Data-Driven Pricing for a New Product

Decisions regarding new products are often difficult to make, and mistakes can have grave consequences to a firm’s bottom line. However, firms often have little foresight on important information about new product demand such as potential market size, rates at which consumers will adopt, and their willingness to pay. One of the most popular frameworks that have been used for modeling new product adoption is the Bass model (Bass 1969). In the original Bass model, sales are temporally influenced by innovators who try a product on their own and imitators who follow earlier adopters. While the original Bass model and its many variants are useful to understand the factors and decisions affecting new product adoptions over time, the vast majority of these models require a priori knowledge of key parameters, which include the market size (denoted by m), the coefficient of innovation (p), and the coefficient of imitation (q). However, these parameters can only be estimated with historical data or guessed based on institutional knowledge.

In this paper, we study the interplay between pricing and learning for a firm whose objective is to maximize the expected revenue of a new product over a finite time horizon. We consider a setting where a firm can learn characteristics of demand by observing sales data at (different) prices over time.

**Markovian Bass Model.** Traditional stochastic diffusion models that add noise to the cumulative demand are not well-behaved when modeling new product adoption. For instance, Brownian diffusion models violate the fact that cumulative sales must be non-decreasing in time. To overcome this technical challenge, we propose a different way to model stochasticity in the adoption process while capturing the features of adoption process modeled in the Bass model. We model the cumulative adoption as a continuous time Markov chain where time between
adoptions depends on price and cumulative sales, which we call as *Markovian Bass Model*. We show that the behavior of a Markovian Bass model converges to the original Bass model as the market size grows. We derive the optimal pricing policy for Markovian Bass model under complete information. We refer to this policy as the Markovian Bass Price (MBP). This policy requires knowing the true demand parameters, hence it will be used as a benchmark when evaluating data-driven pricing policies.

**Demand learning.** We show that the loss resulting from a wrong initial estimation of the demand parameters can grow proportionally in time and can be significant. Hence, we propose a method to utilize historical demand and learn about the key demand parameters using maximum likelihood estimators (MLE). The main challenge arises from the fact that the price influences both current demand and future adoptions. As a result, the samples generated from demand data are not *i.i.d*. This makes standard techniques of proving MLE convergence (which are used to bound the estimation errors) unusable. We circumvent this impediment and still show that the MLEs are asymptotically normally distributed under Markovian Bass Model and that the mean squared error approaches zero as more people buy. While the log-likelihood function is not concave in the demand parameters, we propose a sequence of parameter transformations that can be solved by standard convex optimization.

**Optimal learning and pricing policy.** We show that a firm can utilize real-time demand data to update the parameter using MLE and optimize the price accordingly. We formulate the problem as a stochastic optimal control problem where the demand parameters are updated by maximum likelihood estimators. We derive the optimal pricing policy which consists of two components – one exploits the Markovian Bass Price (MBP) using the most up-to-date parameter estimates; the other is a “trembling hand” experimentation which fades over time as the firm
gathers more data and the estimates become more accurate. To the best of our knowledge, our work is the first to derive an optimal, data-driven pricing policy in a new product adoption problem.

**Performance guarantee for tractable pricing policies.** Since the exact optimal learning and pricing policy is difficult to implement for problems of practical scale, we propose two simple and computationally efficient pricing policies where the firm periodically updates the parameters using the data. The first policy (MBP-MLE) is a tractable approximation of optimal policy and can be used in a setting where a firm can change the price freely. The second policy (MBP-MLE-Limited) reflects a business constraint that the firm can change price a limited number of times. For both policies, we provide analytic performance bounds and show that the worst-case regret is in the order of the log in problem scale.