Economically Motivated Adulteration in Farming Supply Chains

**Problem definition:** Economically motivated adulteration (EMA) is a serious threat to public health. In this paper, we analytically examine farms’ strategic adulteration behavior and the resulting EMA risk in farming supply chains.

**Academic/Practical Relevance:** We develop an analytical framework that realistically models various scenarios of EMA and captures a few major risk drivers, including the uncertainty in the quality of the farms’ output, the dispersion and traceability of the supply chain, and the testing capabilities present in the supply chain. We leverage the framework to derive structural insights that can help to better prioritize and address EMA risk in food supply chains.

**Methodology:** We develop a set of game-theoretic models to study farms adulteration behavior under both “preemptive EMA” (for decreasing the likelihood of producing low-quality output) and “reactive EMA” (for increasing the perceived quality of the output). We fully characterize the farms equilibrium adulteration behavior in both scenarios and analyze how quality uncertainty, supply chain dispersion, traceability, and testing sensitivity (in detecting adulteration) jointly impact the equilibrium behavior. We also examine the manufacturer’s optimal strategy for investing in traceability and testing to meet a certain risk constraint while minimizing total costs.

**Results:** We show that greater supply chain dispersion almost always leads to a higher risk of EMA. Furthermore, we caution that investing in quality improvement without also enhancing testing capabilities may inadvertently increase EMA risk. We validate our models with real cases and field data and show that the model predictions are consistent with empirical evidence.

**Managerial Implications:** For regulators, our results underscore the importance of collecting data and verifying a food manufacturer’s sourcing supply chain to more proactively manage EMA risk. For companies, we recommend addressing risks resulting from supply chain dispersion by mitigating the potential underlying issues in a dispersed supply chain, for example, by enabling better traceability and risk sharing between farms and manufacturers.

**Keywords:** Economically motivated adulteration, farming supply chains, supply chain dispersion, traceability, testing sensitivity, quality risk, food safety
1. Introduction

Food adulteration is a serious threat to public health and a major concern to most governments in both developed and developing countries. Food adulteration can occur in a broad range of scenarios. Unintentional adulteration often occurs as a result of negligence or incompetence, for example, bacterial contamination due to bad hygiene practices. In some scenarios, food adulteration is intentional, for example, motivated by malicious intent to harm the public food system (e.g., bioterrorism). In many other scenarios, intentional adulteration is driven by economic motives and often referred to as economically motivated adulteration (EMA). The U.S. Food and Drug Administration define EMA as the “fraudulent, intentional substitution or addition of a substance in a product for the purpose of increasing the apparent value of the product or reducing the cost of its production, i.e., for economic gain” (Johnson 2014). Over the last several decades there were many publicly-known EMA incidents of food products around the world, and a majority of them originated from developing countries. For example, consumption of melamine-tainted infant formula and milk led to six infant deaths and nearly 300,000 young children severely sickened in China in 2008 (Everstine et al. 2013). Melamine was added to the milk by farmers and collectors to increase the perceived protein content of the milk. In another example, the outbreaks of avian flu had led to extensive use of antibiotics and other illegal drugs in poultry farming in Asia. In particular, the 2012 KFC “Instant Chicken Scandal” in China revealed that the chickens used by KFC were treated with as many as 18 illegal antibiotics on the farms (Pi et al. 2014). In both of these examples, the source of the adulteration was in the upstream parts of the supply chains, specifically farms and collectors.

In this paper, we develop, to the best of our knowledge, the first comprehensive analytical modeling framework to examine strategic adulteration behaviors of farms (and/or collectors) and the resulting EMA risk in a farming supply chain. Our modeling framework captures various major drivers for EMA in a holistic manner, including the uncertainty or variability of the quality of the farms’ output, the dispersion and traceability of the supply chain, and the testing capabilities present in the supply chain. The models allow us to answer a few key questions: (i) What are the farms’ equilibrium adulteration strategies under different EMA scenarios? (ii) How do the above drivers jointly impact the farms’ equilibrium adulteration strategies? (iii) How should a manufacturer most efficiently invest in traceability and testing frequency to satisfy a desirable risk constraint with minimal budget? We validate the models with real cases and field data to ensure that the models are grounded in practice and consistent with empirical evidence. We leverage the analysis of our models to derive important and unique insights that can be used to help both regulators and commercial entities in food supply
chains to better prioritize and address EMA risk more proactively. We next elaborate on the major EMA risk drivers captured in our models.

The first factor is the uncertainty or variability of the quality of a farm’s output. Quality uncertainty can result from issues inherent to the production process; e.g., the quality of milk produced from a cow depends on the health of the cow. It can also be the result of external factors; e.g., epidemics like avian flu affect the quality of chickens raised in a farm. Quality uncertainty can be a major cause of EMA especially in markets with quality-based pricing, i.e., where farms receive a better selling price if the products appear to have higher quality. We divide food adulteration driven by quality uncertainty into two distinct scenarios. The first scenario is called “preemptive EMA” where adulteration occurs before the uncertain quality of the products is realized. The primary goal of preemptive EMA is to decrease the likelihood of producing low-quality output. For example, overuse of antibiotics is a serious concern in pork, poultry, and seafood farming in various countries including Bangladesh, China, India, and Vietnam (Sahoo et al. 2010). Such overuse typically emerges because fears from producing low-quality (e.g., sick or underweight) animals prompt farms to extensively use antibiotics to prevent diseases. The second scenario is called “reactive EMA” where adulteration occurs after the uncertain quality of the products is realized. The primary goal of reactive EMA is to increase the perceived quality of low-quality products and create fake high-quality ones. For example, the intentional adulteration of raw milk with melamine in China emerged in response to price pressure for low-protein milk, which led farms and collectors to add melamine to increase the perceived protein content in the milk (Sharma and Rou 2014). Similar situations occurred in India’s dairy supply chain where farms were found adulterating milk with urea to increase its perceived solids-not-fat (SNF) content and attract higher prices (Handford et al. 2016).

Another factor that may contribute to the risk of EMA in food is the dispersion of a farming supply chain. We define supply chain dispersion as the extent to which agricultural products are sourced from a distributed network of farms, with each farm producing a small fraction of the total quantity (Huang et al. 2017). Dispersed farming supply chains that source from hundreds of thousands of smallholder farms are prevalent in many developing countries (Narrod et al. 2008, Compos 2016). Supply chain dispersion may increase the risk of EMA for at least two possible reasons. First, with a dispersed network of farms, it is difficult (if not impossible) to inspect every farm or to trace every unit of supply back to the producing farm. Instead, manufacturers typically perform inspection on the aggregated supply after the products from all farms are mixed. Due to the lack of traceability, even if the manufacturer detects adulteration in the aggregated supply, it often cannot identify the
problematic farms, nor can it effectively impose penalties on the farms to deter them from adulterating. We capture these challenges in the models we develop. Second, smallholder farms rely on the revenue from selling their products to maintain the livelihood of their families. Hence, when they face substantial uncertainty in quality and the associated price pressure, they are likely to become aggressive and engage in adulteration to ensure their only means of income. In an extension, we consider risk-averse farms and show that such concerns indeed motivate farms to adulterate even more.

Adding to the complexity of a dispersed farming supply chain, the manufacturer’s testing capability in terms of test sensitivity is another factor influencing the risk of EMA in food products. We model both perfect and imperfect testing scenarios. Perfect testing corresponds to scenarios in which a known adulterant is being tested and advanced testing methods exist to identify even a trace amount of the adulterant. For example, there exist highly sensitive methods to detect certain antibiotics in food products (Pikkemaat 2009, Mungroo and Neethirajan 2014). However, due to the large amount of possible adulterants that could penetrate food supply chains, in many cases, the ability to detect adulteration depends on the relative amount of adulterants being used. This situation corresponds to the imperfect testing scenario. For example, the presence of a large quantity of adulterants may change the characteristics of the product (e.g., its texture, smell, or color) or may lead to adverse symptoms among consumers who consume the product, and as a result, signal adulteration. In fact, the melamine-tainted infant formula scandal broke out precisely because the quantity of melamine added to the milk became so high that many children got ill, hence alerting the authorities.

**Contributions:** This paper addresses a timely and crucial topic on food safety and makes important contributions to both research and practice. From a research standpoint, we develop a new modeling framework that realistically captures major risk drivers of EMA in a holistic manner and validate the model predictions with empirical data.\(^1\) Two modeling features differentiate our model from prior works that examine dishonest supplier behavior (e.g., Babich and Tang 2012, Cho et al. 2015, Mu et al. 2014, 2016). First, we explicitly model exogenous quality uncertainty and distinguish it from endogenous adulteration decisions, whereas prior works consider adulteration as equivalent to producing low-quality products at a lower cost. Our approach is essential to correctly capture how quality uncertainty combined with quality-based pricing motivates adulteration. Second, while prior works treat detection of low-quality products as exogenous, we model imperfect testing where the probability of detecting adulteration is endogenously influenced by the farms’ adulteration decisions. This

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\(^1\) Our work is related to the broad literatures on quality management and agricultural supply chains. For a focused discussion, we only mention the most relevant subgroup of research here and refer interested readers to Pla-Aragonés (2015) and Nagurney and Li (2016) for comprehensive reviews.
endogeneity substantially complicates our analysis because we must consider the strategic interactions among farms and the interdependency of their adulteration decisions under quality uncertainty. In the case of reactive EMA with imperfect testing, we must solve for the farms’ equilibrium adulteration strategies in a game of asymmetric information (as the number of realized low-quality units at each farm is the farm’s private information).

From a practical standpoint, the holistic modeling framework allows us to derive tangible insights for addressing EMA risk in food products more proactively. For example, we show that greater supply chain dispersion almost always leads to a higher risk of EMA. Furthermore, under imperfect testing, dispersion increases risk even if there is full traceability in the supply chain. These results contribute to the supply chain risk management literature which has primarily focused on disruption risks (e.g., Sheffi 2005, Van Mieghem 2007, Tomlin 2009, Wang et al. 2009, Simchi-Levi et al. 2014). We focus on quality risks stemming from endogenous actions within the supply chain and demonstrate that the structure of the supply chain can be a major driver of risks. Another new insight emerged from our analysis is that merely investing in quality improvement without enhancing testing capabilities at the same time may backfire and inadvertently increase EMA risk. This can occur when testing is imperfect, and hence, farms feel “safe” to adulterate to a moderate level without worrying about being caught later. Taken together, our results underscore the importance and necessity of applying a systemic, supply chain perspective to enable proactive management of EMA risk in food products. Such a perspective is missing in most current practices, which mainly rely on inspection and information at the product and individual company level to manage risk.

