Dynamic Pricing under a Static Calendar

**Setup.** This paper pertains to the following dynamic pricing problem. A firm is endowed with a finite, discrete, unreplenishable inventory of a single item to sell over a finite time horizon. Demand is stochastic but stationary, and the *demand function*, which specifies the rate of demand as a function of the price offered, is given in advance. The goal is to dynamically control the price offered, based on the exact realizations of demand, to maximize the firm’s revenue before the time horizon or inventory runs out.

We allow for time to be either continuous or discrete. If time is continuous, then the demand function specifies, for each price that the firm could offer, the arrival rate of a Poisson process representing a single unit of demand. If time is in the form of discrete time steps, then the demand function specifies, for each price that the firm could offer, the probability of getting a sale in a time step. For convenience, we focus on the case where the firm has a “price ladder” containing a finite number of “price points” to choose from, and make the standard assumption that the *revenue rate* (price multiplied by rate of demand) is decreasing in price, which does not lose any generality.

This problem was introduced by Gallego and van Ryzin (1994), who derive structural results for the optimal policy, which dynamically adjusts the price based on the remaining inventory level relative to the remaining time horizon. Among other results, the authors also show that, even if the price ladder contains a large number of prices, a simple pricing policy which employs only two prices is asymptotically optimal as the starting inventory level approaches infinity. Since 1994, this problem, often referred to as the *single-leg revenue management* problem, has been foundational in the revenue management literature, generating a wealth of theoretical and practical interest.

**Motivation for our work.** From our collaborations with a large Consumer Packaged Goods company, we have found that while they appreciate the advantages of dynamic pricing, it is operationally beneficial for them to plan out a deterministic price trajectory, i.e. a *calendar* with a price for every point in time, in advance. It is possible to deviate from this calendar as demand is observed, but there is a significant overhead in doing so, and thus deviation should be reserved for
situations where the realized demand was drastically higher or lower than expected. Motivated by this, we formulate the *dynamic* pricing problem under *static* calendar constraints.

Our problem consists of two stages:

1. Set an initial calendar, knowing the starting inventory, time horizon, demand function, and deviation penalty.

2. Derive a dynamic pricing policy based on the calendar initially set.

We use the convention that if inventory runs out, then the selling season ends immediately and no further revenues or deviation penalties are incurred. We refer to a calendar as optimal if it maximizes the expected revenue *assuming* that the optimal dynamic pricing policy (based on this calendar) will be used in the second stage.

**Our results.** First, we consider the problem where *zero* deviation from the initial calendar is allowed (i.e. the deviation penalty is infinity). We first show that an optimal calendar is monotone, in that the prices are non-increasing. We also show that in general (assuming the time horizon is sufficiently long relative to the starting inventory), the optimal calendar uses all \( m \) prices, where \( m \) is the number of prices on the price ladder. Therefore, the optimal calendar is characterized by \( m \) strictly positive numbers summing to 1, which prescribe the fraction of time each price point should be offered, in a high-to-low fashion. We derive structural results for the optimal calendar resembling those of Gallego and van Ryzin (1994) for the optimal pricing policy—specifically, a *majorization* property in the fractions, where if the starting time horizon is made longer or if the starting inventory is made smaller, then the higher prices occupy a larger fraction of the calendar (but in all cases, the calendar still ends with the lowest price).

Second, we consider this problem’s *adaptivity gap*—the fraction of optimal revenue lost when the firm must set an initial calendar and pay a penalty for deviating from it. Using a Deterministic Linear Program (DLP) to upper-bound the optimal revenue from dynamic pricing, we show that by using a simple two-price calendar with a fixed switching point from its higher price to its lower price, and never deviating from this calendar, the best-possible revenue ratio relative to the
DLP (which approaches 1 as the starting inventory level approaches infinity) can be obtained. This extends the classical result of Gallego and van Ryzin (1994), who obtain the same bound for a two-price policy which can switch either from high to low or low to high, using a dynamic switching point based on realized demand. The benefit of our approach is that the switching point is fixed, but the drawback is that the switch must be from high to low; in fact, we show that it is not possible to achieve the best-possible bound if the switch is both static and from low to high. Another feature of our approach is that its analysis can be naturally adapted to the discrete-time setting, leading to a ratio bound which is tight with respect to both the starting inventory level and the total number of time steps.

Finally, we have numerical experiments showing that the optimal calendar, and even its basic structure, is surprisingly sensitive to the exact value of the deviation penalty. For example, we know that when the deviation penalty is infinity, the optimal calendar sequentially drops the price with at most \( m - 1 \) switching points and always ends with the lowest price; but for smaller penalties, it is possible for the optimal calendar to not require the lowest price, and furthermore, it is possible for the optimal calendar to switch between prices arbitrarily many times.