That’s Not Fair: Tariff Structures for Electricity Markets with Rooftop Solar

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(1) Problem definition: Utility regulators are grappling to devise compensation schemes for customers who sell rooftop solar generation back to the grid, while safeguarding environmental interests, and the financial interests of utilities, solar system installers, and retail customers. This is difficult: Regulatory changes introduced in Nevada in 2015 to protect Nevada’s utility induced SolarCity, the market leader in solar systems, to suspend operations in Nevada. We show that the choice of tariff structure is crucial to achieving socially desirable objectives.

(2) Academic/Practical Relevance: It is important for regulators to understand how tariff structure interacts with social objectives. We consider this problem by formulating and analyzing a model to study this interaction, with implications for consumers, regulators and industry.

(3) Methodology: We use a sequential game to analyze the regulator’s social welfare maximization problem in a market with a regulated utility, an unregulated, monopolistic, profit-maximizing solar system installer, and customers who endogenously determine whether to adopt solar or not, based on utility tariffs, solar prices and their heterogeneous usage profiles and generation potentials.

(4) Results: We show that two features characterize an effective tariff structure: the ability to discriminate among customer usage tiers, and the ability to discriminate between customers with and without rooftop solar. Absent any one of these, we show that the regulator’s optimization problem might be infeasible or result in cross-subsidization. We then present a tariff with these two characteristics—featuring full retail price repurchasing from solar customers—that always guarantees feasibility of the regulator’s optimization problem and can avoid cross-subsidization. We illustrate our findings numerically using data from Nevada and New Mexico, two states currently grappling with this issue.

(5) Managerial Implications: Many utilities in the U.S.A. operate tariff structures that are missing at least one of the two identified features. Regulators must overhaul these tariff structures to adequately safeguard all stake-holders.

Key words: rooftop solar, net-metering, energy policy, game theory
1. Introduction

Rooftop solar has seen a boom in recent years, with residential rooftop solar installations consistently growing at over 20% year-on-year (Mike Munsell 2017). One of the main catalysts for this growth has been the practice of utility companies offering “retail net-metering” to customers with solar panels: Under this scheme, these customers can sell any excess electricity their panels generate to their utility for full retail credit. Effectively, such customers pay only for their “net” usage.

While such an incentive is useful because a move to rooftop solar is environmentally desirable, retail net-metering threatens the profitability of utility companies, who are forced to buy excess energy from customers at retail rates which are significantly higher than their prevailing wholesale rates. A utility might commonly combat this by raising retail electricity rates for all users, or by reducing the rate at which utilities repurchase excess generation from solar households. However, both these solutions are problematic: If the utility company raises rates, (typically) poorer non-solar households would bear some part of the burden imposed by (typically) wealthier solar households (Krysti Shallenberger 2017a); this would result in cross-subsidization, a phenomenon under which one set of customers benefits at the cost of another set of customers. Alternately, if the repurchase rate is reduced, customers may no longer be incentivized to install solar, and solar installers could be put out of business. This latter dynamic played out recently in Nevada (Buhayar 2016).

In each of the thirty-three U.S. states with regulated electricity markets, a body called the Public Utilities Commission (PUC) has the charge of solving this complex problem: In each of these states, the PUC balances the welfare of the various stake-holders by regulating the rates and services of public utilities. The PUCs can therefore be thought of as social welfare maximizers; in the context of rooftop solar, this means protecting utility company profitability and ensuring fair rates for customers, while providing a nourishing environment for rooftop solar in order to protect environmental interests (PUCs’ stated objectives often explicitly include environmental stewardship; see California Public Utilities Commission (2017)). The PUC’s task is further complicated by the fact that solar system installers (henceforth solar companies) are typically unregulated; therefore, regulatory interventions must account for such solar companies making self-interested decisions.

As might be expected, the effect of increased rooftop solar adoption on utility companies’ profits has resulted in considerable regulatory flux: in the U.S., 42 of the 50 states took some action related to net-metering, rate design or solar ownership during the third quarter of 2015 alone (NC Clean Energy Technology Center and Meister Consultants Group 2015). The PUC’s regulatory tight-robe
walk of balancing customer, societal and utility welfare is a tricky affair that has, on occasion, gone awry: NV Energy, the utility company in Nevada, imposes a simple two-part tariff: a monthly fixed charge and a variable “energy” charge per kWh of energy consumed. After being negatively impacted by increased rooftop solar adoption, NV Energy initiated a prolonged dialogue with their PUC, in which they raised the spectre of cross-subsidization (Chediak and Buhayar 2015). The outcome of this dialogue was a ruling that solar customers would eventually pay thrice as high a fixed charge as non-solar customers, and would be credited for excess generation at wholesale rates (significantly lower than the existing retail rate). This announcement prompted SolarCity, the market leader in solar systems, to suspend operations in Nevada and cut over 500 jobs (Buhayar 2016). In December 2016, a year after this ruling was made, the PUC reversed its stand, voting to restore retail net-metering and the original rate schedule in the Sierra Pacific territory (Pyper 2016). Meanwhile in February 2017 in Maine, the PUC passed a bill to phase down compensation paid to customers for their excess generation. In July 2017, a new bill that aimed to roll back this decision in order to boost solar growth was vetoed by the Governor, who cited cross-subsidization as the reason for his decision: he said that net-metering subsidizes the cost of solar panels “at the expense of the elderly and poor who can least afford it” (Krysti Shallenberger 2017b).

There is no evident consensus on the structural properties a tariff should have in order to be effective: utilities in different states operate a variety of tariff structures. For instance, NV Energy in Nevada and Duke Energy in North Carolina have only a single tier in their tariff structures, whereas utilities such as PNM Energy in New Mexico and Idaho Power in Idaho have 3 tiers. Utility tariffs also vary in whether they discriminate between solar and non-solar customers. Nevada permits NV Energy to have solar and non-solar customers on different rate schedules, but states such as New Mexico and Washington explicitly disallow this. Meanwhile, Arizona Public Service (APS) has a tiered tariff structure that pays solar customers less than the retail rate for excess energy sold back, effectively putting them on different rate schedules from non-solar customers.

Motivated by such developments, we explore this delicate problem faced by the PUC’s, focusing on how effective different tariff structures are in enabling the PUC to induce socially optimal welfare outcomes in an electricity market with rooftop solar. We do so by explicitly modeling the regulator’s social welfare optimization problem and demonstrating that some common tariff forms are potentially inadequate to the task. We model a monopolistic, vertically integrated utility company (like NV Energy) whose tariffs are set by the PUC; a monopolistic, price-setting solar company (similar to SolarCity); and residential customers who are heterogeneous in their demands
and generation capability (available roof space). After the PUC fixes a tariff (upon negotiation with the utility company), the solar company and customers play a sequential game: the solar company sets the price of solar systems to maximize its profit, anticipating customers’ decisions (made endogenously) to install solar or not. Customers make their self-interested installation decision based on their demand, rooftop solar generation potential, and excess generation that they expect to sell back to the grid (taken together, these determine a customer’s “usage tier,” or equivalently, “usage profile”), the tariff set by the PUC, and the solar company’s declared price. Naturally, the regulator takes the behavior of the solar company and customers into account when deciding on a tariff.

A key element of our model is the endogenization of the (monopolistic) solar company’s pricing decision, a departure from most existing literature. This is in keeping with the observation that over a third of the market share in the U.S. was claimed by one private company (SolarCity) in 2014 and 2015 (Roselund 2015), that wields significant pricing power because of lower costs (Shahan 2016). The endogenized solar pricing decision creates additional incentive compatibility constraints the regulator must account for, failing which the solar company may exit the market.

Using our model, we find two attributes that a tariff structure must have in order to guarantee effectiveness: the ability to discriminate between customers based on their usage tier, and the ability to discriminate between customers with and without rooftop solar. While APS’s tariff structure has both these features, at least one of them is absent in the tariff structures of many other utilities, for example, NV Energy in Nevada and PNM Energy in New Mexico. In the absence of either one of these attributes, we show that the regulator might not be able to induce a socially optimal outcome, and often will have to resort to an outcome with cross-subsidization. We then show that a simple two-part tariff with these attributes, featuring full retail price repurchasing from residential solar customers, always guarantees feasibility of the regulator’s social welfare optimization problem. We also show that if solar adoption generates an overall customer surplus (possibly at the expense of the utility), this tariff structure can guarantee an outcome with no cross-subsidization.

Complementary to our analysis, using consumer survey data obtained at the household level (U.S. Energy Information Administration 2009), we estimate customer usage profile parameters and numerically illustrate how our suggested tariff structure compares to the tariff structures currently in use in Nevada and New Mexico, two states wrestling with this issue. We find that

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1 We ignore commercial customers for tractability.
both states’ tariffs perform poorly compared to our suggested tariff: while our tariff is able to avoid cross-subsidization in all test cases, the current tariffs in both states are not. Notably, in Nevada, use of the existing tariff structure most adversely affects (possibly low income) customers living in the smallest houses.

2. Literature Review

There is a substantial body of Operations Management literature exploring various aspects related to the management of renewable energy resources. Aflaki and Netessine (2016) and Hu et al. (2015) study capacity investment decisions for renewable resources such as wind and solar. The effect of tariff structures on such investments have been studied in Alizamir et al. (2016), Ritzenhofen et al. (2016), and Kok et al. (2015). The operational aspects of managing renewable energy resources are studied in Zhou et al. (2014), Wu and Kapuscinski (2013), and Al-Gwaiz et al. (2016).

The energy policy literature contains a stream of work investigating regulatory considerations arising from the increase in distributed generation. Some of these papers, such as Keyes and Rábago (2013) and Lehr (2013) provide frameworks for regulation. Linvill et al. (2013) qualitatively discuss the challenges a regulator might face when implementing net-metering or feed-in tariffs for compensating distributed generation. Some studies such as Blackburn et al. (2014), Borlick and Wood (2014), Brown and Bunyan (2014) and Moore et al. (2016) find that current subsidy levels in tandem with net-metering tend to overcompensate customers for excess generation. NC Clean Energy Technology Center and Meister Consultants Group (2015) provides a comprehensive view of recent solar-related regulatory changes that have been effected in the 50 states in the U.S.

Bird et al. (2013) describe the role of the regulator in a changing electricity landscape as: (1) Keeping the utility company viable, resulting in relatively stable cash flows and revenues from year to year; (2) Fairly apportioning the utility’s cost of service among customers, without undue discrimination; and (3) Promotion of economic efficiency in the use of energy as well as competing products and services, without compromising on reliability. Our work draws on this description of the regulator’s role; we formulate and solve an analytical model to explore which tariff structures enable the regulator to induce market outcomes in keeping with these criteria.

Another stream within the energy policy literature studies the diffusion of solar among customers. Simulation approaches are common (Denholm et al. 2009), and are used as a building block for many other pieces of research like Gagnon and Sigrin (2015) and Drury et al. (2013). There is also a large body of empirical literature in this stream; see Ong et al. (2010), Lobel and Perakis (2011),
One possible response to solar adoption eroding their profits is for utility companies to raise electricity rates, making solar energy an even more attractive possibility further undermining utility profitability. This death spiral behavior is studied in Satchwell et al. (2015), Costello and Hemphill (2014), Denholm et al. (2009) and Darghouth et al. (2016).

Closer to our work, Babich et al. (2017) take the perspective of a government entity deciding between offering a feed-in-tariff and a tax-rebate policy for rooftop solar installation. They study how the policy in place affects the solar panel investment decisions of a representative household in the presence of exogenous shocks that affect generation efficiency, variability in electricity price and solar panel investment cost (i.e. they have a dynamic model, but with exogenously given solar prices). Similar to Babich et al. (2017), our work also deals with aspects of renewable energy that involve decisions by a principal (the PUC) and customers; however, our paper presents a static (rather than dynamic) model of solar adoption among heterogeneous customers who make potentially heterogeneous investment decisions, with a solar company that makes an endogenous pricing decision. Our model’s static setting allows us to study the question of what the regulator’s welfare-optimal choice of tariff should be, and our heterogeneous customer model allows us to study the customer equity implications of solar adoption. Goodarzi et al. (2015) take the perspective of a regulator who seeks to minimize utility costs by choosing an appropriate feed-in-tariff rate paid to customers who sell all rooftop generation back to the grid at that rate. In their model, customers are homogeneous in their demand characteristics but heterogeneous in their discount rates, and make a solar adoption decision based on the feed-in-tariff rate. Similar to our model, the solar system price in their model is also chosen endogenously by a profit maximizing solar company. However, in contrast to their paper, we model customers who are heterogeneous in their demands and generation capabilities, and who sell only excess generation back to the grid. As mentioned above, our heterogeneous customer model allows us to study the customer equity implications of the policies in place. Further, our regulator chooses the tariff structure and parameters: this generalizes the feed-in-tariff rate that Goodarzi et al.’s analysis is restricted to.

Our work also relates to the extensive literature on uniform versus non-linear pricing: we study what features a tariff structure must have in order for a regulator to be able to induce a socially optimal outcome. Varian (1989) and the references therein provide an elaborate discussion on
various issues related price discrimination: tiered tariffs are a tool for second-degree price discrimination. While other papers such as Sundararajan (2004) and Choudhary et al. (2005) discuss non-linear pricing for certain specific situations, their findings are not directly applicable to our setting because of our model’s unique characteristics: the tariff chosen by the regulator interacts with customers’ strategic behavior through the price of solar (which customers use to decide whether to adopt solar or not). This tariff must be chosen so as to induce the monopolistic solar company to set a price of solar that will induce a socially optimal outcome.