The remainder of the paper is organized as follows. In §2, we present our modeling framework and how it maps to various practical scenarios. In §3, we first analyze the farms’ adulteration behavior under preemptive EMA for both perfect and imperfect testing. We then study how supply chain dispersion, traceability, and quality uncertainty impact the resulting EMA risk in the supply chain. In §4, we perform the same analysis for the reactive EMA scenario. In §5, we examine the manufacturer’s optimal investments in traceability and testing frequency to mitigate EMA risk. In §6, we calibrate our model parameters with field data and demonstrate that our model predictions align with empirical observations. In §7, we analyze a number of model extensions and confirm the robustness of our results. Finally in §8, we conclude the paper and discuss the practical implications of our research.

2. Modeling the Farming Supply Chain

We study a farming supply chain in which a single manufacturer (she) procures in total $k$ units of an agricultural product from $n$ homogeneous farms (he). Based on farming supply chain data we collected
from China’s government agencies, we observe in multiple industries that when a manufacturer sources from multiple farms, the output level of each farm is very similar. Therefore, analyzing homogeneous farms is reasonable. Let \( m \) denote the number of output units from each farm, i.e., \( m = k/n \). Each farm is paid based on the average quality of his output. The quality of a farm’s output is uncertain ex ante. Specifically, each unit of output is of low quality with probability \( p_L \) and of high quality with probability \( (1 - p_L) \). Therefore, the total number of low-quality units at a farm follows a binomial distribution with parameters \( m \) (number of units) and \( p_L \) (probability of each unit being low-quality).

Following Huang et al. (2017), we define supply chain dispersion as

\[
D = -\sum_{i=1}^{n} f_i \log(f_i),
\]

where \( f_i \) is the fraction of the total output supplied by farm \( i \). In our setup, \( f_i = m/k \), and hence, \( D = \log(n) \).

By keeping the total supply chain output \( k \) constant and changing \( n \), we effectively change the total number of farms and the fraction of the total output supplied by each farm in the supply chain. This enables us to study the effect of supply chain dispersion on the risk of EMA. In particular, a large \( n \) (i.e., a small \( m \)) represents high supply chain dispersion, i.e., a supply chain with a large number of small farms each supplying a small fraction of the total output. Conversely, a small \( n \) (i.e., a large \( m \)) represents low supply chain dispersion, i.e., a supply chain with a small number of large farms each supplying a large fraction of the total output. Another parameter of interest is the probability of a unit being low-quality, \( p_L \). Changing \( p_L \) allows us to study how a farm’s adulteration behavior is affected by an increased or a decreased likelihood of producing low-quality output. A high \( p_L \) means there is a greater chance for the farm to produce low-quality output (e.g., during epidemics).

We study both preemptive and reactive EMA. Under preemptive EMA, farms make adulteration decisions before the uncertain quality of their output is realized. Engaging in adulteration reduces the value of \( p_L \), i.e., decreases the chance of producing low-quality output. A representative example is the excessive use of illegal antibiotics in poultry farming to prevent diseases in chickens. Under preemptive EMA, we measure EMA risk in the supply chain based on the amount of adulterants added by the farms. Conversely, under reactive EMA, farms make adulteration decisions after the uncertain quality of their output is realized. Engaging in adulteration increases the perceived quality of the output and creates fake, high-quality units. A representative example is adding melamine (urea) to artificially increase the perceived protein (SNF) content in milk. Under reactive EMA, farms can condition their adulteration decisions on the realized number of low-quality units. Therefore, we measure EMA risk in the supply chain through both the probability of an individual farm adulterating his output and the expected total number of adulterated units within the total supply chain output.
Figure 1 illustrates the sequence of events in our model for both preemptive and reactive EMA. The key differences between the two scenarios are in the first two steps. For preemptive EMA, the model dynamics are as follows: (i) Each farm simultaneously and individually decides the amount of adulterants to add to reduce the likelihood of producing low-quality output ($p_r$). (ii) The uncertain quality of each unit of output is realized. (iii) The manufacturer purchases from all farms and pays each farm based on the average quality of his output. (iv) The manufacturer stores samples of the output from $t$ randomly-chosen farms, where $t \in [0,n]$. (v) The manufacturer aggregates the output from all farms; with probability $q$, she tests the aggregated supply for adulteration. (vi) If adulteration is detected in the aggregated supply, then the manufacturer tests each of the stored samples. (vii) If a farm is found to have adulterated his output, then he is charged a penalty of $cm$, where $c$ is the per-unit penalty. We show that all of our results remain the same if the total penalty a farm incurs by adulterating his output is convex increasing in $m$ ($\S 7$, Proposition 12).

For reactive EMA, the first two steps in the model dynamics are instead as follows: (i) The uncertain quality of each unit of output is realized. That is, each farm knows the number of high- and low-quality units he produces, and this is private information to each farm. (ii) Each farm simultaneously and individually decides whether or not to adulterate all of the realized low-quality units to create fake high-quality ones. The remaining steps proceed exactly the same as in preemptive EMA. To map steps (i) and (ii) to practice, consider for preemptive EMA that step (i) corresponds to a farm adding (or not) antibiotics to the feeds of the chickens, and step (ii) corresponds to some chickens growing healthily while others suffering from diseases. For reactive EMA, step (i) corresponds to some cows producing milk with good protein content while others producing milk with lower protein level, and
step (ii) corresponds to the farm adding (or not) melamine to the milk. Note that the “farm” and “manufacturer” notation in our model represents more generally a player in the upstream (e.g., a farm or a collector) and downstream (e.g., a manufacturer or a wholesaler) of the supply chain.

We also note a key difference in the farm’s adulteration decision between preemptive and reactive EMA. In the former setting, the farm chooses how much to adulterate, i.e., the amount of adulterants to add. In the latter setting, the farm chooses whether or not to adulterate; in the case of adulterating, he adulterates all of the realized low-quality units. Both are reasonable assumptions from the practical examples discussed above. In §7.1, we allow farms to adulterate a fraction of their low-quality units under reactive EMA and obtain similar results.

We next explain steps (iii)–(vii) in Figure 1 in more detail. First, in step (iii), we model quality-based pricing – a very common payment scheme in many agricultural industries (Bennett et al. 2001). In particular, the price for each high-quality (low-quality) unit is $r_H (r_L)$ with $r_H > r_L$. This price difference is an important economic motive for farms to engage in adulteration. For products that are mixed before being sold to the manufacturer (e.g., milk), our model captures linear pricing based on average quality. This pricing scheme is common in the dairy industry (Draaiyer 2002). Second, in step (iv), we capture the traceability of the supply chain by modeling the number of randomly-chosen farms, $t \in [0, n]$, from which the manufacturer stores samples. We call the fraction, $t/n$, the traceability factor of the supply chain. In a fully traceable supply chain (i.e., one with a traceability factor of 1), samples from all farms are stored and hence, can be tested later for adulteration if needed. Conversely, in a partially traceable supply chain (i.e., one with a traceability factor smaller than 1), only some of the farms’ samples are stored and tested if necessary. Therefore, the traceability of the supply chain significantly impacts the manufacturer’s ability to identify all of the adulterating farms. Only those farms that are traced could potentially incur a penalty from adulterating. Our way of modeling traceability captures known practices where food manufacturers store samples from individual farms before aggregating all of the supply. For example, Bright Dairy in China stores milk sample from each cow in separate jars before aggregating the milk. If a quality problem is detected in the aggregated milk, then the company tests each jar and notifies those farms with quality problems (Flynn and Zhao 2014). Bright Dairy’s practice represents high traceability in the supply chain and is (unsurprisingly) rare in the dairy industry in China. Similar examples exist for the preemptive EMA scenario where poultry companies store frozen chickens or pieces of frozen meat before aggregating all chickens for processing (Rajić et al. 2007, Zhang and Bhatt 2014).
Third, in steps (v) and (vi), we model the manufacturer’s two-step process of testing for adulteration. Specifically, with some probability $q$ (which captures inspection frequency), the manufacturer tests the aggregated supply for adulteration. Only if the aggregated supply is found to have been adulterated will the manufacturer then test the stored samples. This two-step process is common in practice due to its cost effectiveness and potential resource constraints of the manufacturer (Draaiyer 2002, Mu et al. 2016). It also captures scenarios where detection of adulteration happens in the downstream of the supply chain when products are inspected by an external party (e.g., government agencies) or when consumers develop adverse symptoms due to consumption of adulterated products. As discussed earlier, an important factor we capture related to testing is the sensitivity of the test in detecting adulteration. We model both perfect and imperfect testing scenarios. With perfect testing, the test is very sensitive and can detect even a trace amount of the adulterants. An example are advanced testing methods targeted for certain antibiotics. With imperfect testing, the sensitivity of the test depends on the relative amount of adulterants in the total supply chain output. The larger the relative amount of adulterants in the output, the more likely that the test can detect adulteration. We model this dependence by considering the detection probability to be linear (in our baseline setup) or convex (in §7.2) increasing in the relative amount of adulterants in the total output.

In the following sections, we examine a farm’s adulteration behavior and the resulting EMA risk in the supply chain in four different settings: preemptive or reactive EMA with perfect or imperfect testing. Table 1 summarizes the key parameters in our model.

