The Value of State Dependent Control in Ridesharing Systems

Recently there is an increasing interest in the control of ridesharing platforms such as Uber and Lyft. These platforms are dynamic two-sided markets where customers (demands) arrive at different physical locations stochastically over time, and vehicles (supplies) circulate in the system as a result of driving demands to their destinations. The platform's goal is to maximize throughput (proportion of demands fulfilled), revenue or other objectives by employing various types of controls. The main inefficiency comes from the geographic mismatch of vehicles and customers: when a customer arrives, she has to be matched immediately with a nearby vehicle, otherwise the customer will abandon the request due to impatience.

We study the dispatch policy as an exclusive control lever to mitigate the loss of demand, where the platform can choose which vehicle to assign when a customer request comes in. Previous work has studied dispatch decisions made in a state-independent manner through randomized policy based on fluid limit solution. However, this approach requires exact knowledge of arrival rates (which is infeasible in practice), fails to react to the stochastic variation in the system. Although this control guarantees asymptotic optimality in the Law of Large Numbers sense, it converges only slowly to the fluid limit. To counter these issues, we study the state-dependent dispatch control of ridesharing systems.

We model the system as a closed queueing network with \( n \) servers representing physical locations, and \( K \) “jobs” that stand for vehicles. This is a common model for ridesharing systems. For each location \( i \) there are some compatible supply locations that are close enough, from where the platform can dispatch vehicles to serve demand at \( i \). Demands arrives at the system stochastically, each has a destination in mind. Each time a customer arrives, the platform makes
a dispatch decision from a compatible supply location *based on the current spatial distribution of available supplies*. After a vehicle picks up a customer, it drops her at the destination and becomes available again. (Supplies do not enter or leave the system.) The platform’s goal is to maintain adequate supply in all neighborhoods and hence meet as much as demand as possible, therefore we adopt the *(global) proportion of dropped demands* as a measure of efficiency. To obtain tight characterizations, we further consider the asymptotic regime where the number of vehicles in the system $K$ goes to infinity.

A main assumption in our model is a complete resource pooling (CRP) condition. For the model considered in this paper, the CRP condition is closely related to the condition in Hall’s marriage theorem in bipartite matching theory. We show that the CRP condition is *necessary* for any dispatch policy to have demand dropping probability that converges to zero. To keep the state space manageable while retaining the essence of our focal challenge (to ensure that all neighborhoods have supplies at all/most times), we assume instantaneous pickups and dropoffs.

**Main Results and Contributions:**

As a function of system primitives, we derive a *large deviation rate-optimal* dispatch policy that *minimizes dropped demand* (maximizes throughput). Our optimal policy is strikingly simple and its parameters depend in a natural way on demand arrival rates. Our contribution is threefold:

1. **Achievability:** We propose a family of state-dependent dispatch policies called scaled MaxWeight (SMW) policies, and prove that *all of them* guarantee *exponential decay* of demand-dropping probability in $K$ under the CRP condition. An SMW policy is parameterized by an $n$-dimensional vector consisting of a scaling factor for each location; each demand is served by dispatching a supply from the *compatible location with the largest scaled number of cars*. We obtain an *explicit* and intuitive specification for the optimal scaling factors based on
location compatibilities and demand arrival rates. SMW policies are simple, explicit and appear promising for practical applications.

2. **Converse bounds:** We provide lower bound of demand dropping probability for any dispatch policy using random-walk related inequalities. We first show that no state-independent dispatch policy can achieve exponential decay rate, which demonstrates the value of state-dependent control --- even a naive state-dependent dispatch policy with no knowledge of demand arrival rates beats the best state-independent dispatch policy asymptotically. Then we justify the CRP assumption by showing that it is a necessary condition for exponential convergence. Further, we obtain the surprising finding that the optimal SMW policy is, in fact, exponent-optimal among all state-dependent policies.

3. **Qualitative insights:** We characterize the system behavior under SMW policies as $K$ goes to infinity, which is technically challenging since the problem remains $n$-dimensional even in the limit. We solve the multi-dimensional variation problem that characterizes the exponent base on a family of novel Lyapunov functions (a different one for each SMW policy). We establish the critical subset property of the problem: given a system state (in the limit), there exists a (state-dependent) subset $J$ of demand locations that are most likely to be depleted of supply in compatible locations, hence leading to demand dropping. The optimal SMW policy avoids using supply from locations compatible with $J$ for demand arising outside of $J$.

   Our working paper has generated talk invitations from Ola and Uber, and interest from Lyft.