Managing Inventory in the Presence of Lead Time and Demand Correlation

1. Motivation and Research Questions

Motivation and Research Gap. According to classical inventory management models, two key elements of an inventory system, lead time and daily demand, are independent. However, in practice, lead time and demand may be correlated. For instance, an unseen increase or decrease in demand may cause the delivery delay or may motivate the supplier to speed up or slow down the delivery; consequently, lead time and demand could be positively or negatively correlated.

The correlation between demand and lead time is documented in few studies (see, e.g., Ryu and Lee (2003), Feng et al. (2013), Hayya et al. (2011) and De Treville et al. (2004)). Wang, et al. (2010) indicate the reorder point and safety stock should be updated in light of this relationship and derive a set of equations for them when demand and lead time is linearly correlated. Although the importance of updating inventory requirements in the presence of this relationship acknowledged, there has been little work in inventory and supply chain management literature studying the impact of this correlation on an inventory system’s total cost. The literature on demand and supply variability is closely related to our work. Some efforts have been made to reduce demand variability (Lee et al. 1997; Cachon, 1999; Wu, et al., 2008; Chen, et al., 2012; Hosoda and Disney, 2012 (1,2)). On the other hand, some research emphasizes on the supply side, mainly focusing in reducing the length of the supply lead time or the variance of it (Bagchi, et al., 1986, Tersine and Hummingbird, 1995, Leng, and Parlar, 2009, Hayya, et al., 2011). This paper focuses on a conventional inventory system of order quantity/reorder point ($Q,r$) policy for a single product with demand and lead time variability. We analyze the relationship between the total cost of the system the marginal value of demand variance, lead time mean, and lead time variance, under the assumption of the correlated relationship between lead time and demand.

Research Questions. In view of this gap, this paper has three primary objectives. First, we incorporate the correlation between demand and lead time in a continuous review inventory model and benchmark it with a case in which demand and lead time are independent. Second, we use this model and analyze the marginal value of demand variance, lead time mean, and lead time variance. Here, we investigate how a firm should adjust these uncertainties to minimize the total inventory cost. Third, we extend our results for a distribution-free demand and lead time case.

2. Model

We relax the assumption of the independence between demand and lead time for a continuous review inventory system and analyze the characteristics of the optimal ordering policy. The resulting inventory model is new and despite its complexity, a closed-form solution can be determined.
We assume there is a linear correlation between demand \((D)\) and lead time \((T)\) given by:

\[
E(D|T) = a + bT
\]

Given this assumption, the firm’s objective is to minimize the total inventory cost by selecting the best ordering and reordering point.

3. Some Main Results and Their Significance

Some of the main results of the paper are the following:

R1. When demand and lead time are negatively correlated, a higher demand (lead time) uncertainty can generate a lower total inventory cost

The first result that we obtain is that when demand and lead time are negatively correlated, a higher uncertainty in demand generates a lower total inventory cost. This reason may seem counterintuitive. An increase in demand variance has implied a decrease in lead time such that the variance of the demand in a lead time decreases.

R2. When demand and lead time are positively correlated, a higher demand (lead time) uncertainty generates a higher total inventory cost

The second result that we show is that when demand and lead time are positively correlated, a higher uncertainty in demand generates a higher total inventory cost.

R3. Existence of optimal demand variance:

For a given \(\rho\), there exists a unique demand variance, \(\bar{\sigma}_D\), such that it minimizes the total cost of inventory policy. \(\bar{\sigma}_D\) is given by:

\[
\bar{\sigma}_D = \left[ \frac{-\mu_D}{\rho \left( \frac{2\sigma_t}{\mu_t} + \mu_t - 1 \right) + \frac{1}{\rho \sigma_t}} \right]^{+}.
\]

R4. Existence of optimal lead time mean:

For a given \(\rho\), there exists a unique lead time mean, \(\bar{\mu}_t\), such that it minimizes the total cost of inventory policy. \(\bar{\sigma}_D\) is given by:

\[
\bar{\mu}_t = \left[ \frac{1}{2} \left( 1 - \frac{1}{\rho^2} \right) - \frac{\mu_D \sigma_t}{\rho \sigma_D} \right]^{+}.
\]

The last two results present the closed form solutions for best values of demand variance and lead time mean. Decreasing the demand uncertainty or supply time for values higher than these thresholds are beneficial. Interestingly, improvements in demand uncertainty or supply waiting time for values lower than these values are not beneficial.

4. References


• Fang, X., Zhang, C., Robb, D.J. and Blackburn, J.D., 2013. Decision support for lead time and demand variability reduction. *Omega*, 41(2), 390-396.