### 3. Preemptive EMA

Let $x \in [0, 1]$ be a farm’s adulteration decision, where $x = 0$ means the farm does not adulterate his output and $x = 1$ means the farm adulterates with the maximum dosage. Any $x > 0$ means the farm adulterates his output to some extent, and a larger $x$ means a larger quantity of adulterants being used. We assume that adding more adulterants beyond the maximum dosage can no longer decrease

<table>
<thead>
<tr>
<th>Variable</th>
<th>Definition</th>
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<tbody>
<tr>
<td>$k$</td>
<td>total supply chain output</td>
</tr>
<tr>
<td>$n$</td>
<td>total number of farms in the supply chain</td>
</tr>
<tr>
<td>$m$</td>
<td>Number of units supplied by each farm; $m = k/n$</td>
</tr>
<tr>
<td>$p_L$</td>
<td>Probability that a unit of output is of low quality</td>
</tr>
<tr>
<td>$r_H$</td>
<td>Per-unit price paid by the manufacturer to the farm for high-quality output</td>
</tr>
<tr>
<td>$r_L$</td>
<td>Per-unit price paid by the manufacturer to the farm for low-quality output</td>
</tr>
<tr>
<td>$t$</td>
<td>Number of farms whose samples are stored by the manufacturer; $t \in [0, n]$</td>
</tr>
<tr>
<td>$q$</td>
<td>Probability that the manufacturer tests the aggregated supply for adulteration</td>
</tr>
<tr>
<td>$c$</td>
<td>Per-unit penalty charged to a farm that is found adulterating</td>
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the likelihood of producing low-quality output. Therefore, \( x \) can be interpreted as the relative quantity of adulterants as compared to the maximum dosage that the farm uses. Let \( h(x) \) denote the resulting probability that a unit of output is low-quality given \( x \). We assume that \( h(x) \) is convex decreasing in \( x \) with \( h(0) = p_{L}^{\text{max}}, h(1) = p_{L}^{\text{min}}, \) and \( p_{L}^{\text{max}} > p_{L}^{\text{min}} \). The notations \( p_{L}^{\text{max}} \) and \( p_{L}^{\text{min}} \) represent the largest and smallest probability that a unit of output is low-quality given the farm’s adulteration decision. With a large (small) \( p_{L}^{\text{max}} \), the farms face a higher (lower) risk of producing low-quality units without adulteration. During epidemics, for example, we can expect \( p_{L}^{\text{max}} \) to be large. The convexity of \( h(x) \) implies that the effectiveness of the adulteration to reduce the likelihood of producing low-quality output is marginally decreasing. In what follows, we first characterize the farms’ optimal adulteration strategies under perfect or imperfect testing. We then examine how supply chain dispersion, traceability, and quality uncertainty impact the optimal strategies and the resulting EMA risk.

3.1. When Will Farms Adulterate?

3.1.1. Perfect testing. Given a farm’s adulteration decision \( x \), the probability that each unit of his output is of low quality is \( h(x) \). Because a farm produces \( m \) units in total, the expected total number of low-quality units is \( mh(x) \). If the farm chooses not to adulterate (i.e., \( x = 0 \)), then he earns an expected payoff of \( r_H m (1 - p_{L}^{\text{max}}) + r_L m p_{L}^{\text{max}} \). If instead the farm chooses to adulterate, then he faces the risk of incurring a penalty if he is eventually caught by the manufacturer. Therefore, to characterize the farms’ adulteration strategies, we need to first analyze the probability that an adulterating farm will be caught by the manufacturer.

Recall from §2 that the manufacturer employs a two-step testing process. Suppose in the case that the manufacturer tests the aggregated supply (which happens with probability \( q \)), she randomly picks one unit of the aggregated supply (e.g., a chicken) to test for adulteration. With perfect testing, the manufacturer detects adulteration as long as the tested unit has any trace amount of adulterants. Let \( n_a \) be the total number of farms that have adulterated their output. The chance that the manufacturer picks an adulterated unit is thus \( n_a/n \). If the manufacturer detects adulteration in the first step, then she further tests the \( t \) samples. Since samples are taken from randomly-chosen farms, the chance that an adulterating farm’s sample has been stored is \( t/n \). Summarizing the above, the probability that an adulterating farm will eventually be caught by the manufacturer is thus \( q(n_a/n)(t/n) \).

We make two important observations from the analysis thus far. First, since the manufacturer can detect any trace amount of adulterants under perfect testing, the farm faces the same level of penalty for any \( x > 0 \). In addition, his expected revenue without considering the potential penalty is increasing
in $x$. Hence, if the farm decides to adulterate, it is in his best interest to adulterate with the maximum dosage, i.e., choosing $x = 1$. Given this observation, an adulterating farm’s expected payoff is equal to $r_H m (1 - p_{L}^{\min}) + r_L m p_{L}^{\min} - q(n_a/n)(t/n)c$. Second, we observe that a farm’s adulteration decision depends on how many other farms are also adulterating (i.e., it depends on $n_a$). Therefore, the farms’ adulteration decisions impact each other’s payoffs, and hence, we solve for a Nash equilibrium (NE) in this static game of complete information (Fudenberg and Tirole 1991, Chapter I). Theorem 1 below characterizes the set of Nash equilibria in the game. All proofs are deferred to the online appendix.

**Theorem 1.** For preemptive EMA with perfect testing, the total number of adulterating farms in any Nash equilibrium of the game is characterized by $n_a^* = \min\left\{ n, \left\lceil \frac{(r_H - r_L)(p_{L}^{\max} - p_{L}^{\min})}{cqt} n^2 - 1 \right\rceil \right\}$, where $\lceil \cdot \rceil$ denotes the smallest integer greater than the argument. We have the following three cases.

1. The farm’s decision on whether or not to adulterate is driven by the tradeoff between the expected revenue gain and the potential penalty from adulteration. By adulterating, the farm increases his expected revenue by $(r_H - r_L)(p_{L}^{\max} - p_{L}^{\min})$ per unit of output, while in the meantime facing a penalty of $q(n_a/n)(t/n)c$ per unit. The equilibrium value $n_a^*$ defines the threshold number such that none of the adulterating farms find it profitable to not adulterate, and none of the non-adulterating farms find it profitable to adulterate.

2. Imperfect testing. The key difference under imperfect testing versus perfect testing is that the detection of adulteration depends on the relative amount of adulterants in the output. In this section, we model this dependency to be linearly increasing. In §7.2, we extend to a convex increasing function. Index the farms by $i = 1, \ldots, n$ and let $x_i \in [0, 1]$ be farm $i$’s adulteration decision. Due to the imperfect sensitivity of the test, the amount of adulterants added (i.e., $x_i$) affects the chance that an adulterating farm will be caught. As a result, it is not necessarily optimal for the farm to adulterate to the maximum dosage (unlike in perfect testing). We first derive the chance of an adulterating farm eventually being caught by the manufacturer. Similar to §3.1.1, suppose with probability $q$, the manufacturer picks one unit (e.g., a chicken) from the aggregated supply to test for adulteration. In this case, the probability that the manufacturer detects adulteration in the aggregated supply can be derived as follows: $P(\text{detection}) = \sum_j \left[ P(\text{detection}|\text{farm } j\text{'s output is picked})P(\text{farm } j\text{'s output is picked}) \right] = \sum_j [x_j(1/n)] = \sum_j x_j/n$. The conditional probability in the bracket is equal to $x_j$ because (i) under preemptive EMA, the added adulterants affect all units of output at a farm, and (ii) the detection sensitivity is linearly increasing in the relative amount of adulterants added. If the manufacturer detects

\[ \text{We assume that when a farm is indifferent between adulterating or not, he chooses not to adulterate.} \]
adulteration in the aggregated supply, then she will test the \( t \) samples. An adulterating farm’s sample will be stored with probability \( \frac{t}{n} \). Finally, the chance that the manufacturer detects adulteration in the sample again depends on the amount of adulterants added, \( x_i \). To summarize, the chance that an adulterating farm \( i \) will eventually be caught by the manufacturer is equal to 
\[
q \left( \sum_{j \neq i} x_j n / t \right) \left( \frac{t}{n} \right) x_i.
\]
Therefore, farm \( i \)'s expected payoff given his adulteration decision \( x_i \) can be characterized as follows.

\[
\pi_{PV}^{PV}(x_i, x_{-i}) = r_H m (1 - h(x_i)) + r_L mh(x_i) - q \left( \frac{x_i + \sum_{j \neq i} x_j}{n} \right) \left( \frac{t}{n} \right) x_i c m,
\]
where \( -i \) denotes all farms other than \( i \). Since all the farms are homogeneous, we focus on analyzing symmetric NE of the game; i.e., NE in which the equilibrium strategy \( x_{PV}^* \) has the same structure for all \( i \). This approach is common in game-theoretic analysis with homogeneous players (e.g., Che 1993, Lee et al. 1997, Wang and Zender 2002, Golosov et al. 2014). We drop the subscript \( i \) in the equilibrium strategy in our subsequent discussion to simplify notation. The next theorem characterizes the unique symmetric NE in the game.

**Theorem 2.** For preemptive EMA with imperfect testing, there exists a unique symmetric NE in which \( x_{PV}^* \) is determined as follows: (a) If \( c < -h'(1)(r_H - r_L) / [q(t/n)((n + 1)/n)] \), then all farms adulterate to the maximum level; i.e., \( x_{PV}^* = 1 \). (b) If \( c \geq -h'(1)(r_H - r_L) / [q(t/n)((n + 1)/n)] \), then the farms adulterate to some extent; i.e., \( x_{PV}^* \in (0, 1) \) and is the solution to the following equation:

\[
-h'(x)/x = q(t/n)((n + 1)/n)c/(r_H - r_L).
\]

Theorem 2 captures a similar tradeoff as under perfect testing. When the per-unit expected penalty is small compared to the per-unit expected revenue gain, all farms adulterate to the maximum level. However, different from before, farms in equilibrium adulterate to some extent but not to the maximum level when the per-unit expected penalty is large (Theorem 2(b)). This is because as farms increase the amount of adulterants, both the expected revenue gain and the chance of being caught increase.

### 3.2. Effects of Supply Chain Dispersion, Traceability, and Quality Uncertainty on Preemptive EMA

We measure the risk of preemptive EMA in the supply chain by the fraction of adulterating farms, \( n_a^*/n \), under perfect testing, and the farms’ adulteration decisions, \( x_{PV}^* \), under imperfect testing. A larger (smaller) value of these terms indicates a larger (smaller) amount of adulterants being used in the total supply chain output, and hence, a higher risk of preemptive EMA. Our first result shows how the level of supply chain dispersion (captured by \( n \)) impacts the risk of preemptive EMA.

**Proposition 1.** (i) \( n_a^*/n \) is increasing (constant) in \( n \) when \( t < n \) (\( t = n \)).
(ii) \( x^{PV^*} \) is always increasing in \( n \).