3. Model

Our model considers residential customers who are heterogeneous in their usage profiles, a monopolistic solar company \( S \), and a regulator \( R \) (the PUC). In addition, we model a vertically integrated utility company \( U \) that is subject to regulation by \( R \) (which makes decisions on behalf of \( U \)).

We define our base case scenario as one with no solar systems, i.e. where all customers depend on \( U \) to satisfy all their demand for electricity. Customers are subject to flat-rate (rather than time-of-use, for tractability reasons) pricing. After \( S \) makes its product available, customers have the option of continuing to depend solely on \( U \) for their energy requirements, or installing solar systems, thereby reducing their dependence on \( U \). We study how \( R \)'s regulatory actions influence social welfare moving from the base case to the post-solar scenario. \( R \)'s social welfare measurement takes into account financial and non-financial (i.e. environmental) considerations. However, customers and \( S \) are modeled as being self-interested, i.e. they maximize their own financial objectives.

We first detail the parameters that characterize each entity and define their decision variables.

- **Customers:** Customers are heterogeneous and have different demands and potentials to generate solar electricity (e.g. because of heterogeneous availability of roof space). We consider \( I \) classes (equivalently, tiers) of customers indexed by \( i \in \{1 \ldots I\} \). Customer class \( i \) has annual demand \( d_i \) kWh and annual generation capability \( g_i \) kWh. We do not capture demand response in our model, and as such, treat \( d_i \) as fixed for a given tier \( i \). A household can estimate \( g_i \) using tools such as Google's Project Sunroof (Google 2018). If a class \( i \) customer installs a solar system, her excess generation (the amount of her generation that she does not consume) is \( e_i \leq g_i \); this excess generation is sold back to the grid. Modeling \( e_i \) as a separate parameter allows us to capture any potential temporal relationship between generation and demand, i.e., a customer’s demand and generation do not necessarily follow the same profile, and their temporal relationship, whatever it is, determines \( e_i \). Under our framework, a class \( i \) customer
depends on the grid for an amount of energy \( d'_i = d_i - (g_i - e_i) \) (her grid usage) and sells back an amount of energy \( e_i \), pegging her net usage at \( d'_i - e_i = d_i - g_i \). We arrange customer classes in order of increasing generation, so that \( g_i < g_{i+1} \). We do not impose a relationship between \( d_i \) and \( g_i \): a customer may generate more than she demands. We let \( h_i \) denote the number of class \( i \) households.

A type \( i \) household has an adopt/do not adopt decision that we encode by binary variable \( s_i \); \( s_i \) takes the value of 1 if the customer chooses to adopt solar and 0 otherwise. All customer parameters are assumed to be fixed, and known with certainty by all decision making entities. In Section 4 we discuss how our model may be extended to accommodate uncertainty in \( d, g, \) and \( e \). Our base model also assumes that all households in a certain class make the same adoption decision. We also discuss in Section 4 how this assumption may be relaxed.

- **Solar Company:** We consider a self-interested, monopolistic, solar company that sets prices for its solar systems. This assumption is driven by prevailing market conditions: in the U.S. SolarCity is the established market leader that has held a stable 34% market share in 2014 and 2015 (Roselund (2015), about 3 times the market share of the nearest competitor) and lower costs, giving it the power to set market prices. We assume that solar panels are infinitely divisible (i.e. we ignore the topography, or solar panel-roof compatibility). \( S \)'s decision variable is \( p_s \), the price that a customer who adopts solar must pay to the solar company per unit of electricity she generates using the installed solar system. SolarCity offers such a contract (this is called a Power Purchase Agreement), under which customers are only assessed a variable charge per kWh of generation, rather than having to pay a lump-sum amount for system purchase and installation (SolarCity 2016). For customer-owned systems that involve upfront payments, this price \( p_s \) can be interpreted as a levelized price—the lifetime adjusted price per kWh that the system generates; this accounts for average sunlight received by the panel, efficiency considerations, down-payments, and maintenance costs. Since these are equivalent from a modeling standpoint, we consider a Power Purchase Agreement setup. Corresponding to the levelized price \( p_s \) that \( S \) chooses, we assume that the levelized cost to \( S \) is \( c_s \) per kWh of generation. Once set, \( p_s \) is assumed to remain fixed. Customers in the U.S. can avail themselves of an investment tax credit of 30% on solar system purchases (Energy.gov 2017).

At the end of Section 3, we show how our model may be adjusted to accommodate this.

- **Regulator:** We consider a socially interested regulator \( R \) whose decision is a tariff function \( T(d', e, s) \) that governs the annual rate that the utility company charges a customer who draws
an amount of energy \( d' \) kWh/year from the grid, sells back \( e \) kWh/year to the grid and either adopts solar \((s = 1)\), or does not adopt solar \((s = 0)\). Note that both \( d' \) and \( e \) are measurable by \( U \) with an appropriate metering system. Also observe that if \( s = 0 \), \( d' = d \) and \( e = 0 \).

- **Utility Company:** We consider a monopolistic utility company \( U \) that faces a fixed annual grid maintenance cost \( f_u \) and an average per unit cost of electricity \( c_u^x / \text{kWh} \), where \( x \) is the amount of electricity it supplies. This framework allows us to capture the non-linear cost functions that utility companies typically face because changes in the amount of electricity they supply alter the mix of generation sources they use. The utility company uses its existing architecture to redistribute excess generation that it purchases from customers across the grid. \( U \) does not take any decisions.

The game played by \( R, S \) and the customers proceeds in the following fashion: Let Period 0 be the base case scenario, when no households have rooftop solar. Under the Period 0 tariff structure, these customers pay a per unit energy cost of \( p_{r0} \), and an annual fixed cost that we normalize to 0, without loss of generality. We use this particular base case tariff structure for simplicity, but our approach readily extends to any general base case tariff structure. In Period 1, \( R \) imposes tariff structure \( T(\cdot) \). In response to tariff \( T(\cdot) \), \( S \) sets a per unit solar rate \( p_s \) in Period 2. In Period 3, individual customers, with knowledge of their demand \( d_i \), generation capability \( g_i \), and excess \( e_i \) observe the tariff \( T(\cdot) \) and the solar price \( p_s \) and then endogenously decide to adopt solar \((s^*_i = 1)\) or not to adopt solar \((s^*_i = 0)\). Since the agents take actions sequentially, each agent’s decision is taken anticipating other agents’ responses in future periods.

We now present the objective functions that govern the decisions of each entity in our model:

- **Customers:** Customers wish to minimize their spend on electricity. Therefore, a class \( i \) customer solves the following problem:

\[
\max_{s_i \in \{0, 1\}} \left( 1 - s_i \right) T(d_i, 0, 0) + s_i \left( T(d'_i, e_i, 1) + p_s g_i \right)
\]  

(1)

- **Solar Company:** \( S \) maximizes profit by choosing an appropriate price of solar.

\[
\max_{p_s > c_s} (p_s - c_s) \sum_{i=1}^{I} s^*_i h_i g_i,
\]

(2)

where \( s^*_i \) is the optimal adoption decision taken by a customer in tier \( i \). We restrict \( p_s \) to being larger than \( c_s \) because \( S \) is unregulated may freely exit the market.
**Regulator:** The regulator $R$ wishes to maximize social welfare improvement. Since $R$ takes a systemic view, it is useful to think of the customers, $U$ and $S$ as belonging to a “system,” we will consider two components of this social welfare improvement: Financial and Environmental.

1. **Financial:** Note that all cash flows except the purchase of electricity at the purchase costs $c_u$ and $c_s$ occur within the system and can therefore be ignored from the system’s perspective. Define $E_0 = \sum_{i=1}^{I} h_i d_i$ as the total amount of energy that customers depend on the utility for in the base case, and $E_1 = E_0 - \sum_{i=1}^{I} s_i^* h_i g_i$ as the total amount of energy that customers depend on the utility for in the post-solar case (here, we assume all excess rooftop electricity is redistributed to other customers). It is useful to define $\Delta E = \sum_{i=1}^{I} s_i^* h_i g_i$ as the amount of energy dependence migrated to rooftop solar. The net decrease in cash flows going out of the system (and hence the financial welfare improvement of the system) is therefore $c_u E_0 - c_u E_0 - \Delta E (T(\cdot)) - c_s (\Delta E) = (c_u E_0 - c_u E_0 - \Delta E) E_0 + (c_u E_0 - \Delta E - c_s) \Delta E$.

2. **Environmental:** In addition to this financial welfare consideration, the regulator considers the environmental benefit accrued by sourcing $\Delta E$ kWh of energy from rooftop solar rather than from the utility. Let $m_u^x$ be the (monetized) average environmental cost of the utility generating one kWh of electricity when the total amount it generates is $x$, and $m_s$ be the (monetized) environmental cost of a rooftop solar panel generating one kWh of electricity.

Again, using $x$ to parameterize $m_u^x$ allows us to capture the non-linear relationship between environmental cost imposed and amount of electricity supplied by the utility because migration to solar potentially changes the mix of generation sources for $U$. This environmental cost can, for instance, be estimated using the social cost of carbon. So, we write $R$’s objective function as:

$$\max_{T(\cdot)} \left( c_u E_0 - c_u E_0 - \Delta E (T(\cdot)) (E_0 - \Delta E (T(\cdot))) - c_s (\Delta E (T(\cdot))) + \right. $$

(3)

$$\left. m_u^x E_0 - m_u^x E_0 - \Delta E (T(\cdot)) (E_0 - \Delta E (T(\cdot))) - m_s \Delta E (T(\cdot)) \right),$$

where $\Delta E (T(\cdot))$ is the extent of migration to rooftop solar induced by tariff choice $T(\cdot)$.

The financial benefit from solar adoption crucially depends on the values of $c_u E_0 - \Delta E (T(\cdot))$ and $c_u E_0$ relative to $c_s$. Based on publicly available information, we estimate that $c_s$ is approximately $0.064$ (the details of this estimation procedure are presented in Section 5). The average wholesale rate of

$^2$This does not depend on the total amount of electricity generated by solar.
electricity (which, in our case is a good estimate of $c_{E_0}$) in the U.S. is roughly $0.04$ (U.S. Energy Information Administration 2017). Solar production typically peaks around mid-day, creating the so-called duck curve (Jeff, St. John 2016), and thus does not generally shave off the peak load (which typically occurs in the late evening) or displace the base load generators during low load periods (early in the morning). Therefore, we do not expect $c_{E_0} - \Delta E(T(\cdot))$ to be significantly different from $0.04$. If $c_{E_0} - \Delta E(T(\cdot))$ and $c_{E_0}$ are equal, the financial benefit simplifies to $(c_u - c_s)\Delta E(T(\cdot))$. Therefore, as long as $c_{E_0} - \Delta E(T(\cdot))$ and $c_{E_0}$ are close enough, we expect the financial benefit from solar adoption to be negative (because $0.04 < 0.065$); this can be interpreted as a cost that society must bear in order to encourage solar adoption. As technological improvements cause $c_s$ to drop, the sign this benefit could flip. Our model is robust to either case.

It is worth pointing out that although we do not explicitly model the decisions of the utility company in response to $R$’s actions, we implicitly capture any capacity changes that the utility company would need to make in response to $R$’s actions through the parameters $c_{E_0} - \Delta E(T(\cdot))$ and $m_{E_0} - \Delta E(T(\cdot))$: since these depend on $T(\cdot)$, the impact of these capacity decisions on utility and system welfare are captured. The utility company has another role in our model: while choosing an adoption level that maximizes social welfare improvement, regulator $R$ must ensure a specified rate-of-return to the utility company. We codify this by denoting the permissible increase in utility profit going from the base case to the post-solar case as $\Delta_U$ (note that this could be negative)\(^3\). Further, she would like to ensure a profit of $\Delta_S$ to the solar company (for example, to encourage further technological innovation; we assume $\Delta_S > 0$). Since the regulator is also responsible for making sure that customers do not overpay for electricity, she would like to choose, from among the functions $T(\cdot)$ that maximize her objective and respect the other constraints she faces, the one(s) that minimize the maximum cash outflow seen by any class of customers. We call this the fairness constraint. This is a common fairness criterion used in game theory and ethics credited to Rawls (Rawls 1974). If this minimized maximum cash outflow is negative, then all classes of customers benefit, and there is no cross-subsidization because no class of customer is hurt by introduction of solar to the market. Formally, we represent these as constraints in $R$’s optimization problem as follows:

$$
\sum_{i=1}^{l} h_i \left( s_i^* T(d_i^*, e_i, 1) + (1 - s_i^*) T(d_i, 0, 0) - c_{E_0}^{E_0 - \Delta E(T(\cdot))} (s_i^* (d_i - g_i) + (1 - s_i^*)d_i) \right) - \sum_{i=1}^{l} h_i (p_{i0} - c_{E_0})d_i = \Delta_U
$$

\(^3\)This value of $\Delta_U$ may be padded to account for transmission losses. For clarity of exposition, we do not model this.
\[(p_s - c_s) \sum_{i=1}^{I} s_i h_i g_i = \Delta_S \]  
\[ (5) \]

\[
\max_i s_i^* (T(d_i', e_i, 1) + p_s g_i) + (1 - s_i^*) T(d_i, 0, 0) - p_{\tau_0} d_i 
= \min_{T(\cdot) \in \tau} \max_i s_i^* (T(d_i', e_i, 1) + p_s g_i) + (1 - s_i^*) T(d_i, 0, 0) - p_{\tau_0} d_i, \]
\[ (6) \]

where \(\tau\) is the set of tariff structures that maximize the regulator’s objective and satisfy all other constraints that she must account for. In Section 4, we precisely characterize \(\tau\).

