Overview: The sales revenue of E-commerce has been growing rapidly since last decade, for instance, the total revenue of E-commerce was $453.46 billion, 13% of total retail sales in 2017 in the U.S., as reported in (Zaroban 2018). Many E-commerce supply chains typically involve products with many alternatives. As observed by researchers (Ganesh et al. 2008), often, these products can substitute each other proportionally, in particular, when some products are out-of-stock. For instance, some customers might turn to iPhone 8 if iPhone X is unavailable. Along with this product substitution effect, the stochasticity and unpredictability of customers’ demand further complicate the production planning as well as the revenue management of supply chain companies. Therefore, in this research, we propose to model and analyze these aspects and develop efficient methods for implementation in multi-product supply chain environments.

The product substitution problem in the multi-product supply chain contains the following three unique aspects. First, the customers’ demand might be highly uncertain. However, even if the demand were known, because of substitution, it would be challenging to distribute it among different products. Second, due to substitution effects, the order quantities of some products can be reduced while those of the others might be increased, which causes the entire supply chain to be completely different from the single product supply chain, that is typically addressed in the literature. Third, the remaining products at the end of each period have to be salvaged at a relatively low price, which requires a sophisticated predetermined optimal ordering policy. These aspects result in severe modeling and algorithmic challenges that require sophisticated methodologies to address them. In our proposed research, we plan to directly address these features. In particular, the problem that we address can be concisely stated as follows:

*Given a number of different products, their demand pattern (e.g., deterministic or stochastic) as well as substitution rates, determine their best order quantities to maximize the overall profit.*
There have been quite a few efforts reported on inventory planning with multi-product substitution. For example, Inderfurth (2004) attempted to maximize the expected profit with multiple products whose demand is stochastic. Stavrulaki (2011) showed that product substitution along with demand stimulation can incur significantly higher profits. Similar efforts can be also found in Zhang et al. (2014). However, most of these efforts have focused on deriving managerial insights of the proposed models, while to the best of our knowledge, none of the existing works have proposed a systematic way to determine an effective inventory plan under product substitution for a multi-product company. In this work, we will fill this gap and propose new mathematical formulations, which are amenable to develop efficient solution algorithms for use by the managers in their decision-making.

**Summary of Contributions:** The objective of this study, motivated by E-commerce, is to determine optimal order quantities of a multi-product supply chain under product substitution and stochastic demand, which maximizes the overall expected profit including total sales profit and salvage value. The main contributions of this work are summarized as below:

(i) When the demand is deterministic, we demonstrate an exact characterization of the optimal order quantity for each product, i.e., the optimal order quantity of each product will be either 0 or equal to the substituted demand. This characterization allows us to reformulate the entire model as a submodular maximization problem. This reformulation admits a polynomial-time approximation algorithm with approximation ratio 1/2, i.e., the output solution of the approximation algorithms will be at most 1/2 away from the true optimal.

(ii) When the demand is stochastic, we first apply sampling average approximation (SAA) to approximate the model, i.e., we formulate the model as a two-stage stochastic program with finite support. We derive first order necessary conditions of the optimal order quantities, and base on these conditions, we give tight lower and upper bounds of optimal order quantities. We also establish the sufficient conditions under which a product can be substituted by the others. We next prove that the profit function is submodular in the order quantities, i.e., the
marginal benefit of increasing one product’s order quantity decreases as the order quantity of another product’s increases.

(iii) The model properties in Part (ii) further motivate us to derive efficient solution algorithms. First of all, the observation that optimal order quantity of each product is bounded, allows us to derive mixed-integer program MIP formulation, which can be solved to the optimality by the off-the-shelf solvers if the number of products is not very large. The MIP formulation can be further improved by exploring the decomposition structure of the two-stage stochastic program model, i.e., we decompose the stochastic model into a series of deterministic models with Lagrangian relaxation approach. Finally, the submodularity property suggests that double greedy algorithm works well with 1/3 approximation ratio.

(iv) Finally, we conduct numerical experiments to illustrate the managerial insights. In our numerical results, we show that the substitution effects can reduce the risks from demand uncertainty significantly. This result is appealing from a practical perspective, as it suggests that having multiple products mitigate risks in supply chain. We also note that if the number of products becomes large, then some products can be completely substituted by others and thus can be removed. This result suggests the importance of selecting a reasonable number of products.

References