Proposition 1 shows that a supply chain with a higher dispersion involves a higher risk of preemptive EMA. Note that under perfect testing, higher supply chain dispersion leads to a higher risk of preemptive EMA solely through decreased traceability of the supply chain (since \( n^*_a/n \) is constant in \( n \) if \( t = n \)). In §7.3, we show that when farms are risk averse (as opposed to being expected payoff maximizers), then an additional reason for why higher supply chain dispersion leads to higher risk is that a smaller farm faces higher uncertainty in the quality of his output. In contrast, under imperfect testing, higher supply chain dispersion results in a higher risk of preemptive EMA even in a fully traceable supply chain. This is because increased dispersion reduces the chance for an adulterating farm to be caught beyond decreased traceability (see the last term in Equation (1)). As the number of farms in the supply chain increases, each farm’s adulteration has a diminishing impact on the amount of adulterants in the aggregated supply (and hence on the likelihood that the manufacturer would detect adulteration when testing the aggregated supply). Therefore, farms feel less risky to adulterate in a more dispersed supply chain, leading to a higher overall risk of EMA.

Our next result describes how changing traceability (\( t \)) while fixing supply chain dispersion (\( n \)) impacts the risk of preemptive EMA in the supply chain.

**Proposition 2.** Both \( n^*_a/n \) and \( x^{PV^*} \) are decreasing in \( t \).

Intuitively, as the manufacturer stores samples from more farms, each farm faces a higher chance to be caught if he adulterates his output. Therefore, farms are less likely to engage in adulteration when the supply chain has better traceability. Conversely, as the likelihood of producing low-quality output \( (p^L_{max}) \) at a farm increases, the supply chain would face a higher risk of preemptive EMA, summarized in the following proposition.

**Proposition 3.** Both \( n^*_a/n \) and \( x^{PV^*} \) are increasing in \( p^L_{max} \).

4. **Reactive EMA**

We now turn to the analysis of the farms’ adulteration behavior in the reactive EMA setting. Recall from §2 that in this setting, the farms decide whether or not to adulterate his low-quality units to create fake high-quality ones after the uncertain quality of his output is realized. In addition, the total number of low-quality units at a farm follows a binomial distribution with parameters \( m \) (number of units) and \( p_L \) (probability of each unit being low-quality). This distribution can be well approximated by a normal distribution with mean \( mp_L \) and variance \( mp_L(1-p_L) \) if \( mp_L \geq 5 \) and \( m(1-p_L) \geq 5 \) (Ross
To ensure tractability, we restrict our analysis to scenarios where these conditions hold (and hence, normal approximation is appropriate). We use \( f(x,m,p) \) to denote the probability density function (PDF) of a normal distribution with mean \( mp \) and variance \( mp(1 - p) \) evaluated at \( x \). As in §3, we first characterize the farms’ optimal adulteration strategies under perfect and imperfect testing, then analyze how the optimal strategies are affected by supply chain dispersion, traceability, and quality uncertainty.

4.1. When Will Farms Adulterate?

4.1.1. Perfect testing. Let \( n_L \) be the realized number of low-quality units for a farm that produces \( m \) units in total. If the farm does not adulterate, then he earns \( r_H(m - n_L) + r_L n_L \) based on the average quality of his output. Consider a dairy farm for example. The farm sells his total output after mixing all of his high- and low-quality milk, earning a price based on the average quality. If the farm decides to adulterate, then he first earns a revenue of \( r_H m \). However, if the adulteration is detected by the manufacturer, then he would incur a penalty of \( cm \). Note that if a farm adulterates, then the adulterants will be present in his output, the manufacturer’s aggregated supply, and the stored sample. For example, if a dairy farm adds melamine to his raw milk, then melamine will be present in the milk from this farm, the aggregated pool of milk at the manufacturer, and any sample stored from this farmer. Under perfect testing, the manufacturer detects adulteration in the aggregated supply as long as any farm chooses to adulterate and the manufacturer tests the aggregated supply (the latter occurs with probability \( q \)). Thus, a farm’s adulteration decision does not affect other farms’ decisions and can be solved independently. If further the sample of an adulterating farm is stored (this occurs with probability \( t/n \)), then the farm will be caught and incur the penalty. Hence, the probability of a farm’s adulteration being detected is \( q(t/n) \), and the expected payoff of an adulterating farm is equal to \( r_H m - q(t/n)cm \). A farm chooses whether or not to adulterate to maximize his expected payoff. The resulting optimal payoff given \( n_L \) can be characterized as \( \pi^{RP}(n_L) = \max\{r_H(m - n_L) + r_L n_L, r_H m - q(t/n)cm\} \). The following theorem describes the optimal adulteration strategy for a farm with \( n_L \) units of low-quality output.

**Theorem 3.** For reactive EMA with perfect testing, the optimal adulteration strategy for a farm is a threshold strategy: He does not adulterate if \( n_L \in [0, \beta^{RP}] \), and he adulterates if \( n_L \in (\beta^{RP}, m] \).

Similar to preemptive EMA, the intuition underlying Theorem 3 is a tradeoff between revenue gain and the potential penalty from adulteration. By adulterating, the farm earns an additional revenue of \( r_H - r_L \) for each unit of low-quality output, while facing the risk of paying a penalty of \( cm \) if the
adulteration is detected. Since the revenue gain increases with the realized number of low-quality units \(n_L\) and the expected penalty is independent of \(n_L\), the farm finds it beneficial to adulterate when \(n_L\) is sufficiently large.

**Proposition 4.** The threshold \(\beta^{RP}\) is decreasing in \((r_H - r_L)\) and increasing in \(q, t, and c\).

Intuitively, the more frequent the manufacturer tests the aggregated supply or the higher the traceability factor \((t/n)\) of the supply chain, the greater expected penalty a farm faces by adulterating. As a result, farms are less likely to adulterate in a more traceable supply chain. In addition, a greater price difference between high- and low-quality output \((r_H - r_L)\) and a smaller per-unit penalty \((c)\) make the gain from adulteration more attractive and the penalty from it less severe, thus motivating the farms to adulterate more often. Hence, reducing the price difference and imposing stronger penalty can deter farms from adulterating. Our result is in line with qualitative evidence that many adulteration incidents occurred when there was external price pressure for low-quality products. For example, in the early 2000s, the Chinese government imposed new regulatory standard on the minimal protein content of dairy products in the market. This new regulation effectively drove the price for low-protein milk to almost zero, creating a strong incentive for farms and collectors to adulterate milk with melamine so as to sell at positive prices (Gale and Hu 2009, Everstine et al. 2013).

4.1.2. Imperfect testing. To capture imperfect testing in the reactive EMA setting, we model the chance that the manufacturer detects adulteration when testing the aggregated supply to be linearly increasing in the fraction of adulterated supply within the total supply chain output (see §7.2 for extensions with a convex increasing detection probability). In this case, the farms’ adulteration decisions are no longer independent from each other (as in the case of perfect testing). Instead, the farms make their individual adulteration decisions simultaneously while knowing that their decisions affect each other’s final payoffs (through the potential penalty from adulteration). Since the realized number of low-quality units at a farm is the farm’s private information, we model the interactions among the farms as a static game of incomplete information and solve for the Bayesian Nash equilibrium (BNE) of the game (Fudenberg and Tirole 1991, Chapter III).

Formally, index the farms by \(i = 1, \ldots, n\) and let \(n_{L,i}\) denote the realized number of low-quality units at farm \(i\). Let \(a_i(n_{L,i}) : \{1, \ldots, m\} \rightarrow \{0, 1\}\) be farm \(i\)'s adulteration strategy, where a value of 0 (1) means not adulterating (adulterating). That is, farm \(i\)'s adulteration strategy specifies for each realized number of low-quality units, whether or not the farm adulterates his low-quality output. From farm \(i\)'s perspective, given all other farms’ adulteration strategies \(a_{-i}(n_{L,-i})\), the expected total number of
adulterated units from these farms is equal to $\mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]$. The expectation is taken on the (uncertain) number of low-quality units at the other farms, which is not observable by farm $i$.

We next derive the chance of farm $i$ being caught adulterating under imperfect testing. If farm $i$ chooses to adulterate, then the fraction of adulterated supply within the aggregated supply (and hence the chance that the manufacturer detects adulteration when testing the aggregated supply) is equal to $\frac{n_{L,i} + \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]}{k}$. Conditional on adulteration being detected in the aggregated supply, if farm $i$’s sample is stored (which occurs with probability $t/n$), then farm $i$ will be caught adulterating with probability $n_{L,i}/m$. Since the manufacturer tests the aggregated supply with probability $q$, the ultimate probability for farm $i$ to be caught if he adulterates is equal to

$$\gamma_i(n_{L,i},a_{-i}(n_{L,-i})) \equiv q\left(\frac{t}{n}\right)\left(\frac{n_{L,i} + \mathbb{E}_{n_{L,-i}}\left[\sum_{-i} n_{L,-i}a_{-i}(n_{L,-i})\right]}{m}\right).$$

Thus, the final expected payoff if farm $i$ chooses to adulterate is equal to $r_H m - \gamma_i(n_{L,i},a_{-i}(n_{L,-i})) cm$. Conversely, the final payoff if farm $i$ chooses not to adulterate is equal to $r_H (m - n_{L,i}) + r_L n_{L,i}$. Farmer $i$ decides whether or not to adulterate depending on which action yields a higher final payoff.

We again focus on analyzing symmetric BNE of this game; i.e., BNE in which the strategy $a^*_i(n_{L,i})$ has the same structure for all $i$. We also drop the subscript $i$ to simplify notation. The next theorem characterizes the unique BNE in this game.

**Theorem 4.** For reactive EMA with imperfect testing, there exists a unique symmetric BNE of the game in which a farm’s adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = 1$ if $n_{L,i} \in [0, \beta^{RV}]$ and $a^*(n_{L,i}) = 0$ if $n_{L,i} \in [\beta^{RV}, m]$, for all $i$. The threshold $\beta^{RV}$ is unique and determined as follows: (a) If $n < 2(1 - p_L)\left(\frac{1}{\sqrt{2p_L^2 + \frac{4(1-p_L)^2(r_H-r_L)}{cqt} - p_L}}\right)^2$, then $\beta^{RV} \in (0, m)$ and is the solution to the equation: $\beta = \frac{n k (r_H-r_L)}{cqt} - (n - 1) \int_0^\beta xf(x, \frac{k}{n}, p_L)dx$. (b) If $n > 2(1 - p_L)\left(\frac{1}{\sqrt{2p_L^2 + \frac{4(1-p_L)^2(r_H-r_L)}{c qt} - p_L}}\right)^2$, then $\beta^{RV} = m$.