While this set-up doesn’t explicitly take into account the 30% investment tax credit from the federal government, this can easily be accommodated with minor adjustments to the model: we can now treat \(p_s\) as the discounted rate that customers pay for solar power, and the solar company now obtains revenue at a rate \(\frac{p_s}{1.3}\) per kWh of energy. All our results continue to hold under this modification, so we ignore the investment tax credit for the remainder.

4. Analysis

We begin our analysis by examining the optimization problem of a class \(i\) customer, as specified in (1). Such a customer favors adopting solar if and only if \(p_s \leq T(d_i, 0, 0) - T(d_i', e_i, 1) = t(i)\) (we break ties in favor of adoption). These values of \(t(i)\) depend on \(T(\cdot)\): specifying \(T(\cdot)\) induces an ordering among the \(t(i)\) values. For a given solar price \(p_s\) the set of classes that adopt is \(\{i : t(i) \geq p_s\}\).

Now consider \(S\)’s pricing decision. We can assert that \(S\)’s optimal choice of \(p_s\) must be either \(t(i)\) for some \(i \in \{1, \ldots, I\}\) or some price larger than \(\max t(i)\): If \(S\) chose some price \(p_s\) between \(t(i)\) and \(t(j)\) for some \(i\) and \(j\) such that \(t(j) = \min_k t(k) : t(k) > p_s\), she could do strictly better by choosing price \(t(j)\) as doing so increases \(S\)’s margin and does not alter her volume (see equation (2)). Therefore, \(S\)’s optimization problem reduces to choosing an \(i^*\) such that the profit obtained by setting \(p_s = t(i^*)\) is larger than the profit obtained from all other choices \(j \neq i^*\) or choosing \(p_s > \max t(i)\). We clarify that all adopting customers pay the chosen solar price \(p_s\); \(t(i)\) is simply a threshold value of \(p_s\) up to which a class \(i\) customer is induced to adopt solar.

These observations allow us to rewrite \(R\)’s optimization problem, folding in the decisions of \(S\) and households to reflect the sequence in which they are taken. To do so, we define some more notation. Let set \(D = \{x \in \mathbb{R} : x = \bar{s} \cdot (h_1 g_1, h_2 g_2, \ldots, h_I g_I)\}\), where \(\bar{s}\) is any \(I\) dimensional vector whose entries are binary. Here, \(\bar{s}\) is the adoption decision vector \((s_1, s_2, \ldots, s_i, \ldots, s_I)\) whose entries indicate whether class \(i\) adopts or not. The set \(D\) therefore contains all possible values that \(\Delta_E\) (which we refer to as the migration quantity) could take. We are interested in the case that \(R\) wishes for at least one class to adopt, that is, the case where \(\exists i : s_i = 1\), otherwise the problem is
trivial. This is the case we shall assume for the remainder. Apart from this, we do not restrict \( \vec{s} \): \( R \) may choose any arbitrary subset of classes to be adopters in order to maximize her social welfare objective. Let \( z \) index into this set. Corresponding to each migration quantity \( E^{(z)} \), there exists at least one adoption vector \( \vec{s}^{(z)} = (s_1^{(z)}, s_2^{(z)}, \ldots, s_i^{(z)}, \ldots, s_I^{(z)}) \). Define adoption set \( A^{(z)} = \{i : s_i^{(z)} = 1\} \).

Now, notice that \( R \)'s objective function (3) is affected by \( T(\cdot) \) only through \( \Delta_E \). Therefore, \( R \)'s decision can be equivalently modeled as choosing \( z \) optimally from the set \( \{1, 2, \ldots, 2^I\} \). In order for this value of \( z \) to induce adoption outcome \( \vec{s}^{(z)} \), it must be the case that \( p_s = t(i) \). Define a class \( i \) to be ‘marginal’ if \( p_s = t(i) \). Since \( p_s \) is chosen from among the set of \( t(i) \) values, which as functions of \( T(\cdot) \) are themselves variables, \( R \) may allow any (indeed, more than one) of the adopting classes in \( A^{(z)} \) to be marginal. Let \( M \) be the set of indices of the marginal adopting classes. Since we assumed that at least one class adopts, this set must be non-empty. We can now pose \( R \)'s optimization problem, folding in the household and solar company decision as follows:

\[
\text{max}_{t(\cdot), z \in \{1, 2, \ldots, 2^I\}, M} c_u E_0 - c_u E^{(z)} (E_0 - E^{(z)}) - c_s E^{(z)} + m_u E_0 - m_u E^{(z)} (E_0 - E^{(z)}) - m_s E^{(z)},
\]

Subject to constraints:

\[
t(i) = \frac{T(d_i, 0, 0) - T(d_i', e_i, 1)}{g_i}, \quad \forall i
\]

\[
M \subseteq A^{(z)}
\]

\[
\sum_{i=1}^{I} h_i (s_i^* T(d_i', e_i, 1) + (1 - s_i^*) T(d_i, 0, 0) - c_i^* (s_i^* (d_i - g_i) + (1 - s_i^*) d_i)) - \sum_{i=1}^{I} h_i (p_{r0} - c_u^{E_0}) d_i = \Delta_U
\]

\[
\text{max}_{i} s_i^{(z)} (T(d_i', e_i, 1) + t(m)g_i) + (1 - s_i^{(z)}) T(d_i, 0, 0) - p_{r0} d_i
\]

\[
= \min_{t(\cdot) \in \tau} \text{max}_{i} s_i^{(z)} (T(d_i', e_i, 1) + t(m)g_i) + (1 - s_i^{(z)}) T(d_i, 0, 0) - p_{r0} d_i, \quad \forall m \in M
\]

\[
(t(m) - c_s) \sum_{i=1}^{I} s_i^{(z)} h_i g_i = \Delta_s, \quad \forall m \in M
\]

\[
(t(i) - c_s) \sum_{j=1}^{I} \sum_{I(j) \geq (i)} h_j g_j < \Delta_s, \quad \forall i \notin M
\]

\[
t(i) \geq t(m), \quad \forall i \in A^{(z)}, m \in M
\]

\[
t(i) < t(m), \quad \forall i \notin A^{(z)}, m \in M
\]
Here, (8) defines $t(i)$ in terms of the regulator’s decision variables, (9) ensures that the choice of marginal adopting classes is consistent with the choice of $z$, (10) ensures that $U$ receives the specified rate of return implied by $\Delta_U$, (11) imposes the fairness constraint on customer payments, (12) ensures that $S$ achieves the specified profit $\Delta_S$ by choosing solar price $t(m)$, $m \in M$, (13) is a set of incentive compatibility constraints, which ensure that $S$ can do no better than make profit $\Delta_S$ by setting $p_s = t(i)$ for $i \notin M^4$, and (14)-(15) induce customers in $A^{(z)}$ to adopt solar.

This formulation is not expressed in a convenient form, and is therefore not amenable to analysis: $\tau$ is ill-defined in (11), and (13) and (9) are not expressed in canonical form, because they deal with set containment and the indicator function. However, by imposing the following restriction on tariff function $T(\cdot)$, we can convert this problem into a form that is easier to analyze.

**Tariff restriction:** Tariff function $T(\cdot)$ is chosen so that all $t(i)$ values are distinct.

**Proposition 1.** The tariff restriction does not affect the regulator’s ability to maximize her objective (7). Further, this restriction does not compromise her ability to meet all constraints, including the fairness constraint (11).

**Proof:** Presented in Appendix A

Therefore, the regulator can, without loss of optimality, choose a tariff function that ensures that all $t(i)$ values are distinct. We can now decompose $R$’s problem neatly by making the observation that the objective function expressed in (7) depends only on $z$ and can therefore be solved independently of the constraints. Accordingly, we can carry out the following steps:

1. Find the value $z^*$ of $z$ that maximizes the objective function. We call this problem $P_1$.

2. Corresponding to the value of $z^*$ chosen, we can enumerate all underlying orderings over the $t(i)$ values that could have resulted in adoption outcome $\tilde{s}^{(z^*)}$. These feasible underlying orderings can be obtained by permuting the ordering of adopters (which we can do in exactly $|A^{(z^*)}|!$ different ways), and for each of these orderings, permuting the non-adopters (which we can do in $(I - |A^{(z^*)}|)!$ different ways). Let $O^{(z^*)}$ be the set of these orderings and let $o$ be an index into these orderings. Observe that $|O^{(z^*)}| = |A^{(z^*)}|!(I - |A^{(z^*)}|)!$. Note that fixing the ordering over the $t(i)$ values automatically fixes the marginal adopting class: pick the index corresponding to the adopting class chosen to have the lowest $t(i)$ value according to $o$. Let $m(o)$ be the marginal adopting class.

4 We model these incentive compatibility constraints as being strict rather than weak, because if $S$ deviates to a price $p_s \neq t(m)$, the outcome induced is different from the desired $z$. We will show in Section 4.3 that if the tariff structure is appropriately chosen, this strict inequality does not impair the feasibility of the problem.
3. Use the value of $z^*$ so obtained to solve $|O^{(z^*)}|$ different optimization problems (which we can index by $o$), one for each possible ordering, setting (11) as the objective. Choose an ordering with the best objective value. We call this problem $P_2$.

We can formally write these problems as follows:

**Problem $P_1$:**

$$\max_{z \in \{1, 2, \ldots, 2^I\}} \quad c_u^{(z^*)} (E_0 - E(z) - c_s E(z)) + m_u^{(z^*)} (E_0 - E(z) - m_s E(z)).$$

(16)

Once the optimal $z^*$ is obtained, solve the following $|O^{(z^*)}|$ optimization problems (which are now in canonical form) and choose the solution with the best objective value.

**Problem $P_2$:**

$$\min_{T(\cdot)} \quad \max_i s_i^{(z^*)} (T(d'_i, e_i, 1) + p_s g_i) + (1 - s_i^{(z^*)}) T(d_i, 0, 0) - p_r d_i$$

(17)

Subject to constraints:

$$t(i) = \frac{T(d_i, 0, 0) - T(d'_i, e_i, 1)}{g_i}, \forall i$$

(18)

$$\sum_{i=1}^{I} h_i \left( s_i^{(z)} (s_i^{(z)}) T(d'_i, e_i, 1) + (1 - s_i^{(z)}) T(d_i, 0, 0) - c_u^{(z)} (s_i^{(z)} - g_i) + (1 - s_i^{(z)}) d_i \right)$$

$$- \sum_{i=1}^{I} h_i (p_r - c_u^{E_0}) d_i = \Delta_U$$

(19)

$$t(i) \text{ ordering consistent with } o$$

(20)

$$t(m(o) - c_s) \sum_{i=1}^{I} s_i^{(z^*)} h_i g_i = \Delta_S$$

(21)

$$t(i) - c_s \sum_{j=1}^{I} \mathbb{I}_{t(j)>t(i) \text{ in ordering } o} h_j g_j < \Delta_S, \forall i \neq m(o)$$

(22)

Here, (21) is a set of $I - 1$ inequalities that impose an ordering of the $t(i)$ values consistent with $o$.

Observe that optimization problem $P_1$ always has a solution because it is unconstrained. However, it is not clear that $P_2$ is feasible. In particular, it is not immediately clear that the tariff parameters can ensure that the incentive compatibility constraints (23) and the ordering constraints (21) can hold together. In relation to this observation, we answer the following question: Let $\mathcal{T}$ be the set of allowable tariff functions from which $T(\cdot)$ must be chosen. How does the choice of $\mathcal{T}$ affect the feasibility of $P_2$? Further, if $P_2$ is feasible, can it induce an outcome free from cross-subsidization?
Definition 1. Cross-subsidization (CS): A market outcome is said to feature CS if the objective value of $P_2$ is positive, i.e., at least one class of customer is financially worse off in the post-solar case. Similarly, an outcome is free from CS if the objective value of $P_2$ is non-positive.

For an outcome to be free from CS, we naturally require that the total improvement in financial welfare for customers $\Delta_C = c_u E_0 - c_u^{E_u} (E_0 - E^{(u)}) - c_s E^{(s)} - \Delta_S - \Delta_U \geq 0$. We examine how the choice of $T$ affects the regulator’s ability to induce outcomes free from CS when $\Delta_C \geq 0$; specifically, is $\Delta_C \geq 0$ sufficient to induce such a CS-free outcome?

4.1. Non-tiered Tariff Structure that discriminates between adopters and non-adopters

Many states in the U.S. (including Nevada, which we examine more closely in Section 5) have utility companies that administer non-tiered rate schedules for residential customers. Non-tiered tariff structures have the benefit of being simple to administer and therefore simple to modify in the rate case proceedings, the process by which utility companies petition for rate changes to the PUC. These non-tiered structures can, however, discriminate between solar adopters and non-adopters, i.e., these two types of customers may be subject to different rate schedules, as is the case with NV Energy in Nevada. We study such rate structures in this section.

