A Unifying Framework for Consumer Surplus under Demand Uncertainty

Introduction

In the economics literature, the concept of Consumer Surplus has mainly focused on the case where demand is a deterministic function of the price see [3], [2]. However, many Operations Management (OM) settings are modeled via a stochastic demand function. With the goal of accurately measuring customer satisfaction, this work generalizes the notion of Consumer Surplus, and studies the impact of demand uncertainty on consumers. Customer satisfaction has become a very important focus for many businesses as well as for policy makers. More than a century ago, scientists and economists proposed a concept called the Consumer Surplus by measuring the difference between willingness to pay and price. This concept was then extensively studied in the economics literature, and applied to several domains. In most of these applications, however, the modeling assumptions often involve a deterministic demand function.

In most operational systems, demand is stochastic by nature. Data scientists and statisticians always seek to improve demand prediction algorithms, but ultimately no method will consistently provide a perfect prediction. Consequently, firms will do their best to predict demand but errors in prediction remain inevitable. The OM community has proposed to study such settings by considering a stochastic demand function, and is often interested in making optimal decisions under uncertainty. Problems such as supply chain management, revenue management, queuing systems and service operations. While the focus has been mainly on the firm’s perspective, several recent research trends (e.g., sustainable operations) also study the problem from the consumer perspective. Most of these practical settings involve uncertainty in demand. In order to measure the customer satisfaction, one needs a general definition of the notion of Consumer Surplus.

As observed in [1], demand uncertainty renders the welfare analysis more complicated. In particular, including a stochastic term in the demand function may hinder the existence of an underlying representative consumer utility function. Consequently, utility analysis may not be possible. Besides this concern, demand uncertainty may also lead to scarcity and stock-outs. Given a price, the demand function represents the maximum number of units that will be consumed (or at least desired to be consumed) in the market. Such a demand function captures multiple consumers, each endorsed with a particular maximum willingness to pay for the item. When demand exceeds supply, not all the customers who are willing to purchase the item can be served. As a result, the Consumer Surplus will depend on which of those consumers end up receiving the item. We formalize this intuition by using the notion of a capacity allocation rule.
Contributions

1. Present a generalization of the Consumer Surplus (CS) notion under stochastic demand, defining rigorously a capacity allocation rule and its use in the computation of the CS. The latter allows the adequate computation of CS under general settings when demand is stochastic.

2. Study the impact of demand uncertainty on consumers. In particular, we compare the expected Consumer Surplus to the case where demand is deterministic, and identify how this depends on: (i) the uncertainty nature (additive or multiplicative), (ii) the allocation rule, and (iii) demand convexity/concavity.

3. Quantify the increase in Consumer Surplus by using a sharing mechanism. More precisely, we study settings where unsold items can be shared among suppliers. We determine how much this can enhance the net welfare of consumers.

4. Analyze the impact of the supplier information on demand uncertainty on the Consumer Surplus. We determine how the Consumer Surplus is affected by different levels of demand uncertainty from the seller’s standpoint.

Results

First, we provide a mathematical definition of allocation rules, i.e., the way the supplier allocates the available units to consumers when demand exceeds supply. This allows us to consider a general allocation strategy which includes the three most popular rules as special cases (highest/lowest willingness to pay and random allocation). We also generalize this concept to a setting with multiple products. Defining formally allocation rules allows us to extend the notion of Consumer Surplus to general settings, therefore present a general definition of the Consumer Surplus for multiple items under stochastic demand. Since previous definitions cannot always sustain cases when demand exceeds supply, we propose a general systematic way to account for stock-outs. We consider both additive and multiplicative noises, and several allocation rules. We derive this definition when the underlying model primitive can either be an aggregate demand function, or a representative consumer endowed with a utility function. We also naturally ensure that when the demand becomes deterministic, the Consumer Surplus we propose coincides with the existing definition.
Second, using the Consumer Surplus definition we introduce, we study the impact of demand uncertainty on consumers. In particular, we compare the expected Consumer Surplus to the case where demand is deterministic, under the same prices. We show that in many cases, demand uncertainty actually hurts consumers. Under a demand with multiplicative noise, consumers are always better off in the deterministic setting. Interestingly, this result holds for any demand function, any noise distribution and any allocation rule. When the demand noise is additive, we show that the impact of demand uncertainty depends on the allocation rule, and on the convexity/concavity properties of the nominal demand.

Third, we study a sharing mechanism in which the supplier can share its excess capacity (or demand) between its different stores. We then derive an analytical bound on the ratio of the Consumer Surplus relative to the deterministic setting. We show that when demand is linear with an additive i.i.d noise, the expected Consumer Surplus under the random allocation rule can be as far as 50% relative to the case with deterministic demand. However, by allowing the retailer to share its supply between two stores, the Consumer Surplus guarantee increases to 75%.

Fourth, we explore how different levels of demand uncertainty information affect the consumer surplus. In particular, we explore three settings where the seller: (IG) ignores the demand uncertainty, (NV) knows the demand uncertainty distribution as in a price-setting newsvendor, and (CO) a contingent setting where the seller knows the exact demand realization and can adapt the price and quantity decisions accordingly. Under a linear demand and uniform noise, we show that the consumers surplus in (CO) is always higher relative to (IG). However, there is no strict dominance between (NV) and the other two settings. We then provide sufficient conditions on the problem parameters that ensure clear relationships between the consumer surplus in the different settings (under additive and multiplicative noise). Our results allow us to draw some operational insights on the impact of demand uncertainty and supplier information on consumers.

References