems involving whole numbers and fractions. Students use strategies based on the understanding of how to use multiplication and division to solve problems involving fractions.
- **5.NF.7**: Add, subtract, multiply, and divide fractions and mixed numbers by using fraction models and equations. Students should be able to explain their reasoning using fraction models and equations.

Connecting to Decimals
Establishing Community: Cluster 1

Locate Lessons and Tasks

- Go to www.tools4ncteachers.com
- Click on a grade level
- Click on a Cluster
- Search within the lessons by standard number

Fraction Models in Grades 6 - 8

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Operations with Fractions

Transitioning from visual models to algorithms should be gradual and fluid, so that models are still valuable tools.

- Utilize the visual models employed in earlier grades.
- Connect those models to algorithms.
- Don’t change to repeated practice that ignores contextualized problems.

Contexts and interpretations support understanding.

Estimation also supports understanding.

Pattern sniffing helps makes sense of algorithms.

½ of the pan

¾ of the ½
11/3/2018

Operations with Fractions

$\frac{3}{4}$ of the $\frac{1}{2}$

$\$12$ for the $8$ Pieces

**Contexts and interpretations support understanding**

- Refer back to multiplication as repeated addition when appropriate
- Division can be thought of as either fairly sharing or as creating equal groups

**Estimation also supports understanding**

- Make estimation a regular part of your teaching practice (it’s not just for fraction work, either)
- It’s a safe way to get at student thinking and an opportunity to deepen the discourse in class
- It’s an opportunity to utilize benchmarks
Pattern sniffing helps make sense of algorithms

\[
\begin{align*}
1 \div \frac{1}{2} &= 2 & 2 \div \frac{1}{2} &= 4 \\
1 \div \frac{1}{3} &= 3 & 2 \div \frac{1}{3} &= 6 \\
1 \div \frac{1}{4} &= 4 & 2 \div \frac{1}{4} &= 8 \\
1 \div \frac{1}{5} &= 5 & 2 \div \frac{1}{5} &= 10
\end{align*}
\]

Operations with Fractions

**One other important note:**

Successful and efficient use of algorithms to operate with fractions does not equate to being able to reason proportionally.

The math of proportions is predominately determining equivalent fractions and multiplying by a fraction.

Giving Students the Gift of Time
District Considerations

- Use of the Instructional Framework is a choice
- All resources can be accessed through:
  - [www.NC2Ml.org](http://www.NC2Ml.org)
  - [http://Tools4NCteachers.com](http://Tools4NCteachers.com)
- The NC Math Check-ins will be aligned to the Instructional Framework

Locate the Instructional Frameworks

- Go to [www.tools4ncteachers.com](http://www.tools4ncteachers.com)
- Click on a grade level
- Scroll down
- Click on Instructional Framework
Instructional Framework Components

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“...knowledge is something we either have or we don’t have. In contrast, **understanding** can be defined as a measure of the quality and quantity of connections that an idea has to existing ideas.”

- L. Lovin & J. Van de Walle

*Teaching Student-Centered Mathematics, Grades 5-8*, pg. 2

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**Tools for Teachers Staff**

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- Jeane Joyner, IHE and Reviewer
- Katie Mawhinney, IHE and Reviewer
- Drew Polly, IHE and Reviewer
- Wendy Rich, K-5 Coordination and Reviewer
- Catherine Stein, IHE and Project Liaison

Please give appropriate credit to the Tools for Teachers project when using these materials.

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**Tools for Teachers**

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