Let $T$ be the set of linear, non-tiered tariff structures that discriminate between adopters and non-adopters, i.e., they are on different rate schedules. $T$ has the following general specification:

$$T(d', e, 0) = r_d d + r_0; T(d', e, 1) = s_d d' + s_e e + s_0$$ (24)

We now present an analysis of the tariff structure (24). Specifically, we examine whether the feasibility of $P_2$ is guaranteed under this tariff structure. We prove the following propositions:

Proposition 2. Tariff structure (24) cannot guarantee the feasibility of $P_2$: there exist parameters and outcomes $z^*, \Delta_S, \Delta_U$ for which $P_2$ is not feasible for any ordering $o$.

Proof: Please see Appendix B

Proposition 2 implies that the system (8)-(13) does not always have a solution. While this is discouraging, the following proposition shows that under a restriction on $z^*$, there does exist a feasible ordering of $P_2$ if we drop the IC constraints.

Proposition 3. In the absence of the incentive compatibility constraints (13), there exists an ordering $o$ for which tariff structure (24) guarantees the feasibility of optimization problem $P_2$ if $z^* = \{i^*, i^* + 1, \ldots, I\}$ for some $i^*$. 

Proof: Please see Appendix C

This has an important implication: if the solar price $p_s$ were also controlled by the regulator $R$, and $z^*$ prescribes that a contiguous block of high-generation customer tiers adopts, a linear tariff structure with non-tiered rates would suffice to satisfy system (8)-(13), as the regulator would not have to contend with IC constraints (13). Alternatively, if the utility company itself offered rooftop solar rather than an outside firm, the IC constraints could be ignored (as the solar price set by $U$ would now be subject to regulation) and equation (10) suitably modified.

The intuition for this tariff’s failure to achieve feasible outcomes is its limited ability to transfer welfare among customers in different tiers. In particular, its ability to selectively make solar unattractive to some tiers and not to others is limited by not having tier-dependent parameters. The conditions laid out in Proposition 3 remove some of these hurdles.

While dropping the IC constraints is a special case under which the linear tariff structure suffices, the current environment in the US is one with an unregulated solar company. Therefore, a richer class of tariff structures may be required to satisfy the constraints of $P_2$.

4.2. Tiered tariff structure that does not discriminate between solar adopters and non-adopters

In states such as New Mexico (which we study in detail in Section 5) and Washington, the PUCs have mandated that solar customers may not be assessed any additional standby, capacity, interconnection, or other fee or charge by the utility (NC Clean Energy Technology Center 2017a,b). Such a rule serves as an incentive for solar adoption. These tariff structures may be tiered, but operate under a single rate schedule, and feature retail net-metering, whereby customers who adopt solar sell back excess electricity at their retail rate: If the utility repurchased electricity at less than their retail rate this would be considered a fee to solar adopters, and is thus prohibited. Let $T$ be the set of such tariff functions. We will show that operating such a tariff structure, while guaranteeing feasibility, limits the ability of the regulator to induce a CS-free market outcome.

Under such a tariff structure, the appropriate rate class (not to be confused with usage class $i \in \{1, 2, \ldots, I\}$) in the rate schedule is applied based on a household’s net demand. Recall that this net demand is $d - g$ if they adopt solar. For instance, if a tier 1 household does not adopt solar, demands an amount of electricity $d_1$, and is placed in rate class 1, a tier 2 household who adopts solar and has a net demand of $d_2 - g_2 = d_1$ also falls into rate class 1 and is billed as such.

Let $C = \{1, 2, \ldots, |C|\}$ be the set of indices corresponding to rate classes in $U$’s rate schedule. Arrange this set in order of increasing (net) demand, i.e., rate class 1 corresponds to the lowest net
demand and rate class $|C|$ corresponds to the highest net demand. In order to support customers making endogenous solar adoption decisions, this rate schedule must contain enough rate classes to support any possible adoption/non-adoption decision by customers. Therefore, $|C| \geq I + 1$.

Now, consider the tariff function $T(d', e, s) = T(c) = r_c + f$, where $c$ is the index of the rate class to which a customer with usage profile $(d', e, s)$ belongs; this is obtained by mapping the household’s net energy usage ($d$ for a non-adopter and $d' - e = d - g$ for a non-adopter) to a class $c \in C$, and $f$ is a fixed cost that all customers pay. The tariff function is fully defined by choosing $r_c, \forall c \in C$, and $f$. Note that having the fixed cost $f$ also depend on class, i.e., having a different fixed cost $f_c$ for every class $c$, is equivalent to the system currently under consideration, as $r_c$ can be suitably modified for each class $c$ to compensate for the difference $f_c - f$.

**Proposition 4.** Corresponding to every ordering $o$ and outcome $z^*$, there exists a feasible rate schedule $(r_c, f)$ that satisfies the constraints of $\mathcal{P}_2$.

**Proof:** Presented in Appendix D

While this tariff structure can always feasibly induce an outcome characterized by $z^*, \Delta_S, \Delta_U$, we find that it cannot guarantee a CS-free outcomes when $\Delta_C \geq 0$.

**Proposition 5.** The tiered tariff structure that does not discriminate between adopters and non-adopters cannot guarantee CS-free outcomes: there exist parameters and outcomes $z^*, \Delta_S, \Delta_U$ for which no CS-free outcome can be generated for any ordering $o$ even when $\Delta_C \geq 0$.

**Proof:** Presented in Appendix E

Therefore, while this tariff structure is simple and guarantees feasibility, it does not have desirable properties with respect to customer equity. Intuitively, this tariff fares better than the non-tiered tariff studied in Section 4.1 because its tiered nature allows welfare transfer among tiers. However, its ability to shield customers from cross-subsidization is limited by the fact that adopters and non-adopters may be grouped into the same tier. Therefore, a tariff structure that can guarantee CS-free outcomes must (at least) be able to discriminate between solar adopters and non-adopters by placing them in different rate schedules.

### 4.3. Tiered tariff structure that differentiates between solar adopters and non-adopters

As an alternative to the class of tariff structures explored in Sections 4.1 and 4.2, consider a tiered tariff structure that presents different rate schedules to solar and non-solar customers. Both these attributes are present in the tariff structure operated by Arizona’s APS. We propose and study a specific tariff structure $\tau$ with these attributes: one that is non-tiered for non-adopters and tiered.

5 If the Period 0 tariff structure is tiered rather than having the same rate $p_{r0}$ apply to all customers, our proposed tariff structure would require that non-adopters also face tiered rates.
for adopters. Under our structure, solar and non-solar customers are charged the same variable energy charge based on their net energy consumption\(^6\), but a solar customer is also assessed a fixed annual charge that depends on her class \(i\). \(\tau\) has the following general specification:

\[
T(d_i, 0, 0) = p_r d_i; T(d'_i, e_i, 1) = p_r(d'_i) - p_r(e_i) + f_i = p_r(d_i - g_i) + f_i. 
\] (25)

Note that \(U\) can infer a solar customer’s class by observing the value of \(d'_i\), the energy drawn from the grid, and contract on a fixed cost that a customer would pay based on this amount. We now present an analysis of tariff structure (25): we show that the feasibility of \(P_2\) is guaranteed under such a tariff structure, and that it can also guarantee no CS if \(\Delta C \geq 0\).

Consider problem \(P_2\) for a given ordering \(o\) and a given adoption outcome \(z^*\). It will be useful to re-order the indices to be consistent with \(o\), that is, the indices are chosen such that \(t(i) < t(j), \forall i < j\). As a result of this re-ordering, classes 1, \ldots \(m-1\) do not adopt solar, while classes \(m, \ldots, I\) adopt, where \(m\) is the index of the marginal adopter. Note that now, we no longer have \(g_i < g_j, \forall i < j\).

\[
t(i) = p_r - \frac{f_i}{g_i}, \forall i
\] (26)

\[
\sum_{i=1}^{m-1} h_i(p_r - c_u^{(z^*)})d_i + \sum_{i=m}^{l} h_i(p_r(d_i - g_i) + c_u^{(z^*)}(d_i - g_i) - p_r e_i + f_i)
\]

\[
-\sum_{i=1}^{l} h_i (p_e - c_{E0})d_i = \Delta U
\] (27)

\[
t(i) < t(j), \forall i < j
\] (28)

\[
(t(m) - c_s) \sum_{i=m}^{l} h_i g_i = \Delta S
\] (29)

\[
(t(i) - c_s) \sum_{j=i}^{l} h_j g_j < \Delta S, \forall i \neq m
\] (30)

Because this tariff structure has more parameters than the non-tiered tariff structure in (24), it might at first seem intuitive (and trivial) that this structure guarantees feasibility of \(P_2\): one could correctly make the observation that the fixed costs for adopting customer classes \(i\) can be chosen arbitrarily to adjust the \(\Delta U\) up and down. However, these fixed costs must also result in the solar price of \(p_s = t(m)\) being an incentive compatible choice for \(S\): picking a solar price \(p_s \neq t(m)\) must result in a smaller profit (the product of margin and volume, see equation (30)) than her profit from picking \(p_s = t(i)\). This deviation can be made sufficiently unattractive by ensuring that values

\(^6\)Note that since solar adopters are charged based on net energy use, we implicitly prescribe retail net-metering.
have been set, substitute them in (27) to obtain a value for $p_k$ kWh of usage that depend on the usage level. We now study properties of this tariff structure.

This is a commonly used tariff, where a customer’s rate schedule features varying energy rates per a tariff featuring the same fixed costs across tiers, with energy costs varying across adopting tiers. by tier for solar customers. Similar to our discussion in Section 4.1, $R$ could equivalently impose a tariff featuring the same fixed costs across tiers, with energy costs varying across adopting tiers. This is a commonly used tariff, where a customer’s rate schedule features varying energy rates per kWh of usage that depend on the usage level. We now study properties of this tariff structure.

**Proposition 6.** Corresponding to every ordering $o$ and outcome $z^*, \Delta_U, \Delta_S$, there exists a feasible tariff function of the form (25) that satisfies the constraints of $\mathcal{P}_2$.

**Proof:** First, find $f_m$ as a function of $p_r$ by solving equation (29) to obtain $t(m) = \left(c_s + \frac{\Delta_S}{\sum_{j=m}^h g_j} \right)$ and $f_m = g_m (p_r - t(m))$. For all $i < m$, set $t(i) = \epsilon i$ for some arbitrarily small $\epsilon > 0$ by setting $f_i = g_i (p_r - \epsilon i)$. For all $i > m$, set $t(i) = \left(c_s + \frac{\Delta_S}{\sum_{j=i}^h g_j} \right)$ and $f_i = g_i (p_r - t(i))$. Note that by definition, this set of $t(i)$ values satisfies (26) and (29). Further, IC constraints (30) are satisfied because the $t(i)$ values for $i > m$ are set $\epsilon$ smaller than what they would have to be to obtain a profit of $\Delta_S$, and for $i < m$ are set so low that they generate a negative profit. Finally, the ordering constraints (28) are satisfied for $i < m$ by definition, and for $i \geq m$ (if the $\epsilon$ term is sufficiently small) because $h_j g_j > 0, \forall j$, leading to $\sum_{j=i}^h g_j$ decreasing in $i$. Once these $f_i$ values have been set, substitute them in (27) to obtain a value for $p_r$. Q.E.D.

This tariff (25) features the same energy (variable) cost across tiers, and varying fixed costs $f_i$ by tier for solar customers. Similar to our discussion in Section 4.1, $R$ could equivalently impose a tariff featuring the same fixed costs across tiers, with energy costs varying across adopting tiers. This is a commonly used tariff, where a customer’s rate schedule features varying energy rates per kWh of usage that depend on the usage level. We now study properties of this tariff structure.

**Lemma 1.** Consider an adopting tier $i$. Then, we have that $g_i p_r - g_i p_s - f_i \geq 0$.

**Proof:** If $m$ is the index of the marginal class, $p_s = p_r - \frac{I_m}{g_m}$. For all tiers $i$ that adopt, it must be the case that $t(m) \leq t(i) \Leftrightarrow \frac{I_m}{g_m} \geq \frac{I_i}{g_i}$ (from equations (26)). Therefore, we have $g_i p_r - g_i p_s - f_i = g_i p_r - g_i (p_r - \frac{I_m}{g_m}) - f_i = g_i \frac{I_m}{g_m} - f_i \geq 0$.

Q.E.D.

We now address properties that this tariff structure exhibits with respect to customer equity and CS. Lemma 1 will be useful in examining these properties.

**Proposition 7.** When $\Delta_C > 0$, there exists an ordering $o$ such that tariff structure (25) can induce an outcome that is CS-free. When $\Delta_C = 0$, tariff structure (25) can induce an outcome that is arbitrarily close to being CS-free.

**Proof:** Presented in Appendix F.

Under some conditions, we can provide a closed form characterization of the solution to $\mathcal{P}_2$. 
Proposition 8. The solution to \( P_2 \) can be characterized in closed form when \( \Delta_C > 0 \) and \( l : d_i \leq d_i \forall i \) is such that \( s_i^{(z^*)} = 0 \), that is, the lowest demand tier does not adopt solar.