**Proposition 5.** The threshold $\beta^{RV}$ is increasing in $(r_H - r_L)$ and decreasing in $q$, $t$, and $c$.

Figure 2 contrasts the farms’ optimal adulteration strategies under perfect testing (Theorem 3) versus imperfect testing (Theorem 4) in the reactive EMA scenario. Under perfect testing, farms adulterate when the realized number of low-quality units is greater than the threshold $\beta^{RP}$. To the contrary, under imperfect testing, farms adulterate when the realized number of low-quality units is smaller than the threshold $\beta^{RV}$. This contrasting pattern is due to the fact that under imperfect testing, the chance of adulteration being detected increases with the realized number of low-quality units at a farm. Observe from Equation (2) that the chance of farm $i$ being caught adulterating increases with $n_{L,i}$ quadratically, whereas the revenue gain from adulterating, $(r_H - r_L)n_{L,i}$, increases with $n_{L,i}$ linearly.
Because the expected penalty increases faster than the revenue gain, the farms find it more beneficial to adulterate when the realized number of low-quality units is low. Due to this contrasting pattern, Proposition 5 shows that how the threshold $\beta^{RV}$ changes with the price difference, testing frequency, traceability, and per-unit penalty has the opposite direction as how $\beta^{RP}$ changes with these parameters (see Proposition 4). Finally, Theorem 4 part (b) shows that when the number of farms in the supply chain (and hence supply chain dispersion) is sufficiently large, all farms adulterate all the time. In the next section, we further analyze how the extent of supply chain dispersion and traceability impact the overall risk of reactive EMA in the supply chain.

4.2. Effects of Supply Chain Dispersion and Traceability on Reactive EMA

We measure the risk of reactive EMA in the supply chain in two ways: $P_n$ denotes the probability of an individual farm adulterating in a supply chain with $n$ farms, and $E_n$ denotes the expected total amount of adulterated output in the supply chain. Under perfect testing, $P_n = \text{Prob}(n_L > \beta^{RP})$ and $E_n = n \int_{\beta^{RP}}^{k/n} x f(x) dx$, where $\beta^{RP}$ is defined in Theorem 3. For the analysis in this section, we treat $n$ and $m$ to be continuous variables to ensure tractability. The following proposition characterizes how $P_n$ and $E_n$ change with $n$ under perfect testing.

**Proposition 6.** For reactive EMA with perfect testing,

(i) In a fully traceable supply chain, i.e., when $t = n$, we have:

(a) $\frac{\partial P_n}{\partial n} = \left( \frac{k}{n^2} \right) f \left( \frac{qck}{n(r_H - r_L)}, \frac{k}{n} p_L \right)$.

(b) If $c \geq p_L (r_H - r_L)/q$, then $\frac{\partial E_n}{\partial n} \geq 0$.

(ii) In a partially traceable supply chain, i.e., when $t < n$, we have:

(a) $\frac{\partial P_n}{\partial n} = \left( \frac{k}{n^2} \right) f \left( \frac{q(t/n)ck}{n(r_H - r_L)}, \frac{k}{n} p_L \right)$.

(b) If $c \geq p_L (r_H - r_L)/3(t/n)$, then $\frac{\partial E_n}{\partial n} \geq 0$.

Proposition 6 part (i) shows that for a fully traceable supply chain, both the probability of an individual farm adulterating and the expected total amount of adulterated output increase with supply chain dispersion if the per-unit expected penalty ($qc$) is greater than the per-unit expected revenue gain from adulterating, $p_L (r_H - r_L)$. Similar results hold for a partially traceable supply chain (Proposition 6 part (ii)), with the expected penalty adjusted for the traceability factor $(t/n)$ of the supply chain.
These results imply that supply chain dispersion aggravates the risk of reactive EMA in the supply chain when the penalty associated with adulteration is large. This implication can be explained by the following dynamics. Imagine that the manufacturer diversifies her supply base by procuring one fewer unit from each of the existing farms and procuring the resulting gap from a new farm (recall from §2 that the total procurement quantity $k$ is fixed in our setup). When the per-unit penalty is large, this last unit originally procured from each existing farm would not be adulterated in expectation. However, when they are instead procured from a new farm, some of them would be adulterated at least sometimes in expectation. Therefore, the expected amount of adulterated output increases in a more dispersed supply chain. Our next result demonstrates how $P_n$ and $E_n$ are affected by supply chain dispersion under imperfect testing. In this case, $P_n = \text{Prob}(n_L < \beta_{RV})$ and $E_n = n \int_0^{\beta_{RV}} x f(x, k/n, p_L) dx$, where $\beta_{RV}$ is defined in Theorem 4.

**Proposition 7.** For reactive EMA with imperfect testing, $\frac{\partial P_n}{\partial n} \geq 0$ and $\frac{\partial E_n}{\partial n} \geq 0$.

In contrast to perfect testing, we find that when testing is imperfect, dispersed supply chains are always at a greater risk of reactive EMA, even under full traceability. The intuition behind Proposition 7 is the following. When there are a lot of farms in the supply chain, the chance that many of these farms simultaneously produce low-quality output and choose to adulterate is low (as quality realizations are independent across farms). In addition, each individual farm produces only a small fraction of the total output in a dispersed supply chain. Therefore, the adulteration decision of a small farm has a limited impact on whether or not the manufacturer would detect adulteration when testing the aggregated supply. In sharp contrast, if the supply chain consists of only two farms each producing half of the total output, then the adulteration decision of each farm would have a much more prominent impact on the testing result. Hence, these larger farms would be more cautious in their adulteration decisions.

Summarizing these discussions, farms in a more dispersed supply chain have greater incentives to adulterate, thus resulting in a higher risk of EMA in the supply chain.

**Proposition 8 below shows that as traceability in the supply chain increases, both the probability of adulteration by an individual farm and the expected total amount of adulterated output decrease.**

**Proposition 8.** For reactive EMA with either perfect or imperfect testing, $\frac{\partial P_n}{\partial t} \leq 0$ and $\frac{\partial E_n}{\partial t} \leq 0$.

Because the sample of an adulterating farm is stored with probability $t/n$ under either testing scenario, a higher traceability factor $(t/n)$ of the supply chain would result in a greater expected penalty from adulteration. As a result, farms are less likely to adulterate in a more traceable supply chain.
4.3. The Effect of Quality Uncertainty on Reactive EMA

We next show how the likelihood of producing low-quality output \( (p_L) \) affects an individual farm’s probability of adulteration and the expected total amount of adulterated output in the supply chain.

**Proposition 9.** For reactive EMA, (a) Under perfect testing, \( \frac{\partial P_n}{\partial p_L} \geq 0 \) and \( \frac{\partial E_n}{\partial p_L} \geq 0 \). (b) Under imperfect testing, \( \frac{\partial P_n}{\partial p_L} \leq 0 \); if \( \frac{\partial \beta_{RV}}{\partial p_L} \geq 0 \), then \( \frac{\partial E_n}{\partial p_L} \leq 0 \).

Recall from Theorem 3 that under perfect testing, a farm would adulterate if the realized number of low-quality units \( (n_L) \) exceeds a threshold \( \beta_{RP} \), where \( \beta_{RP} \) is independent of \( p_L \). Since \( n_L \) follows a binomial distribution with parameters \( m \) and \( p_L \), a higher value of \( p_L \) increases the chance that \( n_L > \beta_{RP} \). Because farms’ adulteration decisions do not affect each other under perfect testing, a higher \( p_L \) would motivate all farms to adulterate more often, thus resulting in a higher risk of reactive EMA in the supply chain.

In sharp contrast, under imperfect testing, the probability that an individual farm adulterates decreases as the probability of producing low-quality output increases. This result is jointly driven by the strategic interactions among farms and the fact that the chance for the manufacturer to detect adulteration depends on the fraction of adulterated supply within the aggregated supply. In particular, when \( p_L \) is low, an individual farm expects that the other farms would have many high-quality units. As a result, even if this farm chooses to adulterate, the fraction of adulterated supply in the aggregated supply can be sufficiently low that the manufacturer would not be able to detect adulteration when testing the aggregated supply. In addition, the farms’ equilibrium adulteration strategy under imperfect testing is such that they adulterate if the realized number of low-quality units is below a threshold (see Theorem 4). Therefore, a lower value of \( p_L \) would lead to a greater chance that each farm engages in reactive EMA.

Note that investing in quality improvement to reduce the probability of producing low-quality output is generally believed to be beneficial. Our analysis highlights that this strategy may backfire if it is not accompanied by also improving the supply chain’s capability in detecting adulteration. This is because a lower chance of producing low-quality output can inadvertently motivate some parties in the supply chain to endogenously adulterate their output (if possible) and create fake high-quality units. Without the capability to differentiate fake high-quality units from truly high-quality ones, consumers could suffer from consuming adulterated products.
5. Investing in Traceability and Testing Frequency to Mitigate EMA Risk

Given a supply network of farms, the manufacturer has two levers to mitigate the risk of EMA in the supply chain: increasing supply chain traceability and the frequency of testing the aggregated supply. Developing these capabilities can be costly. For example, it is very difficult to trace and inspect every individual farm in a supply chain sourcing from thousands of farms (Nestlé 2015). Therefore, the manufacturer needs to balance between the cost of investing in these capabilities and the benefit of reducing EMA risk in the supply chain. To address this tradeoff, we develop an optimization model from the manufacturer’s perspective, where the objective is to minimize total investment costs while satisfying a constraint that the resulting risk of EMA in the supply chain cannot exceed a certain level.

First consider preemptive EMA. In this setting, the overall risk of EMA in the supply chain is measured by $n_a^*/n$ under perfect testing and $x^{PV*}$ under imperfect testing (see §3.2). Define $l(q)$ and $g(t)$ as the manufacturer’s investment costs for increasing testing frequency and traceability, both of which are convex and increasing functions. The manufacturer’s optimization problem under preemptive EMA can be characterized as follows.

$$\Pi_{PP}(q,t) = \min_{q,t} \{ l(q) + g(t) | n_a^*/n \leq \alpha, q \in [0,1], t \in [0,n] \},$$

$$\Pi_{PV}(q,t) = \min_{q,t} \{ l(q) + g(t) | x^{PV*} \leq \alpha, q \in [0,1], t \in [0,n] \},$$

where $n_a^*$ and $x^{PV*}$ are defined in Theorems 1 and 2, and $\alpha$ is the maximum level of risk allowed. For reactive EMA, the manufacturer’s optimization problem can be modeled similarly as follows.

$$\Pi_{RI}(q,t) = \min_{q,t} \{ l(q) + g(t) | P_n \leq \alpha, q \in [0,1], t \in [0,n] \},$$

$$\Pi_{RE}(q,t) = \min_{q,t} \{ l(q) + g(t) | E_n \leq \alpha, q \in [0,1], t \in [0,n] \},$$

where $P_n$ and $E_n$ are defined in §4.2 given the farms’ optimal adulteration strategies under perfect and imperfect testing, characterized in Theorems 3 and 4. The key difference is that we measure the risk of reactive EMA in the supply chain in two ways: the probability of an individual farm adulterating (i.e., $P_n$ as in Model (5)) and the expected total amount of adulterated output in the supply chain (i.e., $E_n$ as in Model (6)).

