Robust Dual Sourcing Inventory Management: Optimality of Capped Dual Index Policies

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We provide closed-form solutions to a robust optimization model for inventory management with two supply sources or modes with general lead times. The fast source is more expensive than the slow source. While the optimal stochastic policy for non-consecutive lead times has been unknown for over 50 years, we prove that the optimal robust policy is a dual index, dual base-stock policy that constrains or “caps” the slow order. Optimality is established in a rolling horizon model that can accommodate non-stationary demand. As the lead time difference grows, the capped dual index policy increasingly smoothes slow orders and, for stationary demand, converges to the tailored base surge policy, which places a constant slow order and has been shown to be asymptotically optimal. In an extensive simulation study, the capped dual index policy performs as well as, and can even outperform, the best heuristics presented in the stochastic inventory literature.

Dual sourcing refers to the practice of replenishing inventory using two sources or shipping modes. Often the primary source (mode) is cheaper but slower than the secondary source (mode). Dual sourcing combines cost-efficiency with responsiveness to mitigate demand and supply risk.

Despite the relevance of dual sourcing in practice, characterizing the optimal dual sourcing policy is extremely difficult. In the early 1960s, researchers showed that a dual base stock policy defined using a single index (namely net inventory-on-hand) is optimal when the lead time difference between the two sources equals 1. Yet more than 50 years later, the structure of the optimal policy for general, non-consecutive lead times—arguably the more relevant case in practice—remains unknown. The challenge stems from the curse of dimensionality because the state space of the classical stochastic dynamic programming formulation grows exponentially in the lead time difference. Recent dual sourcing literature (reviewed below) therefore has proposed several heuristic policies with numerically demonstrated good performance.
Robust optimization presents an alternative approach to studying dual sourcing. Classical stochastic analysis models random variables by a probability distribution over a support set. Any resulting performance analysis is invariably tied to properties of that distribution. In contrast, robust optimization models random variables solely by a (possibly restricted) support set, called the “uncertainty set.” Robust performance analysis involves a deterministic maximization of the worst-case outcome, which is invariably linked to the choice of the uncertainty set. While robust optimization is non-parametric in that it does not assume a probability distribution, its key parameter is the degree of conservatism by which the original support of the random variable is restricted.

We present and solve robust optimization models for periodically reviewed, dual sourcing inventory management that minimize the total cost of purchasing, inventory holding and backlogging costs. We adopt uncertainty sets inspired by the central limit theorem (CLT) which provide a direct link to stochastic models and allow a closed-form characterization of the robust optimal dual sourcing policy. Our main results and contributions can be summarized as follows:

(1) We present a robust rolling horizon model for inventory management with two supply sources with general lead times, and solve its optimal dual sourcing policy in closed form. To the best of our knowledge, this is the first study to model dual sourcing using robust optimization and identify the optimal policy for non-consecutive lead times.

(2) We prove that the robust optimal policy is a dual-index, dual-base-stock policy that constrains or “caps” the slow order. This optimality is established in a rolling horizon model that can accommodate non-stationary demand. The capped dual index policy tracks two inventory positions (over the short and the long lead time, respectively) and is characterized by three parameters: two base-stock or order-up-to levels (one per source) augmented by an upper bound or “cap” on the slow order quantity. In each period, if the inventory position of either source falls below its corresponding base-stock level, it places an order to each source to raise each inventory position back to its base stock, but the slow order quantity cannot exceed its cap. The presence of the cap is the distinguishing feature compared to the dual index policy in Veeraraghavan and Scheller-Wolf (2008) and provides crucial order-smoothing; indeed, as the lead time difference grows, the capped dual index policy converges to the tailored base surge policy, which Xin and Goldberg (2017) have proved to be asymptotically optimal.
(3) We present closed-form expressions for the base stock levels and the cap of the robust optimal capped dual index policy for the CLT uncertainty set, following Bandi and Bertsimas (2012) and Mamani et al. (2016), which can accommodate non-stationary and correlated demand: the CLT uncertainty set constrains each partial sum of demands over the planning horizon by a CLT constraint and restricts demand in each period by an upper and lower bound. We also consider two special cases: (i) the full-sum CLT uncertainty set constrains only the full sum (instead of all partial sums) of demands over the planning horizon; (ii) the hypercube uncertainty set only restricts demand in each period by an upper and lower bound.

(4) We present extensive simulation studies that show that the capped dual index policy performs as well as, and can even outperform each of, the best heuristics presented in the stochastic inventory literature, which is reviewed next.

References