Proof: Presented in Appendix G.

Therefore, our proposed tariff structure (25) is guaranteed to be feasible, has desirable properties with respect to customer equity, and its solution can even be characterized in closed form when adopting solar creates a customer surplus.

4.3.1. Practical Issues/Extensions Some practical hurdles stand in the way of implementing tariff structure (25). We now discuss these issues and how they may be remedied.

1. Demand, generation and excess are not actually deterministic: Our model assumes deterministic values for households \( h_i \), demand \( d_i \), generation capacity \( g_i \), and excess \( e_i \). However, this will not be true in practice: households’ usage and generation parameters are random variables. We can relax the deterministic assumption by allowing individual households to draw \( d, g, \) and \( e \) from a discrete distribution with \( I \) points in its support. We define a joint probability mass function \( \phi(d, g, e) \) over the space of possible values of \( d, g, \) and \( e \). Let us index the support of \( \phi(\cdot) \) by \( i \). Now, \( h_i = H \ast \phi(d_i, g_i, e_i) \), where \( H \) is the total number of households in the market. In expectation \( h_i \) customers will have demand \( d_i \), generation \( g_i \) and excess \( e_i \). Thus, with this definition of \( h_i \), we can reformulate programs \( P_1 \) and \( P_2 \) in terms of expectations, and \( h_i \) customers will make a decision consistent with the deterministic tier \( i \) decision.

2. Not all customers belonging to a class that is induced to adopt solar will actually adopt: For various reasons including access to liquidity and inertia to change, some customers belonging to a tier designated to adopt solar might not actually make the adoption decision. We can easily relax this assumption. Let \( \pi_i \) be an exogenously given expected fraction of households in tier \( i \) that would adopt solar if economically viable (endogenous determination of \( \pi_i \) is beyond the scope of this model). The remaining \( 1 - \pi_i \) fraction of households in tiers \( i : s_i^{(z^*)} = 1 \) continue to fulfill their demand from the utility directly, because they are unwilling to adopt solar. We can then reformulate our model by replacing \( h_i \) by \( \pi_i h_i \) in the solar profit equations, and adjusting the utility company’s rate-of-return equation appropriately.

3. The tariff structure may incentivize customers to exaggerate their demand to generate bill savings: It is undesirable for a tariff structure to incentivize a type \( i \) customer to “spoof” a type \( j \) customer by exaggerating her demand in order to generate bill savings,
that is, her total outflow (to $U$ and $S$) is reduced by spoofing another class\textsuperscript{7}. Therefore, we must consider the four possible ways a customer type may spoof another customer type.

(i) **Non-Solar to Non-Solar**: Non-solar customer $i$ can appear to be non-solar customer $j$ by exaggerating her demand to $d_i = d_j > d_i$. However, doing so increases her bill from $p_i d_i$ to $p_i d_j > p_i d_i$, so she will not do so.

(ii) **Non-Solar to Solar**: Non-solar customer $i$ can appear to belong to a tier $j$ that adopts solar by installing solar that generates $g_i$ and appearing to have a grid usage of $d_j' = d_j - g_j + e_j$. To do so, she must alter her demand to $d_i' = (d_j - g_j + e_j) + g_i - e_i$. This is undesirable if $d_i' > d_i \Rightarrow (d_j - g_j + e_j) > (d_i - g_i + e_i)$. To prevent this from happening, we must ensure that $p_r d_i < p_r (d_i - g_i) + f_i + p_s g_i = p_r (d_i - g_i + e_i) + f_j + p_s g_i$. But we have that $p_r d_i < p_r (d_i - g_i) + f_i + p_s g_i$. Therefore, it is sufficient for us to specify parameters such that:

\begin{equation}
    p_r (d_i - g_i) + f_i < p_r (d_j - g_j + e_j - e_i) + f_j.
\end{equation}

(iii) **Solar to Non-Solar**: Solar customer $i$ can appear to be a non-solar customer $j$ by altering her demand $d_i$ to $d_j$. This is undesirable if $d_j > d_i$. To prevent this from happening, we must ensure that $p_r (d_i - g_i) + f_i + p_s g_i < p_r d_j$. But since tier $i$ adopts, we have that $p_r (d_i - g_i) + f_i + p_s g_i \leq p_r d_j$. Since $d_j > d_i$, $p_r d_i < p_r d_j$, and therefore, such a customer $i$ will not spoof a non-solar customer $j$.

(iv) **Solar to Solar**: Solar customer $i$ can appear to be another solar customer $j$ by appearing to have a grid usage of $d_j' = d_j - g_j + e_j$. To do so, she must alter her demand to $d_i' = (d_j - g_j + e_j) + g_i - e_i$. This is undesirable if $d_i' > d_i \Rightarrow (d_j - g_j + e_j) > (d_i - g_i + e_i)$. To prevent this from happening, we must ensure that $p_r (d_i - g_i) + f_i + p_s g_i < p_r (d_i - g_i) + f_j + p_s g_i$, which is identical to the condition in inequality (31).

Such spoofing behaviors can be eliminated in various ways. For example, constraint (31) could be added to $\mathcal{P}_2$ for classes falling into (ii) and (iv) above, that is, for all classes $i, j$ such that $d_j - g_j + e_j > d_i - g_i + e_i$ and either $i$ is a non-adopter and $j$ is an adopter, or both $i$ and $j$ are adopters. Whether $\mathcal{P}_2$ remains feasible under these constraints depends on the specific parameters under consideration. We show in Section 5 that $\mathcal{P}_2$'s feasibility and CS outcome is unaffected by the inclusion of these constraints for realistic parameter values. If the regulator

\textsuperscript{7} We do not discuss the case of a customer curtailing her demand in order to generate bill savings because we do not model demand response and the costs associated with curtailing demand.
chooses not to enforce this constraint in $P_2$, she can use it to check if the solution obtained is exposed to such spoofing behavior. Alternatively, $U$ could assign a class not just by measuring grid usage, but also by measuring net demand $d_i - g_i$ and assigning a class on the basis of both these measurements. Such a measurement would deter spoofing behavior, because except under pathological parameter values, simply exaggerating demand will not allow a type $i$ customer to mimic a type $j$ customer on both these dimensions.

4. **The tariff structure may incentivize customers to install solar capacity strictly less than $g_i$:** It is undesirable for this tariff structure to induce solar customers to install solar capacity smaller than $g_i$. Observe that if a tier $i$ customer’s generation is reduced to $\tilde{g}_i < g_i$, her excess also reduces to some $\tilde{\epsilon}_i \leq \epsilon_i$. This might or might not alter her tier (which, recall, is measured by measuring her grid usage $d - (g - \epsilon)$). Suppose it does not result in a tier alteration. In this case we can impose the constraint $p_s < p_r$ so that installing less than capacity $g_i$ will increase a customer’s bill by an amount $p_r$ per unit of generation foregone and this bill increase is not compensated for by having to purchase less from $S$. To ensure that $p_s < p_r$, we impose constraint $f_m > 0$, so that $p_s = l(m) = p_r - f_m/g_m < p_r$.

But what if the customer was able to change her tier, and therefore, the applied fixed cost by installing panels to less than capacity? Let us assume that for every possible $\tilde{g}_i < g_i$, $R$ can infer the resulting excess $\tilde{\epsilon}_i$ based on the customer’s usage profile over the day. Such a customer can spoof a tier $j$ customer by choosing capacity $\tilde{g}_i$ such that $d_i - \tilde{g}_i + \tilde{\epsilon}_i = d_j - g_j + \epsilon_j$. For all such pairs of $\tilde{g}_i, \tilde{\epsilon}_i$ values, impose constraint (32), in addition to $f_m > 0$.

$$ p_r(d_i - g_i) + f_i + p_s g_i < p_r(d_i - \tilde{g}_i) + f_j + p_s \tilde{g}_i \quad (32) $$

Note that we need not consider the possibility of a customer altering both $d_i$ and $g_i$ to exaggerate her grid usage $d'_i$: Decreasing $g_i$ by one unit increases her cash outflow by $p_r - p_s$, while increasing $d_i$ by one unit increases her cash outflow by $p_r$, which is larger than $p_r - p_s$.

We shall see in Section 5 that for realistic parameter values, $p_s < p_r$ because the prescribed fixed cost $f_m$ for the marginal tier $m$ is positive. We shall also see in Section 5 that for realistic parameter values, moving to a higher tier causes customers to incur higher fixed costs, and therefore, reducing $g_i$—even to effect a change in tier—is not beneficial.
5. Numerical Analysis

Now, using data from the states of Nevada and New Mexico—where regulatory changes threaten the rooftop solar industry—we study how the tariff structures in operation compare to the two-part tiered tariff structure presented in (25). We first discuss our approach to estimating the parameters $d, g, e,$ and $h$ in Section 5.1. Using these parameters, in Section 5.2 we numerically investigate the performance of using the specific versions of the non-tiered, linear tariff structure proposed in the states of Nevada and New Mexico. We contrast these to results obtained if the states were to adopt our proposed tiered two-part tariff structure. As we will see, while both states’ tariffs are able to feasibly generate the outcome $z^*$ specified by $\mathcal{P}_1$, they induce poor customer equity outcomes in problem $\mathcal{P}_2$.

5.1. Estimating the Parameters

Our starting point for estimating the parameters of the system is the household micro-data from the US Energy Information Administration’s Residential Energy Consumption 2009 Survey (U.S. Energy Information Administration 2009). This survey contains responses from 112 single-family housing units in the states of Nevada and New Mexico (that appear as a single group in the data). For each housing unit, we estimate the value $g_i$: we take the total area of each house in square feet, as reported in the data set, and divide by the number of stories to obtain an estimate of the total roof area. Then, we assume that each square foot of panel area can generate 9 Watts of electricity when the sun is shining (Solar-Estimate 2012) to obtain the rated power output of solar panel installations. This estimate is an approximation of installed capacity, because total household area reported in the survey also includes basements and attics, where they are present. Moreover, not all available roof space is typically usable for solar panels. Accordingly, we correct this estimate (as a first approximation) by a single multiplicative factor such that installation sizes so obtained are roughly in the 3 kWh - 10 kWh range (Fu et al. 2016). We multiply this installation capacity by 2190, the estimated number of hours of sun in Nevada and by 2471 hours of sun for New Mexico to estimate the generation per year for households in each state (SolarDirect 2016). We group these households into 4 roughly equally sized buckets ($i = 1 \ldots 4$) based on their generation capacity. Within each bucket $i$, we compute the average energy demanded, $d_i$ and the average generation capacity $g_i$. Bucket size $h_i$ is computed as the proportion of these households belonging to generation bucket $i$ (we normalize the total number of households to 1). Since $\sum_i h_i = 1$, $\Delta_s$, $\Delta_C$, and $\Delta_U$ are measured per-household.
Using the US Department of Energy data on hourly residential load in a typical meteorological year for cities in Nevada and New Mexico (US Department of Energy 2013), we find that residences in Nevada typically consume about $\mu = 29\%$ of their demand between the hours of 11 a.m. and 6 p.m., typical hours for solar reliance. This figure is about $\mu = 34\%$ for New Mexico. Since we do not have this data broken up by household class, we assume that all houses consume this proportion of their demand when the sun is shining, and use this to estimate $e_i$, the excess generation they would be able to sell back to the grid as $\max(0, g_i - \mu d_i)$.

The results of this exercise are presented in Appendix H. We use these parameters in our analysis.

5.2. Comparison of tariff structures

We now wish to compare the tariff structures in operation in Nevada and New Mexico to our suggested two-part tiered tariff structure. We are particularly interested in understanding how these tariff structures perform with respect to their ability to induce CS-free outcomes. To do so, we set $\Delta_C = 0$ and compare tariff structures. (If $\Delta_C < 0$, then CS is unavoidable; as $\Delta_C$ increases above zero, CS-free outcomes become easier to induce.) We ignore the federal investment tax credit because it will ramp down starting in 2019 (Energy.gov 2017).

As noted in Section 4, not all customers who are financially incentivized to install solar do so; we attempt to capture this inertia in our experiments, by letting $\pi_i$ be the proportion of class $i$ households that adopt solar if financially prudent. Equivalently, this may be interpreted as the probability that a class $i$ household that is financially incentivized to adopt solar does so. We estimate a reference level of $\pi_i$ for each class and state in our data set using data from the preliminary version of US Energy Information Administration’s Residential Energy Consumption 2015 Survey (U.S. Energy Information Administration 2015)\(^8\), which also has a flag indicating whether the surveyed households have installed rooftop solar. Specifically, we use data from the Pacific Census Region, where households have had significant federal and state incentives to adopt solar (Borlick and Wood 2014).

We scale these $\pi_i$ values to reflect optimistic and pessimistic adoption scenarios. Under the optimistic scenario, if all classes adopt at the specified $\pi_i$ levels, 3% of all residential energy, the current level in California, (California Distributed Generation Statistics 2017, Ivan Penn 2017) will be supplied by rooftop solar. Under the pessimistic scenario, if all classes adopt at these $\pi_i$ levels, 1% of all residential energy will be supplied by rooftop solar. Appendix I shows the resulting $\pi_i$.

