Multi-Item Inventory Management with Opaque Products

Overview. This paper studies the inventory management of multiple items in conjunction with opaque products. An opaque product is defined as a product where some specific attributes are not revealed to the customer until after purchase. Once the purchase of an opaque product is finalized, the retailer allocates one of the items to give to the customer. An increasing number of online retailers offer opaque products in categories such as apparel, toys, baby products, appliances, and shoes, where typically the color or style are not revealed to the customer prior to purchase. For example, the Blender Bottle on Amazon.com comes in 5 color varieties, along with a ”Colors May Vary” option (i.e., the opaque product in this instance). The goal of this work is two-fold. First, we seek to characterize optimal policies for multi-item inventory management with opaque products. Our focus is primarily on the decision of which item to allocate when a customer purchases an opaque product. We prove structural properties of the optimal allocation policy, and show that a very intuitive policy is asymptotically near optimal. Second, we quantify the cost savings benefit of introducing an opaque product option. We show that as long as the fraction of customers who choose the opaque product is above a relatively small threshold, the cost savings can be quite significant.

Model. In our model, the online retailer has $N$ items for sale, which may be regarded as substitutable options (i.e., different colors or styles of the same item). In addition, the retailer offers an opaque product option to the customer. Purchasing customers arrive dynamically to the retailer according to a Poisson process. With probability $q$, the customer purchases the opaque option and with probability $1 - q$ the customer purchases one of the $N$ items, each with equal probability. A customer who purchases the opaque option may be allocated any of the $N$ items at the discretion of the retailer. Once the inventory level for any item reaches zero, the retailer immediately replenishes all products up to an exogenous order-up-to-level $c$. There is no lead time, lost sales, nor backlogging. The retailer incurs
a fixed joint replenishment cost when order is made, regardless of how many units or item types are ordered. In addition, the retailer incurs per unit holding costs for every unit in the system. In our model, the only decision to be made is which item to allocate to a customer purchasing an opaque product, with the objective being to minimize the combined long-run average ordering and holding costs.

We note that the reorder level, and the policy for when an order is triggered, is provided exogenously to allow us to focus our analysis on the opaque product allocation. In fact, multi-item inventory problems with joint replenishment costs are notoriously difficult, and may have very complex optimal policies even in simple two item settings (see Ignall (1969)). Because of the inherent difficulties within the multi-item inventory problem, we use a simple reordering policy with a fixed order-up-to-level, and focus on how to fulfill demand for opaque products as well as the cost benefit provided by the opaque products.

**Contributions.** We first consider the optimal policy for allocating opaque products. Elmachtoub and Wei (2015) showed that the optimal policy in the case where $N = 2$ is the myopic policy, which allocates the product with the highest on-hand inventory to a customer purchasing an opaque product. Surprisingly, we show that when $N \geq 3$, the myopic policy is no longer optimal. In particular, we show that the myopic policy is suboptimal in a family of inventory states where one item is almost out of inventory but the total inventory is relatively high. In this type of state, it may be better to choose the low inventory item so that an earlier replenishment occurs, in exchange for reducing the holding costs due to the high inventory situation.

Next, we show the myopic policy of allocating the item with the highest on-hand inventory is asymptotically optimal. The relative difference in long run average cost between the myopic and optimal policy is on the order of $\Theta\left(\frac{N}{c}\right)$, which can be relatively small for even moderate values of $c$. Numerically, the cost difference between the two policies is less than 0.01%.
Our third contribution is to offer insights on how the opaque selling strategy impacts the overall costs, in comparison to a strategy without opaque products. The analysis of cost savings relies on studying a key metric: the expected number of customers served between replenishments. Intuitively, if we can serve more customers between consecutive replenishments, the average ordering cost would be lower and the average holding cost in a replenishment cycle would also be smaller. There is a natural connection between our inventory problem and the balls-into-bins model, which we leverage in proving our results. In our model, customers (balls) sequentially arrive and each customer chooses one out of $N$ products (bins). Leveraging results from the balls-into-bins literature, we show that using opaque products can increase the number of customers served between replenishments on the order of $\Theta(N\sqrt{c\log N})$. Furthermore, we show that the total inventory cost savings relative to the no opaque selling strategy are at least $\Theta\left(\frac{\sqrt{\log N}}{\sqrt{c}}\right)$, as long as $q$ exceeds a small threshold. This quantity is significant for relatively small values of $c$, and increases as the number of item types increases. Our numerical results verify our theoretical findings.

Finally, we provide a detailed analysis of a generalization of opaque selling called $k$-opaque products. A customer who buys a $k$-opaque product first chooses $k$ out of $N$ products, and then the retailer will allocate one among the $k$ choices. We show that 2-opaque products offers the same order of magnitude of inventory cost savings as $N$-opaque products. We also numerically show that using 2-opaque products may be more profitable than $N$-opaque products, in particular when a consumer choice model is used to capture consumers decisions with regards to which item or opaque product they prefer.

References
