Pricing Problems under the Markov Chain Choice Model

In this paper, we consider a Markov chain choice model to describe how the customers choose among the products as a function of the prices of all of the available products and we solve pricing problems under this choice model. In our Markov chain choice model, a customer arriving into the system is interested in a certain product with a certain probability. Depending on the price charged for this product, the customer decides whether to purchase the product. If the customer purchases the product, then she leaves the system. If the customer does not purchase the product, then she transitions to another product or to the no purchase option with certain transition probabilities. If the customer transitions to another product, then she decides whether to purchase the other product depending on the price of this product. In this way, the customer transitions between the products until she purchases a product or she reaches the no purchase option. We study three fundamental pricing problems when customers choose according to the Markov chain choice model.

First, we study monopolistic pricing problems, where there is a single firm that controls the prices for all of the products. The goal is to set the prices for the products to maximize the expected profit obtained from each customer. A standard formulation of the problem presents critical difficulties (e.g., objective function is nonconcave and simply computing the objective value of the formulation at particular prices requires solving a system of equations). We develop an approach to find the prices for the products that maximize the expected profit obtained from each customer. This approach requires solving a sequence of single dimensional optimization problems. Although the objective function of the standard formulation is not concave, we show that our approach finds a global maximizer of the objective function. Also, we show that if the unit cost associated with a certain product increases, then the optimal price of this product increases, whereas the optimal prices of all other products decrease. Furthermore, if the unit costs of all products increase by the
same amount, then the optimal prices of all products increase. These results can be of interest independently, but we also use them to characterize the structure of the optimal policy for the dynamic pricing problem with a single resource.

Second, we study competitive pricing problems with multiple firms. Each firm owns a certain subset of the products and controls the prices for the products that it owns. Customers choose among all products owned by all firms. The goal of each firm is to set the prices for the products that it owns to maximize the expected profit it obtains from each customer. We show that a Nash equilibrium exists. We also prove that the prices for the products in any Nash equilibrium are no larger than those when a central planner computes the prices to maximize the expected profit obtained from each customer. Thus, competition between the firms tends to lower the prices. Furthermore, we show that there exists a Nash equilibrium that Pareto dominates any other possible Nash equilibria. Such a Nash equilibrium is simultaneously preferred by all firms. We show that the prices at the Pareto dominating equilibrium decrease as the control of the products are split among a larger number of firms and the intensity of competition increases.

Third, we study dynamic pricing problems with a single resource. Customers arrive randomly over time and choose among the products according to the Markov chain choice model. There is limited inventory of the resource. The sale of a product consumes a unit of the resource. The goal is to find a policy to dynamically decide what prices to charge for the products to maximize the total expected profit over a finite selling horizon. We formulate this problem as a dynamic program and show that if we have more units of the resource at a particular time period, then the optimal prices decrease. Also, if we get closer to the end of the selling horizon with a certain remaining number of units of the resource, then the optimal prices also decrease. Furthermore, we consider a deterministic approximation formulated under the assumption that the demands for the products
take on their expected values. Such a deterministic approximation has a nonconcave objective function and a nonconvex feasible set of solutions. Also, the objective function and the constraints do not have closed form expressions. We give an equivalent reformulation for the deterministic approximation with closed form expressions for the objective functions and the constraints. We characterize when the objective function is concave in the decision variables.

Besides the three fundamental pricing problems above, our formulation of the Markov chain choice model makes useful contributions. It is not a priori clear in the literature how to use the Markov chain choice model when the prices are adjustable and the choice process of the customers reacts to the prices. One can make the transition probabilities a function of the prices, but this approach complicates the choice model to such an extent that the corresponding pricing problems become intractable. We keep the transition probabilities independent of the prices, but when a customer visits a certain product, she decides whether to purchase this product based on the price for the product. Although the transition probabilities are independent of the prices, the purchase probability of a product still depends jointly on the prices of all products. We show that our extension of the Markov chain choice model is compatible with the random utility maximization principle, where each customer associates random utilities with the products and the no purchase option, choosing the alternative with the largest utility.

Markov chain choice model is a more general choice model than the multinomial logit model. We give numerical examples to demonstrate that the modeling flexibility provided by the Markov chain choice model can be beneficial and this choice model can do a better job of predicting the customer purchase behavior when compared with the multinomial logit model. Noting the fact that numerous monopolistic and competitive pricing problems under the Markov chain choice model are tractable, the Markov chain choice model can indeed be a viable option for applications.