\(^8\)The complete version of this data set is still not available.
levels. To study this situation with partial adoption within a class we reformulate $P_1$ so that $E^{(z)}$ values (the migration quantity corresponding to outcome $z$) accurately reflect the change in energy demanded from $U$ given adoption levels $\pi_i$. A reformulated version of $P_2$ is presented in Appendix K.

In our numerical study, we use a $c_s$ value of $0.059074$ for Nevada $0.052356$ for New Mexico. We compute this using a 30 year lifetime (SolarCity 2016c) for solar systems that produce power at 200 Watts per panel (SolarCity 2016b), with output degrading at a rate of 0.5% per year (Jordan et al. 2010) and a per Watt cost of $2.89$.

Next, we turn to estimation of $c_{u}^{E_{0}}$. Using financial disclosures from NV Energy in Nevada (NV Energy 2016a,b) and PNM Energy in New Mexico (PNM Energy 2017), we estimate $c_{u}^{E_{0}}$ to be $0.035$ in Nevada $0.030$ in New Mexico. Since natural gas and other fossil fuels are the major source of fuel in Nevada (81%) and New Mexico (66%), we estimate parameters for the post-solar case by assuming that rooftop solar generation displaces natural gas. We obtain the mix of energy sources used in Nevada from U.S. Energy Information Administration (2016a) and for New Mexico from U.S. Energy Information Administration (2016b). In order to compute $c_{u}^{E_{0}}$, we use the levelized cost estimates of various sources of electricity from Lazard (2017), scaled by a multiplicative factor to make them consistent with $c_{u}^{E_{0}}$. In order to estimate the values of $m_{u}^{z}$, we use the lifecycle greenhouse gas emissions of different energy sources published in Intergovernmental Panel on Climate Change (2014) and a social cost of Carbon of $59.03$ per metric ton of CO$_2$, which we obtain by correcting the 2030 social-cost-of-Carbon estimate of $50$ 2007 dollars per metric ton of CO$_2$ (Environmental Protection Agency 2017) for inflation. Of the years for which estimates are available, 2030 falls closest to the middle of the typical 30 year useable lifetime of a solar panel.

We now individually consider the cases of Nevada and New Mexico. Since we are interested in studying customer equity outcomes, we set $\Delta_S = 0.0001$, and set $\Delta_U$ so that $\Delta_C = 0$ according to equation $\Delta_C = c_{u}^{E_{0}}E_0 - c_{u}^{E_{0}}E^{(z)}(E_0 - E^{(z)}) - c_sE^{(z)} - \Delta_S - \Delta_U$. We compare the performance of our tariff structure and the tariff structure in operation on both the pessimistic and the optimistic adoption scenarios. For each state, for each type of scenario, we carry out the following steps:

1. Enumerate the objective value of $P_1$ for all 16 possible outcomes (each tier may either be incentivized to adopt or not). Choose the $z^*$ with the best objective value.

2. Corresponding to this solution $z^*$, find the solution of $P_2$ for all possible orderings $o$ using the tariff structure in operation in the state. Choose the solution with the best objective value.
3. Corresponding to this solution $z^*$, find the solution of $P_2$ for all possible orderings $o$ using our suggested two-part tiered tariff structure, including the constraints required to ensure that customers are not encouraged to spoof their demands or generation amounts to generate bill savings. Choose the solution with the best objective value.

5.3. Nevada

For both the optimistic and pessimistic scenarios in Nevada, we find that the solution to $P_1$ corresponds to all four classes of customers adopting solar. In the optimistic scenario, the objective value of $P_1$ is $1.46, and for the pessimistic scenario, $0.49. Recall that these are normalized net welfare improvements per-household.

NV Energy in Nevada operates a non-tiered tariff structure. After the changes in 2015, NV Energy published a memo to its customers, providing an overview of the changes they would see in their bill (NV Energy 2015). According to these changes, solar customers would be compensated at less than retail rates for excess energy sold back to the grid, and would pay different fixed costs from non-solar customers. However, there was to be no change to the energy charge (the per-unit) energy rate for solar and non-solar customers. Accordingly, we use tariff structure (24), restricting $r_d$ to be equal to $s_d$. We use a value of $p_{r0} = 0.10182$, the average value of the rates in Northern and Southern Nevada (NV Energy 2017). The results of our experiments are summarized in Table 1. We see that in both the optimistic and pessimistic scenarios, the objective value of $P_2$ using the current non-tiered tariff is quite large—indicating the presence of CS—compared to that of our suggested tariff (where it is near 0). Therefore, this outcome constitutes a situation of CS. We see that tier 1 and 2 customers are adversely affected by a move to solar. Further, the most strongly impacted households are tier 1 customers, who are also potentially the poorest customers with smaller houses and lower demand.

5.4. New Mexico

As was the case with Nevada, for both the optimistic and pessimistic scenarios in New Mexico, we find that the solution to $P_1$ corresponds to all four classes of customers adopting solar. In the

<table>
<thead>
<tr>
<th>Class</th>
<th>Old Bill</th>
<th>Adopter</th>
<th>Non-adopter</th>
<th>Adopter</th>
<th>Non-adopter</th>
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<th>Non-adopter</th>
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<td>$1,349.78</td>
</tr>
</tbody>
</table>

Table 1 Bill Comparisons - Nevada Optimistic and Pessimistic Scenarios
optimistic scenario, the normalized objective value of $P_1$ is $1.97$, and for the pessimistic scenario it is $0.66$.

New Mexico’s net-metering regulations have also seen some recent opposition (Robert Walton 2016). As it stands, New Mexico operates a tiered retail net-metering tariff that does not allow discrimination between solar and non-solar households (NC Clean Energy Technology Center 2017a); the tariff structure treated in Section 4.2. In order to set up this tariff structure, recall that our estimates in Table 4 categorized customers in New Mexico into four tiers. We use values of $p_{r0} = \$0.08822, \$0.09003, \$0.09705, \text{ and } \$0.09858$ corresponding to rates that would apply to tiers 1, 2, 3, and 4 respectively (PNM Energy 2018). For convenience, we choose rate class boundaries close to the average of the tier demand values $d_i$. Specifically, we let rate class 1 apply to customers who demand up to 9300 kWh, rate class 2 apply to customers who consume between 9300 and 11550 kWh, rate class 3 apply to customers who demand between 11550 and 13750 kWh, and rate class 4 apply to customers who demand more than 13750 kWh.

Now, we consider the modifications required to the tariff structure in the post-solar case. Recall that a class $c$ customer’s bill under this tariff structure is of the form $r_c n + f$, where $n$ is the net demand of the customer being considered. From the estimates of $d$ and $g$ in Table 4, we compute the net demand $(d - g)$ values for all tiers, if they were to adopt. A tier 1 customer who adopts has a net demand of 3031 kWh, a tier 2 customer who adopts has a net demand of 1482 kWh, a tier 3 customer who adopts has a net demand of 2356 kWh, and a tier 4 customer who adopts has a net demand of −2001 kWh (tier 4 customers are net suppliers). In order to give additional flexibility to the New Mexico tariff, we add a fifth rate class that applies to net demands of up to 5900 kWh. This fifth rate class will then apply to all households that adopt solar\textsuperscript{9}. The table in Appendix J summarizes the rate classes under consideration.

We compare New Mexico’s tiered tariff structure to a variant of our tariff structure discussed in Section 4.3, where we have fixed costs that vary by tier in order to accommodate the fact that $p_{r0}$ varies by tier (please see footnote 5 in Section 4.3). We also impose constraints to make rates $r_i, \forall i$ positive. The results of our experiments are summarized in Table 2. Under both tariffs, the solutions are identical the pessimistic and optimistic scenarios. These show a similar situation to Nevada’s where tiers 1 and 2 are affected. Here, the most adversely affected customers are those in tier 2.

In summary, although Proposition 2 showed that $P_2$ might not be feasible under the non-tiered

\textsuperscript{9} Adding a new rate class for each net-demand usage level will implicitly enable the New Mexico tariff to distinguish between solar and non-solar customers, making it equivalent to our tariff.
Two-part Tiered Tariff Outflows

<table>
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<th>Class</th>
<th>Old Bill</th>
<th>Adopter</th>
<th>Non-adopter</th>
<th>Current Tiered Tariff Outflows</th>
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</thead>
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<td>$1,406.79</td>
<td>$1,406.79</td>
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</tr>
</tbody>
</table>

Table 2 Bill Comparisons - New Mexico Optimistic and Pessimistic Scenarios

tariff structure (24), we did not encounter infeasibility in our experiments for Nevada. However, while feasible, the non-tiered linear tariff structures in Nevada and New Mexico performed poorly compared to our tiered tariff structure with respect to their ability to avoid CS: In Nevada, under both the pessimistic and optimistic scenarios, the customers most affected by the introduction of solar belonged tier 1. These are the households with the smallest rooftops, possibly housing lower-income residents. In New Mexico, the most adversely affected customers were customers in tier 2, which shows that no class of rate-payers can be assured of not being forced into cross-subsidization. In contrast, our tiered tariff structure was able to avoid CS even after we imposed constraints to prevent customers from exaggerating their demand to realize bill savings. These experiments serve to illustrate the crucial role that tariff structure plays in helping to ensure equity across customers.

6. Conclusions and Future Work

There has been considerable regulatory flux associated with rooftop solar energy in the past few years. The regulator’s task of trading off the interests of the utility company, the solar company, different consumers and society at large is clearly a challenging one. In some cases regulatory changes have had dire consequences: changes introduced in the state of Nevada all but killed the solar industry in the state.

In this paper we study the regulator’s problem of choosing a tariff function to induce a socially optimal outcome. The regulator takes into account financial and environmental considerations, and operates in a setting with a monopolistic, price-setting solar company and customers who individually decide whether or not to install solar. We pose the regulator’s decision as an optimization problem which we show can conveniently be hierarchically separated into two subproblems. We show analytically that the tariff structure chosen must have the ability to discriminate across customer usage tiers and the ability to discriminate between solar and non-solar customers in order to guarantee feasibility. We present a tariff structure with both these features and show that in addition to guaranteeing feasibility, it also can guarantee outcomes free from cross-subsidization when solar adoption generates a surplus for the customer base. The implication is that regulators must migrate to tiered tariff structures and put solar and non-solar customers on different rate
schedules in order to induce socially optimal outcomes. While states such as Arizona already seem
to be doing this, most utility tariffs in the U.S. do not have both these features.

This work lays the foundation for several research extensions. For instance, our model considers
a flat-rate pricing scheme, rather than a time-of-use (TOU) pricing scheme such as the one being
rolled out in California. While TOU is yet to gain significant traction in the U.S., modeling this
explicitly could eventually become critical to analyzing policy decisions related to solar, as having
a TOU pricing scheme incentivizes customers to shift their demand profiles temporally (demand
response). Capturing demand response would also modify our model’s current assumption that the
aggregate demand for electricity is unaffected by solar adoption. In addition, our work models the
solar company as being monopolistic. Another possible avenue for future research is to explicitly
model competition in the solar marketplace, with the utility itself potentially being a competitor
in the solar domain. Similarly, while our model implicitly captures the impact of utility capacity
investments in response to solar adoption through $c_u^x$ and $m_u^z$, one could consider a model in which
these decisions are explicit outputs of the model. And, on the customer side, we could modify
the assumption that all customers (or an exogenously defined proportion) who are incentivized to
adopt solar do so. An interesting extension would be to endogenously determine the proportion of
customers in a tier that would adopt solar as a function of savings generated.

While this work explores a static setting, related questions can be explored in a dynamic setting,
complementing literature such as Babich et al. (2017) and Lobel and Perakis (2011). Such work
would require a significantly different model that captures the diffusion of solar among customers,
the interaction between solar adoption penetration and solar cost, and utility capacity investment
decisions. If such work also continued to capture heterogeneity in the customer base, this would
necessitate more nuanced modeling of the regulator’s optimization problem.

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7. Supplementary Material
Appendix A: Proof of Proposition 1

First, notice that $R$’s objective function does not depend directly on $T(\cdot)$, but only on $z$. Therefore, if she can choose a value of $z$ that maximizes her objective function, and find $M$ and $T(\cdot)$ that satisfy her constraints (and therefore induce adoption outcome $z$), her ability to maximize her objective is not compromised by this restriction. Fixed a value of $z$ and consider a solution that does not follow this restriction. We now show that this solution can be converted to one that respects this restriction all of $R$’s constraints.

Let $O$ be the set of indices $i$ such that $\exists j : t(i) = t(j)$. Partition $O$ into $O_n$, the set of indices in $O$ corresponding to non-adopting classes, $O_{an}$, the set of indices in $O$ corresponding to adopting classes that are not marginal classes, and $O_{am}$, the set of indices in $O$ corresponding to adopting classes that are marginal. Now, carry out the following steps.

1. Sequentially, for each $i \in O_n$, decrease $t(i)$ by a small value $\epsilon_i > 0$ by appropriately increasing $T(d_i', e_i, 1)$. Note that doing this does not affect (11) because this equation is only affected by $T(d_i, 0, 0)$ for non-adopting classes. Further, since $\epsilon_i > 0$, we can ensure that the tariff restriction is respected, the inequalities in (13) continue to be respected, and the inequalities in (15) continue to be respected.