Before characterizing the manufacturer’s optimal investment strategy under each of these model scenarios, we first define the following useful constants.

(i) For Model (3):

$$u_{PP} = \frac{(r_H - r_L)(p_{\text{max}} - p_{\text{min}})n^2}{c(1 + [n\alpha])},$$

(7)
(ii) For Model (4):
\[ u^{PV} \equiv -h'(\alpha)(r_H - r_L)n \frac{\alpha c(n+1)/n}{\alpha c(n+1)/n}. \] (8)

(iii) For Model (5) under perfect testing:
\[ u^{RP} \equiv \left( \frac{n^2(r_H - r_L)}{ck} \right) \left( \frac{p_Lk}{n} + \phi^{-1}(1 - \alpha)\sqrt{\frac{kp_L(1-p_L)}{n}} \right). \] (9)

(iv) For Model (5) under imperfect testing:
\[ u^{RV} \equiv \left( \frac{kn(r_H - r_L)}{c} \right) \left( \frac{p_Lk}{n} + \phi^{-1}(1 - \alpha)\sqrt{\frac{kp_L(1-p_L)}{n}} \right) + \left( n - 1 \right) \int_0^{\frac{p_Lk}{n} + \phi^{-1}(1 - \alpha)\sqrt{\frac{kp_L(1-p_L)}{n}}} xf(x, k/n, p_L)dx \left( \frac{p_Lk}{n} + \phi^{-1}(1 - \alpha)\sqrt{\frac{kp_L(1-p_L)}{n}} \right)^{-1}. \] (10)

The notation \( \phi \) represents the PDF of the standard normal distribution. These constants are the values of \( q_t \) when the risk constraint in the corresponding models indicated is binding. Note that in the optimal solution to these models, the risk constraint must be binding because \( n^*_a, x^{PV^*}, \) and \( P_n \) are all decreasing in \( q \) and \( t \), whereas the investment costs are increasing in \( q \) and \( t \). The following theorem summarizes the manufacturer’s optimal investment strategy for Models (3), (4), and (5) under perfect and imperfect testing.

**Theorem 5.** Given the constants \( u^j \) with \( j \in \{ PP, PV, RP, RV \} \) defined in Equations (7)–(10), we have the following results for \( \alpha \leq 0.5 \).

(i) If \( u^j > n \), then the corresponding manufacturer problem is infeasible.

(ii) If \( u^j \leq n \), then the optimal solution to the corresponding manufacturer problem \( (q^*, t^*) \) can be characterized as follows. (a) If \( l'(1) \leq u^j g'(u^j) \), then \( (q^*, t^*) = (1, u^j) \). (b) If \( g'(n) \leq \frac{u^j}{n^2} l'(\frac{u^j}{n}) \), then \( (q^*, t^*) = \left( \frac{u^j}{n}, n \right) \). (c) If \( l'(1) > u^j g'(u^j) \) and \( g'(n) > \frac{u^j}{n^2} l'(\frac{u^j}{n}) \), then \( (q^*, t^*) \in (0, 1) \times (0, n) \)

and satisfy the following first-order conditions: \( q^* = \sqrt{\frac{u^j g'(u^j/q^*)}{l'(q^*)}} \) and \( t^* = \frac{u^j}{q^*} \).

Theorem 5 part (i) suggests that if the manufacturer cannot satisfy the risk constraint even when the supply chain is fully traceable and she always tests the aggregated supply, then additional levers are necessary to meet the risk constraint. Given our earlier discussions in §3.2 and §4.2, one possible solution is to reduce supply chain dispersion. Theorem 5 part (ii) shows that when a feasible solution exists, the manufacturer always chooses the solution with the best cost-effectiveness. If the marginal cost at maximum testing frequency is lower than the marginal cost at the minimum necessary traceability to satisfy the risk constraint (i.e., increasing testing frequency is in general more cost effective than
increasing traceability; Theorem 5 part (ii-a)), then it is optimal for the manufacturer to always test the aggregated supply and build just enough traceability given the risk constraint. Conversely, if increasing traceability is in general more cost effective than increasing testing frequency (Theorem 5 part (ii-b)), then it is optimal for the manufacturer to build full traceability in the supply chain and test just enough given the risk constraint. If neither of the above is true (Theorem 5 part (ii-c)), then the optimal investment is an interior solution that achieves the best cost balance between investing in the two levers.

Our next proposition characterizes how the optimal investment solution \((q^*, t^*)\) described in Theorem 5 and the resulting optimal cost change with supply chain dispersion.

**Proposition 10.** Let \(SC_H\) and \(SC_L\) be two supply chains such that the supply chain dispersion in \(SC_H\) is greater than that in \(SC_L\) (i.e., \(n_H > n_L\)). Consider each of the manufacturer’s optimization problems formulated in Models (3), (4), and (5) under perfect and imperfect testing. We have the following results for \(\alpha \leq 0.5\).

(i) If the manufacturer’s problem is infeasible for \(SC_L\), then it is also infeasible for \(SC_H\).

(ii) If the manufacturer’s problem is feasible for \(SC_H\), then it is also feasible for \(SC_L\).

(iii) Assume that the manufacturer’s problem is feasible for \(SC_H\). Let \((q_{H}^*, t_{H}^*)\) and \((q_{L}^*, t_{L}^*)\) be the optimal solution for \(SC_H\) and \(SC_L\) respectively. Then, \(q_{L}^* \leq q_{H}^*, t_{L}^* \leq t_{H}^*\), and the resulting optimal cost for the manufacturer is lower in \(SC_L\) than in \(SC_H\).

Proposition 10 highlights two results. First, given a desirable risk constraint, it is always more difficult for a manufacturer with a more dispersed supply chain to satisfy the constraint (parts (i) and (ii)). Second, conditional on being able to satisfy the risk constraint, it is always more costly for a manufacturer with a more dispersed supply chain to do so (part (iii)). Therefore, higher supply chain dispersion results in greater challenges for a manufacturer to manage and mitigate the risk of individual farms adulterating, from both feasibility and financial standpoints.

Finally, we consider the manufacturer’s problem formulated in Model (6). The key difference in this model versus the others is that the risk constraint is imposed on \(E_n\), the expected total amount of adulterated output in the supply chain. Since \(E_n\) aggregates all farms’ adulteration decisions, we cannot derive the manufacturer’s optimal decisions analytically. Nevertheless, consistent with Proposition 10, we show that higher supply chain dispersion again makes it more costly for the manufacturer to satisfy a desirable risk constraint, regardless of testing sensitivity (perfect or imperfect testing).
Table 2 Parameter Values for Guangdong and Shandong Provinces

<table>
<thead>
<tr>
<th>Province</th>
<th>n</th>
<th>m</th>
<th>r_H</th>
<th>r_L</th>
<th>c</th>
<th>q</th>
<th>t</th>
<th>p_L^{max}</th>
<th>p_L^{min}</th>
<th>p_L</th>
</tr>
</thead>
<tbody>
<tr>
<td>Guangdong</td>
<td>8</td>
<td>84,000</td>
<td>19.07</td>
<td>0</td>
<td>19.07</td>
<td>(0.1,..,0.9)</td>
<td>{1,..,8}</td>
<td>0.2,..,0.9</td>
<td>0.1</td>
<td></td>
</tr>
<tr>
<td>Shandong</td>
<td>13</td>
<td>67,000</td>
<td>19.07</td>
<td>0</td>
<td>19.07</td>
<td>(0.1,..,0.9)</td>
<td>{1,..,13}</td>
<td>0.2,..,0.9</td>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

Proposition 11. Let SC_H and SC_L be two supply chains such that the supply chain dispersion in SC_H is greater than that in SC_L, i.e., n_H > n_L. Consider the manufacturer’s optimization problem formulated in Model (6) and assume that it is feasible for SC_H. If \( \alpha \leq n \int_{m/p/3} x f(x,m,p)dx \), then for both perfect and imperfect testing, the optimal cost for the manufacturer is lower in SC_L than in SC_H.

6. Relating Model Predictions to Empirical Observations

In this section, we calibrate our model parameters with field data to the extent possible to examine how well our models’ predictions align with empirical evidence in various EMA scenarios.

6.1. Preemptive EMA: Misuse of Antibiotics in Poultry Farming in China

In China’s poultry industry, the distributed farming model in which chickens are sourced from a large number of smallholder farms is still very common (Pi et al. 2014). Due to multiple outbreaks of avian flu, there has been a rise in the misuse of antibiotics, antivirals, and Chinese traditional medicines by Chinese poultry farms (Huang et al. 2017). In 2012, CCTV undercover reporters found that farmers working for two major suppliers of KFC restaurants in China were feeding their chickens with as many as 18 antibiotics (CCTV 2012). We collected farming supply chain data published by China’s General Administration of Quality Supervision, Inspection, and Quarantine (AQSIQ), a government agency responsible for entry-exit commodity inspection, certification, accreditation, and import-export food safety. By utilizing this farming data and various market data, we calibrate our model parameters and predict the risk levels of poultry manufacturers in different production regions in China. Our analysis focuses on two leading provinces of poultry production, Shandong and Guangdong, which account for 15% and 8% of the total production (Inouye 2017).

Table 2 summarizes the parameter values used in our analysis for the two provinces. The values of n and m are derived by averaging the number of farms and the sizes of farms (in the number of chickens produced annually) supplying to different poultry manufacturers in each province (18 manufacturers in Guangdong and 34 in Shandong). Chinese regulations require that adulterating farms pay the market value of their output as penalty (National People’s Congress 2015). Therefore, we set \( c = r_H \).

