Duopoly Competition with Network Effects in Discrete Choice Models

Network effects are widely documented when human beings engage in social interactions. They arise from the “payoff externality,” in the sense that an individual’s payoff depends not only on her own action but also on the actions chosen by others. Examples abound, from the classical videotapes, keyboards, operating systems, telecommunication networks, to recent online games, cloud services, and mobile apps. For instance, Dropbox allows users to conveniently share their files across platforms; users are likely to choose Dropbox over other service providers (e.g., Google Drive and OneDrive) if many of their colleagues have already adopted Dropbox. As another example, an increasing amount of users on one side of a two-sided market (such as Airbnb) attracts more users on the other side, and thus generate network effects for the users on the same side.

Because network effects largely enhance individuals’ willingness to pay through the cascade of externalities, the aforementioned industries are highly profitable and network effects lift the potential and competition there to the next level. Competition over networks gives rise to various phenomena that could not be observed in conventional industry structures. In particular, it is widely believed that network effects create the winner-take-all phenomenon: a product (such as Dropbox) may capture a dominant market share over its seemingly identical alternatives (such as OneDrive). It is also pointed out in several studies that the network structure is related to market segmentation: product adoption may vary tremendously from segment to segment in the network (e.g., WeChat in China, Facebook’s Messenger in North America, and WhatsApp in other regions). Such market dominance and market segmentation do not arise if the market were monopolized
by a single firm.

This paper attempts to gain deep understanding of the firms’ competitive strategies when customers’ purchasing decisions are influenced by network effects. We pay particular attention to the emergence of market dominance and market segmentation in the form of “asymmetric equilibria.” In pursuit of this goal, we consider two firms that sell their substitutable products to a market of network-connected customers. The customers choose between the two products, or leave the market without purchasing. They make their decisions based on price, quality, and the anticipated network effect, in the sense that they are influenced by the choices of their neighbors in the network. We describe the choice process by a multinomial logit (MNL) model.

Despite the parsimony of our model, the equilibrium analysis turns out to be highly intractable. The complexity can be attributed to three sources: the network interactions, the non-concave payoff function arising from the MNL model, and asymmetric equilibria. To circumvent the difficulty, we adopt a novel approach that focuses on the inverses of the best-response functions. It allows us to obtain analytical results whereas selecting best responses from the local maxima of a non-concave function is virtually impossible.

We show that depending on the products’ qualities and the strength of the network effects, the Nash equilibria exhibit highly distinct features. When the products are symmetric and customers are homogeneous, i.e., the network that connects customers is a complete graph, a single symmetric Nash equilibrium arises if the network effects are weak. At the other extreme, when the network effects are strong enough, there exist three Nash equilibria: two stable asymmetric Nash equilibria in which one firm captures almost all the entire market and the other firm is left with little market share, and an unstable symmetric Nash equilibrium. The stable equilibria exhibit some form of market dominance, and it emerges because of strengthened competition.

More surprisingly, under the same assumption of homogeneous customers and products, when the product quality is low and the network effects are neither too weak nor too strong, the resulting market equilibrium is never symmetric even if the firms are ex ante symmetric. This result is somewhat paradoxical but particularly robust, because it
implies the non-existence of even unstable symmetric equilibria. In this sense, our result demonstrates a strong rebuttal to the conventional, and perhaps naive, intuition that symmetric equilibria shall exist for symmetric firms. We are not aware of any prior work that proves this in the context of network effects with competition.

We next consider products with heterogeneous qualities. When the network effects are strong enough, we establish the existence of two Nash equilibria corresponding to the respective market dominance positions of each firm, regardless of their quality difference. This implies that even though one product is far inferior to its competitor, it can still retain market dominance due to the strong network effects. This could happen, for example, if the product has a first-mover advantage (although in our model the game is one-shot). Our second result justifies the investment in quality improvement: if the quality difference is sufficiently large, the superior product can penetrate the network effects and secure its dominating position, which is the only equilibrium outcome of the duopoly competition.

For a market of customers who are heterogeneous in their price sensitivities and network effects, we are able to characterize the market share in symmetric equilibria. In particular, more price-sensitive customers and customers with unanimously higher network externality are more likely to buy either product in the symmetric equilibria.

Finally, we consider a network with two communities: the connectivity inside each community is homogeneous, which is different from the externality between communities. This network structure encompasses many special networks that are investigated in the literature, such as star graphs and complete bipartite graphs. We provide that market segmentation can arise under the network effects: one product is dominating in one community, while the other product is popular in the other community. This market segmentation emerges if the network effects inside communities are relatively strong in comparison to those between communities. In the opposite case wherein the network effects between communities is stronger than those within communities, the aforementioned segmentation is no longer feasible. Notably, market dominance may still exist, in the sense that one firm dominates the other in both communities.