2. Sequentially, for each $i \in O_{an}$, decrease $t(i)$ by a small value $\epsilon_i > 0$ by appropriately decreasing $T(d_i, 0, 0)$. Note that doing this does not affect (11) because this equation is only affected by $T(d_i', e_i, 1)$ for adopting classes. Further, if $\epsilon_i$ is small enough and appropriately chosen, we can ensure that the tariff restriction is respected, the inequalities in (13) continue to be respected, and the inequalities in (14) continue to be respected because $t(i)$ is still larger than $t(m)$.

Now, consider the set $O_{am}$. Of these, choose exactly one class $m$ to be the marginal class. For all $i \in O_{am} \setminus m$, sequentially increase $t(i)$ by a small value $\epsilon_i$ by appropriately increasing $T(d_i, 0, 0)$. Note that doing this does not affect (11) because this equation is only affected by $T(d_i', e_i, 1)$ for adopting classes. Further, if $\epsilon_i$ is small enough and appropriately chosen, we can ensure that the tariff restriction is respected. Since these $t(i)$ values are larger than $t(m)$ the incentive compatibility constraints (13) corresponding to these classes will now be respected for a small enough $\epsilon_i$, because if $S$ sets a price $t(i)$, since $t(i)$ is now larger than $t(m)$ class $m$ does not adopt, therefore decreasing the quantity in the summation term of (13) by at least $h_m g_m$. Naturally, (14) continues to hold as $t(i)$ is now larger than $t(m)$. Q.E.D.
Appendix B: Proof of Proposition 2

We prove Proposition 2 by showing the following counter-example with three classes of customers. Under this tariff structure, we have \( t(i) = \frac{d_i r_d + r_o - ((d_i + s_e - g_i) s_d + c_i e_s + s_o)}{g_i} \). Let \( z^* \) specify that classes 2 and 3 adopt and class 1 does not adopt. Let \( h_1 = 4000, h_2 = 250, h_3 = 1000, d_1 = 1000, d_2 = 2000, d_3 = 3000, g_1 = 500, g_2 = 1000, g_3 = 1500, e_1 = 200, e_2 = 400, e_3 = 600. \) \( R \) now has the choice between two possible orderings: Under ordering \( o_1, t(1) < t(2) < t(3) \), and \( m(o_1) \) is 2, while under ordering \( o_2, t(1) < t(3) < t(2) \), and \( m(o_2) \) is 3. We show that \( P_2 \) is infeasible under both these orderings.

System of constraints under \( o_1 \):

\[
\begin{align*}
1000r_d - 700s_d - 200s_e + r_0 - s_0 & \quad < \quad 2000r_d - 1400s_d - 400s_e + r_0 - s_0 & \quad < \quad 3000r_d - 2100s_d - 600s_e + r_0 - s_0 \\
500 & \quad < \quad 1000 & \quad < \quad 1500
\end{align*}
\]

(33)

\[
\begin{align*}
1750000 \left( \frac{2000r_d - 1400s_d - 400s_e + r_0 - s_0 - c_s}{1000} \right) & = \Delta_S \\
3750000 \left( \frac{1000r_d - 700s_d - 200s_e + r_0 - s_0 - c_s}{500} \right) & < \Delta_S \\
1500000 \left( \frac{3000r_d - 2100s_d - 600s_e + r_0 - s_0 - c_s}{1500} \right) & < \Delta_S
\end{align*}
\]

(34) \hspace{1cm} (35) \hspace{1cm} (36)

Solving for \( r_0 - s_0 \) using equation (34), and substituting in (35) and (36), we obtain

\[
\frac{(5c_s - 10r_d + 7s_d + 2s_e)}{2} + 23\Delta_S < 0 \quad \text{and} \quad 700000 \left( \frac{5c_s - 10r_d + 7s_d + 2s_e}{2} \right) + 3\Delta_S > 0. \]

Let \( \frac{(5c_s - 10r_d + 7s_d)}{2} = v \) The above inequalities simplify to \( s_e + v < \frac{-23}{2(5250000)}\Delta_S = -2.19048 \cdot 10^{-6}\Delta_S \) and \( s_e + v > \frac{-3}{2(7000000)} = -2.14286 \cdot 10^{-6}\Delta_S \), which is not possible, because \( \Delta_S > 0 \). Therefore, \( P_2 \) is infeasible under \( o_1 \).

System of constraints under \( o_2 \):

\[
\begin{align*}
1000r_d - 700s_d - 200s_e + r_0 - s_0 & \quad < \quad 3000r_d - 2100s_d - 600s_e + r_0 - s_0 & \quad < \quad 2000r_d - 1400s_d - 400s_e + r_0 - s_0 \\
500 & \quad < \quad 1500 & \quad < \quad 1000
\end{align*}
\]

(37)

\[
\begin{align*}
1750000 \left( \frac{3000r_d - 2100s_d - 600s_e + r_0 - s_0 - c_s}{1500} \right) & = \Delta_S \\
3750000 \left( \frac{1000r_d - 700s_d - 200s_e + r_0 - s_0 - c_s}{500} \right) & < \Delta_S \\
250000 \left( \frac{2000r_d - 1400s_d - 400s_e + r_0 - s_0 - c_s}{1000} \right) & < \Delta_S
\end{align*}
\]

(38) \hspace{1cm} (39) \hspace{1cm} (40)

Solving for \( r_0 - s_0 \) using equation (38), and substituting in (37), we obtain:

\[
3c_s - 4r_d + \frac{14s_d}{5} + \frac{4s_e}{5} + \frac{3\Delta_S}{1750000} < c_s + \frac{\Delta_S}{1750000} < \frac{3c_s}{2} - r_d + \frac{7s_d}{10} + \frac{s_e}{5} + \frac{3\Delta_S}{3500000}
\]
However, \( c_s + \frac{\Delta_s}{1750000} < \frac{3s_d}{2} - r_d + \frac{7s_d}{10} + \frac{s_e}{5} + \frac{3\Delta_s}{3500000} \) can be rewritten as \(350000(5c_s - 10r_d + 7s_d + 2s_e) + \Delta_s > 0\) and \(3c_s - 4r_d + \frac{14s_d}{5} + \frac{4s_e}{5} + \frac{3\Delta_s}{3500000} < \frac{3c_s}{2} - r_d + \frac{7s_d}{10} + \frac{s_e}{5} + \frac{3\Delta_s}{3500000}\) can be rewritten as \(350000(5c_s - 10r_d + 7s_d + 2s_e) + \Delta_s < 0\), which contradict each other. Therefore, \(\mathcal{P}_2\) is also infeasible under ordering \(o_2\). Q.E.D.

**Appendix C: Proof of Proposition 3**

To prove this property, we choose a number \(\epsilon > 0\) and consider a net-metering tariff system (where customers pay a variable charge proportional to their net energy usage). We choose the ordering \(t(i) < t(j), \forall i < j\), which is consistent with classes \(m, m+1, \ldots, I\) being adopters. Setting \(r_d = s_d, s_e = -s_d\), and \(r_0 = s_0 - \epsilon\), the ordering constraints \(t(i) < t(j)\) simplify to \(s_d - \frac{\epsilon}{g_i} < s_d - \frac{\epsilon}{g_j}\), which is true, because \(g_i < g_j\). This leaves us with a linear system of the form \(Ax = b\) with five unknowns: \(r_d, s_d, s_e, r_0\), and \(s_0\). Since the rows of \(A\) are linearly independent, there exists a solution \(x\) to this equation. Therefore \(\mathcal{P}_2\) is feasible under this ordering.

Q.E.D.

**Appendix D: Proof of Proposition 4**

This proof proceeds in two parts. In the first part, we will show that ignoring the set of equations (19) and (20), we can always find a set of \(t(i)\) values that satisfy (21)-(23). In the second part, we will show that corresponding to the set of \(t(i)\) values found, we can find \(r_c, \forall c \in C\) and \(f\) that satisfy (19) and (20).

**Part 1:** First, re-index the usage tiers so the index matches the ordering \(o\). Note that after re-ordering, we no longer have the property that \(g_i < g_{i+1}\) \(\forall i \in \{1, 2, \ldots, I - 1\}\). Now, observe that (21) can be rewritten as \(t(i) < t(i+1), \forall i \in \{1, 2, \ldots, I - 1\}\). Equation (22) can be rewritten as \((t(a(o)) - c_s) \sum_{i=a(o)}^{I} h_i g_i = \Delta_s\) and constraints (23) can be rewritten as \((t(i) - c_s) \sum_{j=1}^{I} h_j g_j < \Delta_s \forall i \neq a(o)\). With some manipulation, we obtain:

\[
t(i) < t(i+1) \forall i \in \{1, 2, \ldots, I - 1\} \tag{41}
\]

\[
t(a(o)) = c_s + \frac{\Delta_s}{\sum_{i=a(o)}^{I} h_i g_i} \tag{42}
\]

\[
t(i) < c_s + \frac{\Delta_s}{\sum_{j=1}^{I} h_j g_j} \forall i \neq a(o) \tag{43}
\]

The set of inequalities in (43) provide upper bounds on all the \(t(i)\) values, and (42) pins down the value of \(t(a(o))\). Notice that these upper bounds are increasing in \(i\), because \(\Delta_s > 0\), and \(\sum_{j=1}^{I} h_j g_j\) is decreasing in \(i\). Therefore, there exist values \(t(i)\) \(\forall i \in \{1, 2, \ldots, I - 1\}\) that respect (41) and the specified upper bounds.

**Part 2:** Once a set of values \(t(i)\) is found, we need to map them to rate class tariffs \(r_c\). To do this, we examine the set of equations (19). Each of these \(I\) equations takes the form:

\[
t(i) = r_m d_i + f - (r_n(d_i - g_i) + f) = r_m \frac{d_i}{g_i} + r_n \left( \frac{d_i}{g_i} - 1 \right), \tag{44}
\]
where \( m \) is the index of the rate class corresponding to net usage level \( d_i \), and \( n \) is the rate class corresponding to net usage level \( d_i - g_i \). Taken together, these \( I \) equations constitute an under-determined linear system of the form \( AR = t \), where \( r \) is a \( |C| \) dimensional vector of \( r_c \) values, \( t \) is an \( I \) dimensional vector of \( t(i) \) values, and \( A \) is a matrix with \( I \) rows and \( |C| > I \) columns. If the rows of \( A \) are linearly independent, then the system has an infinite number of solutions. For the sake of contradiction, assume that the rows are linearly dependent. Then, there must exist an \( I \) dimensional vector \( \lambda \neq 0 \) such that \( \lambda^T A = 0 \). Let \( m_1 = \arg\max d_i \). Let \( c_1 \) be the column in \( A \) corresponding to the rate class into which a tier \( m_1 \) customer would fall if they did not adopt solar. This column has exactly one non-zero entry because no other household can fall into this rate class, whether they adopt solar or not. Let \( w_1 \) be the index of the row in \( A \) that contains this non-zero entry. Then, it must be that the \( w_1^{th} \) entry of \( \lambda \) is 0 in order for the \( c_1^{th} \) entry of \( \lambda^T A \) to be 0. Therefore, the vector \( \lambda^T A \) is unaltered if we replace all entries in the \( w_1 \) row of \( A \) by zero. Now, this same argument can be applied repeatedly to assert that all entries of \( \lambda \) must also be zero: choose \( m_2 = \arg\max_{i \neq m_1} d_i \), find the index \( c_2 \) of the column corresponding to the rate class to which a tier \( m_2 \) customer would belong if it did not adopt solar, and observe that exactly one row corresponding to this column now has a non-zero entry (recall that we changed all entries in row \( w_1 \) to 0). Let \( w_2 \) be the index of this row. We can assert that the \( w_2^{th} \) entry of \( \lambda = 0 \). By repeating this procedure, we can assert that all entries of \( \lambda \) are 0. This contradicts our assumption, and therefore, the rows of \( A \) are linearly independent. Therefore, we can obtain values of \( r_c \) consistent with the equations (44). These values can then be substituted in (20) to find a feasible \( f \). Q.E.D.

**Appendix E: Proof of Proposition 5**

We prove Proposition 5 by showing the following counter-example with three classes of customers. Let \( h_1 = 445, h_2 = 218, h_3 = 1000, d_1 = 500, d_2 = 681, d_3 = 1024, g_1 = 100, g_2 = 181, g_3 = 343, p_{r0} = 0.1, \Delta_s = 1, c_s = 3/40 \). With these parameters, there are four possible values of net demand that a household can have:

1. Class 1 household adopts: Net demand = \( d_1 - g_1 = 400 \) kWh.
2. Class 1 household does not adopt, or class 2 household adopts: Net Demand = \( d_1 = d_2 - g_2 = 500 \) kWh.
3. Class 2 household does not adopt, or class 3 household adopts: Net Demand = \( d_2 = d_3 - g_3 = 681 \) kWh.
4. Class 3 household does not adopt: Net Demand = \( d_3 = 1024 \) kWh.

