Optimal Control of Reverse Logistics in Supply Chains with Multiple Flows of Product

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In contrast to forward logistics that deals with the flow of goods and services from the point of origin to the point of consumption, reverse logistics refers to principles and practices for managing the flow of surplus inventory in the form or material, goods, or equipment back through the supply chain (for reuse, resale, recycling, or disposal). In 2002, the value of reverse logistics was estimated at $100 billion (Stock et al., 2002), while by 2013 that estimate, as provided by the Reverse Logistics Association, grew to between $150 billion and $200 billion annually (Rogers et al., 2013). Even though there has been an increasing recognition of the importance of reverse logistics there remains a lack of clarity in both practice and the research literature regarding precisely what in reverse logistics is so important, how reverse logistics creates such value for companies, and what exactly the drivers of that value are.

The particular aspect of reverse logistics of interest to us in this paper has to do with jointly managing multiple flows of product in the supply chain, as reverse logistics necessarily involves more than just the regular order flow for the product. Formally, we build a model of multi-stage supply chains with stochastic demand and the following four fundamental types of product flows at each stage: regular and expedited flow downstream, reverse flow of product upstream, and lateral flow of product out of the supply chain.

The regular flow of product in our model represents standard, regularly scheduled orders by which each stage in a multi-stage supply chain replenishes its inventory, period by period, from the stock available at the stage immediately upstream. The expedited flow, on the other
hand, allows the product to move downstream through the entire supply chain (or any portion thereof) within the same period. Expediting of inventory is a common practice in industry.

In contrast to regular and expedited orders that flow downstream, the reverse flow of product takes place through returning inventory upstream at each stage in the system. Returning inventory was originally used in the distribution of books and magazines. More recently, reverse logistics has become a widely used supply-chain capability in industries such as apparel, the high tech, and automobile manufacturing. In practice, this reverse flow of orders tends to be managed independently of other product flows in the system, leading to suboptimal outcomes. The fourth flow of product, laterally out of the system, is made possible by the existence of demands external to the supply chain. Those external demands provide the opportunity for supply chains to sell off excess inventories into secondary markets at multiple points in the system.

We formulate a periodic review, finite-horizon, multi-stage inventory model to minimize the total discounted expected cost by jointly optimizing regular ordering, expedited ordering, reverse ordering, and secondary market sales decisions, under stochastic, Markov-modulated demand. The setting of our paper is a supply chain in which the physical transformation of the product is completed at the most upstream location in the system. The finished product is then moved downstream, from one location to another, in order to bring it closer to the customer, without physically transforming the product any further in the process. (Such logistics systems abound, for instance, in the apparel industry or the pharmaceutical industry).

Our formulation belongs to the class of multiechelon models. Such models are generally difficult to analyze due to the curse of dimensionality inherent in the multi-dimensional nature of their objective cost functions. Only a small number of multiechelon models has been completely solved, usually by the standard method of a change of variables from installation quantities to
echelon quantities, first applied in Clark and Scarf (1960). The model considered in our paper has an important additional dimension of complexity not encountered in other multiechelon systems – namely, the joint management of four different product flows in every period, each with its own corresponding decision at every location in the supply chain. This leads to the formation of multi-dimensional boundaries of the feasible region. Further, due to the interaction of those product flows, the standard Clark and Scarf (1960) approach based on reformulating the optimality equations by means of echelon quantities does not bear fruit in our model, and multi-dimensional boundaries of the feasible region persist even under the echelon formulation.

To deal with this challenge, we develop a novel, two-step approach. In the first step, we introduce the notions of selling inventory short and purchasing inventory long. Short sales and long purchases are standard tools of the financial options theory, and in this paper we adapt those concepts to the setting of supply chains with multiple flows of product. We use those notions to derive some key properties of the optimal policy that make it possible to reduce the dimensionality of the feasible region for the problem. We then proceed to identify conditions under which the form of the optimal policy can be established for the problem. The form of the resulting optimal policy achieves the decomposition of the multidimensional objective cost function for the problem into a nested sequence of single-dimensional subproblems, and thus renders the problem analytically and numerically tractable. When external demands, in the form of secondary markets, are restricted to the most upstream stage in the system, this optimal policy can always achieve such decomposition. In that case, we establish additional properties of the optimal policy that are of theoretical and practical interest. Finally, we make use of the attained decomposition to develop an efficient algorithm to compute optimal policies and quantify the value of reverse logistics under a variety of supply chain structures and product characteristics.