Managing Uncertain Capacities for Revenue Optimization: Algorithm and Insights

By the end of the 2000s, a major air carrier in Latin America serving local, regional and long-haul markets, was facing a crucial revenue management (RM) problem. As part of the input parameters of the RM system, the analysts needed to enter the capacities of the aircrafts that were serving the different legs in the network. However, assessing these values—which is usually the outcome of a long-term fleet assignment problem for many airlines—was a major challenge for the RM group there. The airline was undergoing organizational and infrastructural revamps which, among other objectives, were aimed at fixing major operational problems that were turning into frequent delays, cancelation and reschedule of flights, reprogrammed aircraft mechanical services and last-minute fleet changes. In addition, the carrier operated a very heterogeneous fleet, with 12 different aircrafts models ranging from McDonnell Douglas MD-81 and MD-83 to Boeing 747-400 and Airbus A340-300, and even within the same model, the seat configuration exhibited variations.

The management team that took over the reorganization of the RM group made a decision that was acknowledged to have negative revenue implications: When inputting the aircraft capacity in the RM system early in the booking horizon, the analyst had to plug-in the minimum capacity among the aircrafts that could potentially serve a particular flight leg. This of course, had dramatic consequences for the long-haul markets served by either an Airbus A340 with 248 economy seats or a Boeing 747 with 379 economy seats. Certainly, the policy implemented was very conservative, but what would be the right capacity values to set in the IT system to maximize revenues when the effective capacities are highly unpredictable? Of course, the airline had to account for the penalty cost incurred if eventually a number of passengers needed to be bumped. In this paper, we provide a methodology to address this question when the admission of product requests during the booking horizon is based on the widely adopted bid-price controls.

**Background and scope**

Network Revenue Management involves controlling a fixed and perishable capacity of a network of resources over a finite horizon in order to serve an incoming stream of random demand, with the objective of maximizing revenues. In the airline case, each product is an itinerary-fare-class combination, spanning one or more resources (i.e., flight legs). The bid-price strategy sets a threshold price for each leg, which is a proxy for its marginal value of capacity. A request for a product is accepted if the fare exceeds the sum of the bid-prices for these constituent legs. Bid-price controls
are common in airlines, hotel and car rental, among other operations.

Along with the bid-price policy, the current RM systems also control capacity availability at the moment a booking request arrives. The system must be parameterized with virtual capacity values at the leg level. We use the adjective virtual because these capacities do not necessarily match the effective physical capacities assigned to the corresponding flight legs, as it was the case in our motivating example.

As a complement to the volume of demand and supply uncertainties, there is the effective demand uncertainty related to the cancellations and no-shows that occur among accepted reservations. Overbooking is the practice of inflating the physical capacities of the legs in the RM system to hedge against the latter phenomenon, and is a standard practice among passenger carriers. Even though the focus of our study is on the volume of demand and supply uncertainties, our method can easily accommodate the randomness of the final effective demand.

Cargo RM is another area that combines uncertain availabilities while taking reservations, with the need to overbook to make an effective use of the final realized capacities. Passenger carriers devote to cargo the space unused by the passenger load, which is uncertain until the time of departure. In addition, long-term agreements that guarantee a specified amount of space on future flights (so-called allotments) are signed with major clients. Because cargo customers may not use the allotted space, some airlines require that the space be released 48 hours before departure. In the end, the cargo capacity is uncertain until very close to the time of departure.

**Summary of results**

To fix ideas, we will use the airline context as the template. We consider the problem faced by an airline whose RM system operates under a bid-price control policy. Our objective is to come up with bid-prices and virtual capacity values so as to maximize net revenues during the booking horizon. Passengers arrive sequentially over time, and the accept/deny decisions must be made on-the-go. The physical capacities are assigned to the flight legs sometime during the booking horizon (potentially, at the very end). The net revenues are the result of the total fares accepted during the booking horizon, minus the penalty that is incurred in the end when an accepted reservation cannot be accommodated among the boarding passengers because of a potential mismatch between the physical and the virtual capacity of a flight leg. The penalty includes the need to reaccommodate the passenger onto another flight, the need to cover travel expenses (hotel room, complimentary
meals for the passenger), and/or the release of a discount voucher for future bookings. Brand name damage and loss of demand are also part of this penalty cost.

We study two variants of this problem: 1) the resource allocation case, which is a stylized model for our motivating situation, where the set of physical capacities is known in advance but assigned ex-post, during or at the end of the booking horizon; and 2) the random capacity case, where the resources are assigned ex-ante to each of the legs, but their capacities are uncertain while taking reservations.

In this paper, we propose a stochastic gradient algorithm to improve an initial set of joint bid-prices and virtual capacities provided by the existing optimization module in a RM system. In each iteration of the algorithm, given a set of values for the bid-prices and virtual capacities, and for given demand parameters from the forecasting/estimation module, we simulate a sample path of bookings, perform the resource allocation (in case of the first variant), and compute the associated penalty cost incurred. Then, we compute a sample path-based gradient of the net revenue function. Using a step size and the calculated gradient, we update the values of the decision variables and repeat the procedure. The algorithm has a strong theoretical foundation: all its limit points are stationary points of the net revenue function. Furthermore, it captures a broad generality in modeling demand and is consistent with the current infrastructure of real world RM systems that maintain separation between the forecasting/estimation and optimization modules. However, within the latter, the proposal provides a way to integrate what is usually a two-stage decision process (virtual capacities and bid-prices).

The main contribution of our paper is practical and illustrated by running a broad range of computational experiments. For both the resource allocation and random capacity variants of the problem, we benchmark our proposal versus several alternative methods discussed in the literature and implemented in practice. Our method delivers significantly higher revenues than others in remarkably fast computational times, and is robust w.r.to different degrees of capacity uncertainty and resource allocation times.