In addition, the penalties faced by larger farms are often more severe than those faced by smaller ones. We capture this difference by using \( cm^2 \) as the penalty function. Since we cannot find information about the manufacturers’ testing frequency and the traceability of their supply chains, we consider a
wide range of values for both $q$ and $t$. We let $h(x) = (p_L^{max} - p_L^{min})x^2 - 2(p_L^{max} - p_L^{min})x + p_L^{max}$. This functional form ensures that $h(0) = p_L^{max}$ and $h(1) = p_L^{min}$. We fix $p_L^{min}$ to be 0.1 and vary $p_L^{max}$ from 0.2 to 0.9 to capture different levels of exogenous uncertainty in the quality of a farm’s output. Finally, since poultry farms use a large variety of drugs in their practices, we consider the imperfect testing scenario. Given these parameter values, we calculate the risk levels of an average manufacturer in these two provinces with 936 different parameter combinations.

Our results show that the preemptive EMA risk faced by an average poultry manufacturer in Shandong is always higher than that faced by an average manufacturer in Guangdong. Averaging over all parameter instances, the risk in Shandong is twice of that in Guangdong. This prediction is consistent with empirical evidence that more poultry manufacturers in Shandong have been found to be involved in EMA incidents than those in Guangdong (Huang et al. 2017). Among the manufacturers in our data, 4 out of 34 (11.8%) Shandong companies and 1 out of 18 (5.6%) Guangdong companies were caught in EMA incidents. Figure 3 further illustrates how the amount of adulterants added in equilibrium changes as $t$ (left figure) and $p_L^{max}$ (right figure) change. We observe that adulteration increases at a faster rate for Shandong than for Guangdong as $t$ decreases or $p_L^{max}$ increases. This result implies that Shandong is at a greater risk of increased adulteration due to external factors such as quality uncertainty, and manufacturers in Shandong can gain more benefit in risk mitigation by improving the traceability of their supply chains.

### 6.2. Reactive EMA: Melamine-Tainted Infant Formula Scandal

The second empirical case we consider is the melamine-tainted infant formula scandal discussed in §1. In this scandal, six infants died and thousands were hospitalized due to consumption of infant formula tainted with melamine. We use our model to examine the risk levels of two major dairy companies operating in China at that time, Sanlu Group and Bright Dairy. Sanlu used a distributed farming model in which it sourced 2.6 million liters of raw milk from 52,000 small farms primarily through middlemen (Chen et al. 2014). Therefore, traceability was seriously lacking. These small farms were
Table 3 Parameter Values for Sanlu Group and Bright Dairy

<table>
<thead>
<tr>
<th></th>
<th>n</th>
<th>m</th>
<th>(p_L)</th>
<th>t</th>
<th>q</th>
<th>(r_L)</th>
<th>(r_H)</th>
<th>c</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sanlu Group</td>
<td>52,000</td>
<td>50</td>
<td>(0.5,\ldots,0.9)</td>
<td>(520,\ldots,2080)</td>
<td>(0.1,\ldots,0.5)</td>
<td>6.8 (1.05r_L,\ldots,1.25r_L)</td>
<td>(r_H)</td>
<td></td>
</tr>
<tr>
<td>Bright Dairy</td>
<td>2,335</td>
<td>700</td>
<td>(0.1,\ldots,0.5)</td>
<td>(520,\ldots,2080)</td>
<td>(0.5,\ldots,0.9)</td>
<td>6.8 (1.05r_L,\ldots,1.25r_L)</td>
<td>(r_H)</td>
<td></td>
</tr>
</tbody>
</table>

also found to often produce low-quality milk due to poor diets and disease control of the cows (Gale and Hu 2009). In addition, due to an exemption of certification verification from the Chinese government, Sanlu rarely tested the raw milk before production (Flynn and Zhao 2014). All of these data suggest that \(p_L\) is high whereas \(t\) and \(q\) are low in Sanlu’s supply chain. Bright Dairy’s supply chain was in sharp contrast to Sanlu’s (Flynn and Zhao 2014). It adopted a vertically integrated model and sourced 1.625 million liters of raw milk from 2,335 corporate-owned or cooperative farms. The company stored samples from each cow to ensure full traceability. In addition, it also made significant investments in feed and animal health in the farms and frequently conducted quality tests to uphold its high quality standards. Therefore, in Bright Dairy’s supply chain, \(p_L\) is low while both \(t\) and \(q\) are high.

Based on the above discussion, we construct the parameter set as summarized in Table 3 for our analysis. In 2008, the average price of raw milk was 6.8 RMB per liter (Gale and Arnade 2015). We use this price as \(r_L\) and vary the premium for high-quality milk to be 5% to 25% above \(r_L\). We again assume \(c = r_H\), use \(cm^2\) as the penalty function, and consider imperfect testing. Note that qualitative evidence suggests that traceability in Sanlu’s supply chain is almost absent. We nevertheless allow for some traceability for Sanlu in our analysis, and hence, we likely underestimate its risk level. We analyze a total of 500 parameter combinations for each company. Table ?? demonstrates the stark contrast in the reactive EMA risk faced by Sanlu Group and Bright Dairy, reflected in both the probability of an individual farm adulterating \((P_n)\) and the expected fraction of the total supply chain output being adulterated \((E_n/k)\). These predictions match the empirical evidence. In particular, Sanlu Group’s products were the most heavily adulterated in the scandal, whereas Bright Dairy was one of two dairy companies that passed all quality inspections by the authorities during the crackdown (Chen et al. 2014). Figure 4 shows, for each company, how the risk of an individual farm engaging in reactive EMA \((P_n)\) changes as \(t\) (left figure) and \(r_H\) (right figure) change. While Sanlu’s risk increases quickly as \(t\) decreases or \(r_H\) increases, Bright Dairy’s risk is almost 0 in all instances. Despite the lack of perfect testing for melamine (at that time), the less dispersed supply chain, as well as better traceability and quality assurance policies established in Bright Dairy’s supply chain, have acted as important levers in guarding against reactive EMA risk for the company.
7. Model Extensions

We examine multiple extensions to the models discussed so far to demonstrate the robustness of our conclusions to a number of modeling assumptions. First, we find qualitative evidence that larger companies who are caught adulterating face more severe penalty than smaller ones. For example, they face longer jail terms and are fined more heavily (Yan 2017). This observation can be captured by modeling the total penalty from adulterating as convex increasing in $m$. We show that all of our results continue to hold in this alternative setup.

**Proposition 12.** All of our results in §3 and §4 continue to hold if the penalty that a farm incurred for adulterating is convex increasing in the total number of units, $m$, supplied by the farm.

### 7.1. Reactive EMA with Decision on How Much to Adulterate

In §4.1.2, we focus on the setup where farms adulterate either all or none of the realized low-quality units. An alternative setup is for farms to decide how many of the realized low-quality units to adulterate. This setup can capture scenarios in which a farm adulterates to fake the overall quality of his output to a desirable level. We first note that our results for perfect testing will not change under this setup. This is because under perfect testing, any amount of adulteration induces the same level of expected penalty, whereas the revenue gain increases in the number of units being adulterated. Hence, a farm would always adulterate all of his low-quality units if he decides to adulterate. Theorem 6 below characterizes the farms’ equilibrium adulteration strategy under imperfect testing, where $a^*(n_{L,i})$ denotes the number of realized low-quality units that farm $i$ adulterates in equilibrium.

**Theorem 6.** For reactive EMA with imperfect testing where a farm can choose how many of the realized low-quality units to adulterate, there exists a unique symmetric BNE of the game in which a farm’s adulteration strategy is a threshold strategy: $a^*(n_{L,i}) = n_{L,i}$ if $n_{L,i} \in [0, \beta_f^{RV}]$ and $a^*(n_{L,i}) = \beta_f^{RV}$ if $n_{L,i} \in (\beta_f^{RV}, m]$, for all $i$. The threshold $\beta_f^{RV}$ is unique and determined as follows: (a) If $cq\left(\frac{1}{n}\right) \geq \frac{n(r_H - r_L)}{2 + (n - 1)p_L}$, then $\beta_f^{RV} \in (0, m)$ and is the solution to the equation: $2\beta = \frac{n_k(r_H - r_L)}{cq} - (n - 1)\left(\int_{0}^{\beta} x f(x; k, p_L)dx + \int_{\beta}^{m} f(x; \frac{k}{n}, p_L)dx\right)$. (b) If $cq\left(\frac{1}{n}\right) < \frac{n(r_H - r_L)}{2 + (n - 1)p_L}$, then $\beta_f^{RV} = m$. 

Figure 4: Effects of Traceability ($t$) and Price for High-Quality Supply ($r_H$) on Reactive EMA

- **Graph 1:** Probability of an Individual Farm Adulterating vs. Number of Stored Samples
  - Bright Dairy: $0$, $0.1$, $0.2$, $0.3$, $0.4$, $0.5$, $0.6$, $0.7$, $0.8$, $0.9$, $1$
  - Sanlu: $520$, $1040$, $1568$, $2080$

- **Graph 2:** Probability of an Individual Farm Adulterating vs. Per-Unit Price of High-Quality Milk ($r_H$)
  - Bright Dairy: $7.14$, $7.48$, $7.82$, $8.16$, $8.5$
  - Sanlu: $7.14$, $7.48$, $7.82$, $8.16$, $8.5$
Theorem 6 shows that when farms can choose to adulterate a fraction of his low-quality units, then they adulterate all low-quality units up to a threshold, after which they adulterate a constant number of low-quality units. This structure is very similar to that in Theorem 4. The only difference is that in the current setup, when a farm has many low-quality units, the farm would adulterate just enough to make the marginal revenue gain from adulteration equal to the marginal penalty, i.e., adulterating $\beta_i^{RV}$ units (as opposed to not adulterating at all in Theorem 4). Similar to Proposition 7, we show that the risk of reactive EMA as measured by the total expected number of adulterated output in the supply chain is increasing in supply chain dispersion.

**Proposition 13.** For reactive EMA with imperfect testing where a farm can choose how many of the realized low-quality units to adulterate, $\frac{\partial E_n}{\partial n} \geq 0$.

