Asymptotic optimality of constant-order policies in joint pricing and inventory control models

**Model and problem formulation.** We consider a periodic-review, single-item, stochastic joint pricing and inventory control problem with backlogging and positive lead times. In this problem, there are two decisions in each period: pricing (e.g., how much to charge per unit of inventory) and replenishment (e.g., how much more inventory to order). In addition, there is a constant lead time between when an order for additional inventory is placed and when that inventory is received. Unsatisfied demand is fully backlogged. The problem is to determine the best joint pricing and inventory control policy in order to maximize the long-run average profit, which is defined as the revenue minus the total cost consisting of a per-unit stockout cost due to unfulfilled demand within each period and a per-unit cost for holding excess inventory within each period. The demand depends on the price as well as a random noise and the random noise sequence is independent and identically distributed.

**Problem significance and literature review.** The problem of joint pricing and inventory control was first studied in [9] and a considerable amount of research has been conducted since then. A great deal of effort has been spent on investigating the structure of the optimal policy. In particular, the so-called base-stock list price policy was proved to be optimal in the setting without setup costs (e.g., [5]). Each base-stock list price policy is associated with a base-stock level and list price. If the inventory level falls below the base-stock level, then the policy orders up to the base-stock level and charges the list price. Otherwise, nothing is ordered and a price discount is offered. Moreover, the discount price is non-increasing in the inventory level. In the presence of setup costs, the class of (s, S) list price policies naturally extends the class of base-stock list price policies, and its optimality was established in various settings (e.g., [2], [3], [7]). Up to date, all existing results on the optimality of base-stock (or (s, S)) list price policies have been established under the zero replenishment lead time assumption. Indeed, as pointed out in [4], “… it remains a significant challenge to incorporate lead time into stochastic models. Indeed, the zero lead time assumption is required for all the multi-period models reviewed here…” . It is dramatically different from pure inventory control models without
pricing decisions, in which it is well known that a base-stock policy remains optimal for models with positive lead times. Furthermore, such problems with positive lead times are generally considered to be computationally intractable due to the curse of dimensionality and the required computation time to exactly solve an optimal policy grows exponentially fast in the lead time. As an exact optimal solution seems out of reach, researchers have instead studied structural properties of the optimal policy and have been focusing on constructing effective heuristics (e.g., [1]). Although great progress has been made, how to design heuristics with performance guarantee remains a major open question.

Similar to the joint pricing and inventory control problem, the structure of the optimal policy is also poorly understood for many inventory models with lead times but without pricing decisions such as single-sourcing lost-sales and dual-sourcing inventory models. Recently, a simple constant-order policy, which will be the subject of our paper, has been attracting great attention since it was first studied in [8]. Such a constant-order policy always places the same order in every period, independent of the state of the inventory system. It is simple and easy to implement, and seems to perform well especially in the large lead time setting. The intuition is that as the lead time grows large, there exists too much randomness in the inventory system between when an order is placed and when that order is actually received, such that a simple constant-order policy that ignores the state information performs nearly optimally. This phenomenon was recently explained in theory in several inventory control models. In particular, in a single-sourcing lost-sales model, a constant-order policy was proved to be asymptotically optimal as the lead time grows (e.g., [6], [10]). In a more complex dual-sourcing model, [11] proved that a Tailored Base-Surge policy, which is a combination of a constant-order (for the slow source) and a base-stock (for the fast source) policy, is asymptotically optimal as the lead time difference between the two sources grows.

**Main results.** We make significant progress towards resolving the open problem that how to design heuristics with performance guarantee. In particular, we propose a class of constant-order policies. Under such a policy, a constant-order amount of new inventory is ordered every period and a pricing decision is made based on the on-hand inventory. We prove that, under mild assumptions on the price-demand function, the best constant-order policy among the proposed family of policies is asymptotically optimal as the lead time grows large. To the best of our knowledge, it is the first
result of its kind in the joint pricing and inventory control model with positive lead times. We also demonstrate that the best constant-order quantity can be effectively computed. Indeed, one advantage of such a policy is the independence of the lead time. Our main proof technique combines convexity arguments and a non-trivial vanishing discount approach that involves the convergence of an infinite horizon discounted inventory model with random yields, limited production capacity and possibly negative demands, to its long-run average counterpart, significantly extending the methodology and applicability of a novel framework for analyzing inventory models with large lead times introduced in [10] and [11] in the context of single-sourcing lost-sales and dual-sourcing models.

References


