Asymptotic Optimality of Order-Up-To Control for Stochastic Inventory Systems with Sequential Probabilistic Service Level Constraints

The problem of minimizing inventory cost over time while providing a high-quality customer service in the presence of stochastic demand is one of the most fundamental and challenging core problems of inventory management. Depending on whether service level is modeled explicitly, many inventory systems can be categorized as cost-based model or service-based. The former assumes that there is no explicit targeted service level guarantee (i.e., no service level constraints) and the objective is to minimize the expected total ordering, holding, and stock-out costs. The latter includes explicit service-level constraint, This is motivated by the fact that, in practice, the cost of unmet demand is often difficult to quantify (e.g., Chen and Krass (2001)) and, therefore, service level is typically used as a more direct metric for evaluating the performance of inventory controls. For example, some firms such as Walmart use both average on-hand inventory and service level as their two key performance metrics (Xin et al. (2017)). In the literature, the event-oriented $\alpha$-service level, defined as the probability of no stock-out, is one of the most widely used service level criteria (e.g., Snyder and Shen (2011)) and is the focus of our work in this paper.

Our work is closely related to some of the early studies of periodic review inventory models with $\alpha$-service level constraints such as Bitran and Yanasse (1984), Bitran and Sarkar (1988), Bitran and Leong (1992). These papers study a backorder inventory system and show that deterministic models well-approximates the stochastic ones in the regime of high service level. It has an important practical implication: while the multi-period stochastic inventory problem with service level constraints is very difficult to solve, managers can use its deterministic approximation to estimate the total costs.

In this paper, we ask whether similar results can be shown for lost-sales inventory systems. Note that a common belief is that, in the presence of positive lead time (i.e., the time
It is worth noting that our work shares the same spirit as Huh et al. (2009) in the sense that both papers show that the lost-sales model is asymptotically identical to its backorder counterpart. Specifically, we consider both the backorder and lost-sales inventory systems with positive lead time and sequential $\alpha$-service level constraints, and analyze the performance of a simple order-up-to control in the regime of high service-level requirement. The parameters of our heuristic control can be computed using the optimal solution of a deterministic approximation of the stochastic backorder system. We derive theoretical bounds for this order-up-to control, which shows to be asymptotically optimal for both the backorder and lost-sales inventory systems as the service level increases to 100%. Thus, we not only complement the results of Bitran and Yanasse (1984), Bitran and Sarkar (1988), and Bitran and Leong (1992) by providing an asymptotically optimal order-up-to control for the backorder system but also show that this control is still asymptotically optimal for the lost-sales system. In terms of theoretical contribution, our performance analysis for the lost-sales system requires a novel construction of an alternative backorder system whose expected total costs is easily comparable to that of the lost-sales systems. In terms of practical contribution, our result gives credence to the use of deterministic program to approximate complex lost-sales inventory problem with service level constraints. Indeed, since real-world inventory problems with service level constraints are typically very difficult to solve, many works in the literature have simply focused on directly analyzing the deterministic formulation of the problems (e.g., Tarim and Kingsman (2003), Tarim, Dogru, Ozen, and Rossi (2011)). Although our result is specific to the setting that we consider in this paper, we believe that it is an important step for future works on analyzing the quality of deterministic approximation for other inventory problems with service level constraints.
counterpart as the lost-sales penalty (or, equivalently, the service level) increases. However, there are some notable differences. In Huh et al. (2009), demands are assumed to be i.i.d. and there is no explicit rate of convergence; by contrast, in our work, demands across different periods can be highly non-stationary, as long as they share the same support, and we also derive an explicit bound for the optimality gap. In Huh et al. (2009), the order-up-to level is derived from the stochastic backorder system whereas the order-up-to level in our heuristic control is computed using the optimal solution of a linear program. Overall, our results in service-based model complement their results in the cost-based model.

References


