Imagine that you have several spare tickets for a sold-out game starting in less than a few days. You want to resell the tickets but are unsure about how much potential buyers are willing to pay for them. How would you price your tickets and possibly adjust the price in light of market response? In an empirical study, Sweeting (2012) found that sellers have increasingly used dynamic pricing strategies in secondary markets for Major League Baseball tickets. Many professional sports teams have also begun the practice of dynamically adjusting ticket prices based on demand (Rishe 2011). Moreover, the selling of unique assets such as houses, specialty equipment, art pieces, and collection items bear similarity to the above problem. All of these assets have a highly uncertain value, and the customer willingness-to-pay (WTP) is usually unknown to the seller. The pricing problem is further complicated by limited inventory and a short selling horizon, both restricting demand learning opportunities. In this paper, we study the optimal pricing decision of a seller facing these three challenges—unknown customer WTP, limited inventory, and short horizon.

Such a dynamic pricing problem can be naturally formulated as a finite-horizon Bayesian dynamic program that updates the prior of the unknown demand distribution with the observed demand realization in each period, allowing for a joint optimization of learning and pricing (Aviv and Pazgal 2005). Unfortunately, the resulting Bayesian dynamic program is analytically intractable. To circumvent the challenge, Aviv and Pazgal (2005) proposed certain heuristics for the problem and evaluated their performance using a performance upper bound. In contrast, another stream of literature (e.g., Kleinberg and Leighton 2003, Besbes and Zeevi 2009, and Harrison et al. 2012) adopted a frequentist approach consisting of sequential phases of exploration and exploitation. This approach is amenable to asymptotic convergence analysis when both the inventory and demand rate are sufficiently scaled up, such as in a high-sales volume environment with ample inventory (so that there are enough sales observations to ensure convergence).

Adhering to the limited inventory, short horizon feature of the problem, we adopt the Bayesian dynamic program approach of Aviv and Pazgal (2005). To improve tractability, we focus on a special case of their model. Specifically, we assume that the customer WTP is independent and
identically distributed (i.i.d.) and there is a single customer demand arrival in each period (see, e.g., Talluri and van Ryzin 2004 for the same assumption). Thus, in each period the demand is Bernoulli with two possibilities: If a customer makes a purchase, then her (realized) WTP must be greater than or equal to the posted price; otherwise, her WTP is below the posted price. In other words, the posted price serves as an either left- or right-censoring point of the true WTP, leading to an interesting two-sided censoring phenomenon. Our goal is to leverage the relatively simplified problem structure to derive new insights, bounds, and heuristics for the problem.

We take the following innovative approach to studying this problem. After simplifying the problem using an unnormalized prior transform technique, we prove a generalized envelope theorem for the Bayesian dynamic program. This theorem helps us derive an explicit expression of the first-order derivative of the dynamic program objective function for the first time in the literature. This new derivative expression enables us to accomplish three things.

First, an examination of the derivative expression reveals new insights about the intertwined effects of left censoring, right censoring, and limited inventory. We find that left censoring induces the seller to price lower, whereas both right censoring and limited inventory have an opposite effect. As a result, the optimal price in general can be either higher or lower than the myopic price that maximizes the single-period revenue.

Second, we use the derivative expression to develop new, easy-to-compute upper and lower derivative bounds. These derivative bounds enable us to determine the corresponding upper and lower bounds for the optimal price decision. We also find that when there is only one unit of inventory for sale, the solution upper bound is the same as the optimal price for a system in which the customer WTP is observed exactly.

Third, and more importantly, we propose two new derivative-based heuristics for the problem. The first heuristic approximates the first-order derivative by a weighted average of its upper and lower bounds, whereas the second one by implementing an easy-to-compute open-loop policy. To evaluate the heuristic performance, we follow the standard performance bounding approach used in the literature (e.g., Aviv and Pazgal 2005 and Chen 2010). Nevertheless, we derive a new, easy-to-compute and tight value function upper bound for this purpose through our derivative analysis.

To illustrate the challenge of computing the optimal solution as well as to demonstrate how to implement our derivative-based heuristics and performance bound, we provide detailed formulas for the case in which the customer WTP follows an exponential distribution with gamma prior, and show that the computation can be further simplified by a dimensionality-reducing scaling property. Finally, we conduct extensive numerical studies to evaluate the performance of our derivative-based heuristics along with two existing heuristics in the literature: one from Aviv and Pazgal (2005) and
the other from Besbes and Zeevi (2009). Our numerical results show that our second derivative-based heuristic outperforms all other heuristics and that its performance is robust with respect to different WTP distributions.

In summary, this paper studies a challenging dynamic pricing problem with unknown customer WTP, limited inventory, and short horizon. We formulate the problem as a Bayesian dynamic program and make the following contributions to the literature. First, we derive an explicit expression for the first-order derivative of the dynamic program objective function for the first time in the literature. This new derivative expression is a significant advancement in our understanding of this challenging problem. Second, we reveal new insights about the intertwined effects of left censoring, right censoring, and limited inventory in the problem. These insights help explain why the comparison between the optimal price and the myopic price in our problem is inconclusive in general. Third, we make methodological contributions by deriving new, easy-to-compute solution bounds and heuristics for the problem based on the derivative expression. Our numerical studies further show that both of our derivative-based heuristics perform well, with the second one consistently outperforming the existing heuristics in the literature.

References