Therefore, \( U \)'s rate schedule must specify 4 different rate classes that apply at each of these net demand levels. Let the rates corresponding to these rate class levels be \( r_1, r_2, r_3, \) and \( r_4 \) respectively. Accordingly:

\[
t(1) = r_1 + \frac{(r_2 - r_1)}{g_1} d_1; \quad t(2) = r_2 + \frac{(r_3 - r_2)}{g_2} d_2; \quad t(3) = r_3 + \frac{(r_4 - r_3)}{g_3} d_3
\]
Let $\Delta U$ be chosen so that $\Delta C = 0$. Therefore, customers as a whole gain exactly 0. We are interested in seeing how close this tariff structure can come to being CS-free. In particular, can customer classes 1, 2, and 3 be financially worse off by an amount arbitrarily close to 0 after solar adoption?

Let every individual household in class 1 be worse off by an amount $a_1$, and every individual household in class 2 be worse off by an amount $a_2$. Because $\Delta C = 0$, we have that every household in class 3 is worse off by exactly $-\frac{a_1 h_1 + a_2 h_2}{h_3}$. Let $z^*$ specify that class 1 adopts, while classes 2 and 3 do not adopt. Since class 1 is the marginal customer, $r_2 d_1 + f = r_1 (d_1 - g_1) + f + t(1) g_1$: the class 1 customer is equally worse off whether they adopt solar or not. Accordingly, we can write the following system of equations:

$$
\begin{align*}
    r_2 d_1 + f &= p r_0 d_1 + a_1 \\    r_3 d_2 + f &= p r_0 d_2 + a_2 \\    r_4 d_3 + f &= p r_0 d_3 - \frac{a_1 h_1 + a_2 h_2}{h_3}
\end{align*}
$$

We solve these equations for $r_2, r_3,$ and $r_4$. Next, we use the following profit equation to obtain $r_1$:

$$
\Delta S = h_1 g_1 \left( r_1 + (r_2 - r_1) \frac{d_1}{g_1} - c_s \right)
$$

Substituting $r_2$ from equation (45) into equation (48), we obtain an equation for $r_1$. Using these expressions for $r_1, r_2, r_3,$ and $r_4$, we obtain the following expressions for the $t(i)$ values.

$$
\begin{align*}
    t(1) &= \frac{6677}{89000} \\
    t(2) &= \frac{-10a_1 + 10a_2 + 181}{1810} \\
    t(3) &= \frac{89a_1}{68600} - \frac{87a_2}{24500} + \frac{1}{10}
\end{align*}
$$

Now, consider the two possible orderings that $R$ could induce.

1. $t(3) < t(2) < t(1)$: $t(2) < t(1)$ simplifies to the inequality $a_1 > a_2 + \frac{402,363}{89,000}$. Therefore, $a$ and $b$ cannot both be made arbitrarily close to 0.

2. $t(2) < t(3) < t(1)$: Since this ordering also contains the inequality $t(2) < t(1)$, this ordering also does not allow $R$ to induce a CS-free outcome.

Therefore, this tariff structure does not allow $R$ to induce a CS-free outcome. Q.E.D.

**Appendix F: Proof of Proposition 7**

Corresponding to an adoption outcome $z^*$, we will choose the ordering yielding the required $z^*$ whereby the non-adopters are arbitrarily arranged to have the lowest indices, the marginal adopter is the adopting class with the lowest value of $g_i$, and all other adopters are arranged in increasing order of $g_i$.\(^{10}\) Under

\(^{10}\)Recall that there are $i!j!$ such orderings where $i + j = I$, $i$ classes do not adopt, and $j$ classes do adopt.
this re-indexed system, if \( m \) is the index of the marginal adopter, set \( t(m) = c_s + \frac{\Delta g}{\sum_{i=m}^{I} h_{i} g_{i}} \). This leads to \( f_{m} = g_{m}(p_{r} - t(m)) \). For all indices \( i < m \), set \( t(i) = c_{i} \) for some small \( \epsilon \), and appropriately choose the fixed costs \( f_{i} \) that would lead to this. For all indices \( i > m \), set \( f_{i} = \frac{\Delta g}{g_{m}} g_{i} - (i - m)\epsilon g_{i} \), leading to values \( t(i) = p_{r} - \frac{\Delta g}{g_{m}} + (i - m)\epsilon \). Clearly, this schedule of fixed costs respects the ordering that the \( t(i) \) values are required to have. Further, the incentive compatibility constraints are respected: for a small enough \( \epsilon \), setting a solar price of \( t(i), i < m \) brings \( S \) negative profit, and setting a price \( t(i), i > m \) increases her margin by an arbitrarily small amount \( (i - m)\epsilon \), but brings lower volumes, because \( \sum_{j=m}^{I} h_{j} g_{j} > \sum_{j=m}^{I} h_{j} g_{j}, \forall i > m \).

Now, let us examine what happens to the cash outflow of every tier under this schedule of rates. For a non-adopting tier \( i < m \), the decrease in cash outflow (and therefore benefit to a customer in the tier) is \((p_{r0} - p_{r})d_{i}\). For an adopting tier, the decrease in cash outflow is \( p_{r0}d_{i} - (p_{r}(d_{i} - g_{i}) + f_{i} + p_{s}g_{i}) = (p_{r0} - p_{r})d_{i} + p_{r}g_{i} - f_{i} - p_{s}g_{i} > (p_{r0} - p_{r})d_{i} \) from Lemma 1.

Accordingly, it is sufficient for us to now show that \( p_{r} < p_{r0} \) under this schedule when \( \Delta_{C} > 0 \), and \( p_{r} = p_{r0} + \delta \), for some arbitrarily small \( \delta > 0 \) when \( \Delta_{C} = 0 \).

\[
\Delta_{C} = \sum_{i}^{I} h_{i} (p_{r0} - p_{r})d_{i} + \sum_{i}^{I} h_{i} (i - m)\epsilon g_{i} \iff p_{r} = p_{r0} - \frac{\Delta_{C}}{\sum_{i}^{I} h_{i} d_{i}} + \epsilon \frac{\sum_{i}^{I} (i - m)h_{i} g_{i}}{\sum_{i}^{I} h_{i} d_{i}}
\]

Therefore, since the term \( \frac{\sum_{i}^{I} (i - m)h_{i} g_{i}}{\sum_{i}^{I} h_{i} d_{i}} \) is vanishingly small when \( \Delta_{C} > 0, p_{r} < p_{r0} \). When \( \Delta_{C} = 0, p_{r} \) can be set arbitrarily close to \( p_{r0} \) by reducing \( \epsilon \). This completes the proof. Q.E.D.

**Appendix G: Proof of Proposition 8**

When \( \Delta_{C} > 0 \), Proposition 7 showed that all tiers can be made to benefit from solar adoption. For tiers \( i \) that do not adopt solar, the gain is \((p_{r0} - p_{r})d_{i}\). For tiers that do adopt solar, the gain is \((p_{r0} - p_{r})d_{i} + p_{r}g_{i} - f_{i} - p_{s}g_{i}\). From Lemma 1, \( p_{r}g_{i} - f_{i} - p_{s}g_{i} > 0 \). Therefore, the smallest gain accrues to the tier with the lowest \( d_{i} \) value (from the \((p_{r0} - p_{r})d_{i}\) term; since all tiers benefit, \( p_{r} < p_{r0} \)). If this \( d_{i} \) value corresponds to a tier \( l \) that is a non-adopter, a household of this tier gains the most if \( p_{r} \) is as low as possible. Note that the total gain accruing to households (\( \Delta_{C} \)) is fixed, and how it is distributed among the tiers is controlled by the choice of \( p_{r} \) and \( f_{i} \) values for adoptive tiers \( i \). By setting \( f_{i} \) values in order to make the \( p_{r}g_{i} - f_{i} - p_{s}g_{i} \) values arbitrarily close to 0 for adoptive tiers, we can ensure that all of \( \Delta_{C} \) is constituted of the \((p_{r0} - p_{r})d_{i}\) terms. We know exactly how to do this from the proof of Proposition 7. Using the schedule specified in the proof of Proposition 7, adoptive tiers face fixed costs \( f_{i} = \frac{i\Delta g}{g_{m}} - (i - m)\epsilon g_{i} \). Therefore, for these tiers, \( p_{r}g_{i} - f_{i} - p_{s}g_{i} = p_{r}g_{i} - \frac{i\Delta g}{g_{m}} g_{i} + (i - m)\epsilon g_{i} - \left(p_{r} - \frac{i\Delta g}{g_{m}}\right) g_{i} = (i - m)\epsilon g_{i} \). Since this can be made arbitrarily close to 0 by reducing the value of \( \epsilon \), this accomplishes our purpose of ensuring that \( \Delta_{C} \) is (almost) entirely constituted of the \((p_{r0} - p_{r})d_{i}\) terms. Q.E.D.
Appendix H: Data used in Numerical Analysis

<table>
<thead>
<tr>
<th>Generation Class (kWh)</th>
<th>Average Generation $g_i$ (kWh)</th>
<th>Average Demand $d_i$ (kWh)</th>
<th>Average Excess $e_i$ (kWh)</th>
<th>% households $h_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-6500</td>
<td>5272</td>
<td>8896</td>
<td>2636</td>
<td>24.11%</td>
</tr>
<tr>
<td>6500-8700</td>
<td>7653</td>
<td>10383</td>
<td>4577</td>
<td>25.00%</td>
</tr>
<tr>
<td>8700-11000</td>
<td>9907</td>
<td>13204</td>
<td>5995</td>
<td>24.11%</td>
</tr>
<tr>
<td>&gt;11000</td>
<td>14483</td>
<td>14282</td>
<td>10252</td>
<td>26.78%</td>
</tr>
</tbody>
</table>

Table 3 Estimated parameters for the state of Nevada

<table>
<thead>
<tr>
<th>Generation Class (kWh)</th>
<th>Average Generation $g_i$ (kWh)</th>
<th>Average Demand $d_i$ (kWh)</th>
<th>Average Excess $e_i$ (kWh)</th>
<th>% households $h_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-7250</td>
<td>5778</td>
<td>8810</td>
<td>2765</td>
<td>21.42%</td>
</tr>
<tr>
<td>7250-9500</td>
<td>8326</td>
<td>9808</td>
<td>4971</td>
<td>24.11%</td>
</tr>
<tr>
<td>9500-12250</td>
<td>10947</td>
<td>13303</td>
<td>6397</td>
<td>26.78%</td>
</tr>
<tr>
<td>&gt;12250</td>
<td>16212</td>
<td>14210</td>
<td>11352</td>
<td>27.68%</td>
</tr>
</tbody>
</table>

Table 4 Estimated parameters for the state of New Mexico

Appendix I: Values of $\pi_i$ used in Analysis

<table>
<thead>
<tr>
<th>Generation class $(i)$</th>
<th>Nevada</th>
<th>New Mexico</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Optimistic $\pi_i$</td>
<td>Pessimistic $\pi_i$</td>
</tr>
<tr>
<td>1</td>
<td>4.05%</td>
<td>1.35%</td>
</tr>
<tr>
<td>2</td>
<td>4.15%</td>
<td>1.38%</td>
</tr>
<tr>
<td>3</td>
<td>3.83%</td>
<td>1.28%</td>
</tr>
<tr>
<td>4</td>
<td>3.35%</td>
<td>1.12%</td>
</tr>
</tbody>
</table>

Table 5 Estimated $\pi$ values

Appendix J: Rate Classes used for New Mexico

<table>
<thead>
<tr>
<th>Class</th>
<th>Net demand (kWh)</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0-5900</td>
<td>$r_1$</td>
</tr>
<tr>
<td>2</td>
<td>5900-9300</td>
<td>$r_2$</td>
</tr>
<tr>
<td>3</td>
<td>9300-11550</td>
<td>$r_3$</td>
</tr>
<tr>
<td>4</td>
<td>11550-13750</td>
<td>$r_4$</td>
</tr>
<tr>
<td>5</td>
<td>&gt;13750</td>
<td>$r_5$</td>
</tr>
</tbody>
</table>

Table 6 Rate Classes - New Mexico

Appendix K: Reformulation of $P_2$ for Partial Adoption within Classes

\[
\min_{T(i)} \max_i \{T(d_i', e_i, 1) + p_s g_i - p_{r0} d_i, T(d_i, 0, 0) - p_{r0} d_i\}
\]

Subject to constraints:

\[
t(i) = \frac{T(d_i, 0, 0) - T(d_i', e_i, 1)}{g_i}, \forall i
\]

\[
\sum_{i=1}^{I} h_i (s_i^* (\pi_i T(d_i', e_i, 1) + (1 - \pi_i)) T(d_i, 0, 0)) + (1 - s_i^*) T(d_i, 0, 0)
\]

\[
-c_u^* (s_i^* (\pi_i (d_i - g_i) + (1 - \pi_i)d_i) + (1 - s_i^*)d_i)) - \sum_{i=1}^{I} h_i (p_{r0} - c_E^0) d_i = \Delta_U
\]
\begin{align*}
  & t(i) \text{ ordering consistent with } o \\
  & (t(m(o)) - c_s) \sum_{i=1}^{I} s_i^{(r)} \pi_i h_i g_i = \Delta_S \\
  & (t(i) - c_s) \sum_{j=1}^{I} \delta_{t(j)>t(i)} \text{ in ordering } o \cdot \pi_j h_j g_j < \Delta_S, \forall i \neq m(o)
\end{align*}

References


SolarCity (2016b) Solar Energy Production (SolarCity).

SolarCity (2016c) Solar Panel Lifespan (SolarCity).


