Multi-stage Intermediation in Display Advertising

Extended Abstract

In display advertising, advertisers rarely purchase inventory directly from publishers. Instead, they contract with intermediaries to acquire impressions on their behalf. An advertiser may choose to acquire impressions via an intermediary for technological reasons (e.g., setting up the infrastructure for trading with the advertising platforms is costly) or to attain the broadest audience possible (e.g., intermediaries may have access to exclusive inventory by partnering with publishers or other intermediaries). In practice, advertisers end up purchasing inventory via an intermediation network with multiple tiers in which intermediaries are organized in tiers according to their specialization. Along this network, every intermediary (except for the one who directly acquires impressions from the supply side) purchases impressions from an upstream intermediary.

Another salient feature of the Internet advertising industry is that the intermediaries typically purchase impressions from a clearing house or an ad exchange, where potential participants submit bids, and then the winner and the payment are determined via an auction. When an impression becomes available for sale, an intermediary along the network collects bids from downstream agents, bids upstream based on these reports, and in case of winning allocates the acquired impression to a downstream agent. This is in sharp contrast with traditional manufacturer/retailer settings where intermediaries physically purchase a good to later resell to a downstream agent. In Internet advertising, intermediaries auction off contingent goods which they only purchase when downstream buyers have signaled interest.

The presence of intermediaries in this industry introduces several new interesting questions.
How should an intermediary bid on behalf of her customers in the mechanism of an upstream intermediary? How does the structure of the intermediation network affect the profits of its participants? Do intermediaries prefer to be closer to the supply source (ad exchange) or demand source (advertisers)? This paper sheds light on these issues by providing a game theoretic model to study the mechanisms offered by a network of intermediaries connecting an ad exchange with advertisers in a setting where advertisers’ types are private. We characterize the optimal mechanisms within a practically relevant class, and show that economic incentives are not necessarily aligned along the network.

We introduce a model that captures the key characteristics of sequential intermediation in display advertising. In our model captive advertisers seek to purchase a unique indivisible impression from a central advertising exchange through an arbitrary number of intermediaries. The connections among the seller, intermediaries, and advertisers are modeled as a directed tree network. The intermediaries have no value for the impression itself, and profit via purchasing the impression from an upstream agent and selling it to a downstream agent at a higher price. The advertisers have private values for the impression which are drawn from a common-knowledge distribution, and the seller optimizes her revenue by selling the impression via an auction.

We model the induced game among the intermediaries and the seller as an extensive form (Stackelberg) game in which intermediaries move sequentially from upstream to downstream by choosing their mechanisms following the seller. In our setting, each intermediary chooses an optimal mechanism after observing the mechanisms of upstream intermediaries and anticipating that the downstream intermediaries will react accordingly. We study the outcome of the strategic interaction among intermediaries and the seller by focusing on the subgame perfect equilibria (SPE) of the induced game.

Our first contribution is to characterize an equilibrium of this game. We show that the equilibrium mechanisms have a simple and appealing structure. Under a regularity assumption on the distribution of values, we show that each intermediary’s optimal bidding strategy takes a simple structure: the intermediary first determines the *virtual value function* associated with
the distribution of bids received from downstream agents. Then, she computes the virtual value associated with the maximum downstream report and bids this amount in the upstream intermediary’s mechanism, whenever this quantity is positive. We show that these bidding strategies (when complemented with appropriate reserve prices) constitute a subgame perfect equilibrium of the game among intermediaries.

We leverage our equilibrium characterization to determine the impact of an intermediary’s position in the intermediation network on her profits. Specifically, we focus on a chain of intermediaries (which is a special case of the tree network), and investigate whether intermediaries in upstream or downstream tiers profit more. In particular, we establish that when the buyer’s value distribution is exponential, the expected profits of all intermediaries are identical. On the other hand, when the buyer has a heavy-tailed value distribution, such as the Pareto distribution, the downstream intermediaries who are closer to the buyer have higher profits. This result suggests that depending on the buyer’s value distribution for the impressions, intermediaries may prefer to participate in different stages of the intermediation process.

We further analyze the subgame perfect equilibrium of the strategic interaction among intermediaries that are organized in a regular tree network. First, we demonstrate that the relation between advertisers’ value distribution and the optimal position is robust to more general configurations. In particular, we show that being closer to advertisers becomes more profitable as the tail of the advertisers’ value distribution gets heavier in a regular tree with two tiers of intermediation. Second, we establish that depending on the network structure a larger number of intermediaries may be more profitable for the seller even if the number of advertisers is fixed. Lastly, we consider intermediaries’ incentive to merge horizontally (within the same tier) and vertically (across different tiers). We show that a horizontal merger is not always profitable for intermediaries. For consecutive intermediaries organized in a chain, we show that the incentive to vertically merge is larger in the most profitable positions in a chain. Our results suggest that incentive to merge is jointly shaped by the underlying network structure and the value distribution of advertisers.