### 7.2. Convex Increasing Testing Sensitivity under Imperfect Testing

In this extension, we analyze a setting of imperfect testing where the detection probability is convex increasing (rather than linearly increasing as in §3.1.2 and §4.1.2) in the relative amount of adulterated output in the total supply chain output. Formally, let the detection probability $S: [0, 1] \rightarrow [0, 1]$ be a convex increasing function such that $S(0) = 0$ and $S(1) = 1$. That is, the detection probability should be 0 (1) if none (all) of the output is adulterated. Under perfect testing, if farm $i$ adulterates with $x_i$, then the chance that the manufacturer detects adulteration when testing the aggregated supply is equal to $\frac{S(x_i)}{n} + \sum_{-i} \frac{S(x_{-i})}{n}$. If the manufacturer further tests the individual sample of farm $i$, then she detects adulteration in the sample with probability $S(x_i)$. Since the manufacturer tests the aggregated supply with probability $q$, the ultimate probability for farm $i$ to be caught if he adulterates is equal to $\gamma_i(x_i, x_{-i}) \equiv q \left( \frac{1}{n} \right) \left( \frac{S(x_i)}{n} + \sum_{-i} \frac{S(x_{-i})}{n} \right) S(x_i)$. Similarly, the probability for farm $i$ to be caught if he adulterates under imperfect testing is equal to $\gamma_i(n_{L,i}, a_{-i}(n_{L,-i})) \equiv q \left( \frac{1}{n} \right) S\left( \frac{n_{L,i}}{m} \right) \mathbb{E}_{n_{L,-i}} \left[ S\left( \frac{n_{L,i} + \sum_{-i} \sum_{k} n_{L,-i} a_{-i}(n_{L,-i})}{k} \right) \right]$.

We again analyze a setting where a farm adulterates either all or none of his realized low-quality units; i.e., the adulteration strategy can be characterized by the mapping $a_i: \{1, \ldots, m\} \rightarrow \{0, 1\}$. Theorems 7 and 8 below show that the farms’ equilibrium adulteration behavior under either testing scenario follows a very similar structure as in Theorems 2 and 4 in §3.1.2 and §4.1.2 respectively.

**Theorem 7.** For preemptive EMA with imperfect testing and convex increasing testing sensitivity modeled by $S(\cdot)$, there exists a unique symmetric NE in which $x_{PV}^*$ is determined as follows.

(a) If $c < -h'(1)(r_H - r_L)/[S'(1)q(t/n)((n + 1)/n)]$, then all farms adulterate to the maximum level; i.e., $x_{PV}^* = 1$. 
(b) If \( c \geq -h'(1)(r_H - r_L)/[S'(1)q(t/n)((n + 1)/n)] \), then the farms adulterate to some extent; i.e.,

\[ x^{PV^*} \in (0, 1) \] and is the solution to the following equation:

\[ -h'(x) = S(x)S'(x)q(t/n)((n + 1)/n)c/(r_H - r_L). \]

**Theorem 8.** For reactive EMA with imperfect testing and convex increasing testing sensitivity modeled by \( S(\cdot) \), there exists a unique symmetric BNE of the game in which a farm’s adulteration strategy is a threshold strategy: \( a^*(n_{L,i}) = 1 \) if \( n_{L,i} \in [0, \beta^S) \) and \( a^*(n_{L,i}) = 0 \) if \( n_{L,i} \in [\beta^S, m] \), for all \( i \). The threshold \( \beta^S \) is unique and determined as follows:

(a) If \( cq\left( \frac{1}{n} \right) \geq \frac{1}{E_{n_{L,i}}} \left[ \frac{1}{\left( \frac{r_H - r_L}{m + \sum_{i=1}^{n} n_{L,i}} \right)} \right] \), then \( \beta^S \in (0, m) \) and is the solution to the equation:

\[ \beta^S(r_H - r_L) = q\left( \frac{1}{n} \right) S\left( \frac{\beta^S}{m} \right) E_{n_{L,i}} \left[ S\left( \frac{\beta^S + \sum_{i=1}^{n} n_{L,i} [n_{L,i} \leq \beta^S]}{k} \right) \right] cm, \]

where \( \mathbb{I}\{\cdot\} \) is an indicator function whose value is 1 if the argument is true and 0 otherwise.

(b) If \( cq\left( \frac{1}{n} \right) < \frac{1}{E_{n_{L,i}}} \left[ \frac{1}{\left( \frac{r_H - r_L}{m + \sum_{i=1}^{n} n_{L,i}} \right)} \right] \), then \( \beta^S = m \).

### 7.3. Preemptive EMA with Risk-Averse Farms

Our last extension considers risk aversion for farms. This extension is valuable because in practice, farms, especially small ones, rely on selling their output to sustain the livelihood of the family, and hence, tend to be risk averse. Risk aversion may impact farms’ preemptive EMA decisions since a small farm faces greater variability in the realized quality of his output due to the small number of units he produces. We capture such risk aversion by subtracting a risk penalty, modeled as \( \lambda E[\mathbb{I}\{n_L > \gamma m\}] \), from the farm’s expected payoff function formulated in §3.\(^3\) This penalty captures the farm’s aversion to producing too many low-quality units, which can threaten the livelihood of his family. The next two theorems characterize risk-averse farms’ equilibrium adulteration behavior under perfect and imperfect testing.

**Theorem 9.** For preemptive EMA with perfect testing and risk-averse farms, the total number of adulterating farms in any NE of the game is characterized by

\[ n_{RA}^* = \min\{n, [T^{RA} - 1]\}, \]

where \( T^{RA} = n^x\left(\frac{r_H - r_L}{p_L^{max} - p_L^{min}}\right) + (\lambda/m) \left( F(\gamma, m, p_L^{max}) - F(\gamma, m, p_L^{min}) \right) \equiv 1 - \int_{0}^{m} f(x, m, h(x))dx. \)

**Theorem 10.** Define \( T(x) \equiv -h'(x) \left( \frac{r_H - r_L}{cq(t/n)((n + 1)/n)} \right) + \lambda f(\gamma, m, h(x)) \left( \frac{\gamma(1-2h(x) + h(x))}{2h(x)(1-h(x))} \right) \). For preemptive

\(^3\) Our approach follows the spirit of the value-at-risk method (McNeil et al. 2015), where the risk penalty can be viewed as we Lagrangify the value-at-risk constraint in the farm’s objective function.
EMA with imperfect testing and risk-averse farms, if \( \gamma \leq \frac{P_{L_{\min}}}{1 - 2P_{L_{\min}}(1-P_{L_{\min}})} \), then there exists a unique symmetric NE of the game in which \( x_{RA_{PV}}^{*} \) is determined as follows. (a) If \( c < T(1) \), then all farms adulterate to the maximum level; i.e., \( x_{RA_{PV}}^{*} = 1 \). (b) If \( c \geq T(1) \), then the farms adulterate to some extent; i.e., \( x_{RA_{PV}}^{*} \in (0,1] \) and is the solution to the equation: \( x = T(x) \).

We observe that the farms' equilibrium adulteration behavior under either testing scenario follows a very similar pattern as in Theorems 1 and 2 in §3.1. The condition on \( \gamma \) in Theorem 10 means that a farm should begin to exhibit aversion when the number of his low-quality units is not too large. This condition is reasonable given our focus on risk-averse farms. Our next result shows that the risk of preemptive EMA in the supply chain is higher when farms are risk averse than when they are expected-payoff maximizers, under both perfect and imperfect testing. With respect to the effect of supply chain dispersion on preemptive EMA risk in the supply chain, we show as in Proposition 1 that greater dispersion leads to higher risk under imperfect testing.

**Proposition 14.** \( n_{a}^{RA^{*}} \geq n_{a}^{*} \) and \( x_{RA_{PV}}^{*} \geq x_{PV}^{*} \). Further, If \( \gamma \leq \frac{P_{L_{\min}}}{1 - 2P_{L_{\min}}(1-P_{L_{\min}})} \), then \( \frac{\partial x_{RA_{PV}}^{*}}{\partial n} \geq 0.4 \).

For the case of perfect testing, we cannot characterize the effect of dispersion on risk analytically. Therefore, we perform extensive numerical simulation and observe that in a total of 32,000,000 numerical instances we run, greater dispersion always leads to a higher risk.5

8. Conclusions

In this paper, we develop a series of analytical models to investigate how exogenous quality uncertainty, supply chain dispersion, traceability, and testing sensitivity (with regard to detecting adulteration) jointly impact farms’ strategic adulteration behavior in a farming supply chain consisting of a distributed network of farms. We focus on economically motivated adulteration and consider two distinct scenarios: “preemptive EMA” in which adulteration occurs before the uncertain quality of a farm’s output is realized with the primary goal of reducing the probability of producing low-quality output; “reactive EMA” in which adulteration occurs after the uncertain quality of a farm’s output is realized with the primary goal of increasing the perceived quality of the output and creating fake high-quality units. We fully characterize the farms’ equilibrium adulteration strategies for both scenarios. We show how these strategies are impacted by quality uncertainty, the level of dispersion and traceability in the

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4 We treat \( n \) as a continuous variable in this analysis for tractability.

5 We use the following parameter values in the numerical simulation: \( k = 100000, \) 100 \( m \) values in \( \{100,\ldots,1000\} \), 20 \( P_{L_{\min}} \) values in \( \{0.1,\ldots,0.9\} \), \( P_{L_{\min}} = 0.1 \), 20 \( q \) values in \( \{0.1,\ldots,0.9\} \), \( r_{H} = 19.07, \) \( r_{L} = 0, \) \( t = n, \) \( \lambda = \{1,2,\ldots,40\} \), and 20 \( \gamma \) values in \( \{0.1,\ldots,0.9\} \).
supply chain, and the sensitivity of the manufacturer’s test for adulteration. Furthermore, we analyze the manufacturer’s optimal investment decisions in traceability and testing frequency to satisfy certain risk constraints at minimum costs. We also calibrate our model based on real cases and field data and demonstrate that the models’ predictions are in line with empirical observations. Finally, we examine a number of model extensions to confirm that our main conclusions are robust to a few modeling assumptions.

Our analysis offers important and unique insights for policy makers and commercial entities in food supply chains to more proactively address EMA risk. First, we demonstrate that supply chain dispersion increases EMA risk in farming supply chains. This result complements the supply chain risk management literature in which multi-sourcing is found to be desirable for establishing supply chain resilience to guard against disruption risks (e.g., earthquake, fire, hurricane). We study, instead, quality risks related to endogenous decisions by entities within the supply chain (i.e., farms adulterating their output). A recent paper by Huang et al. (2017) empirically confirms the adverse effect of supply chain dispersion on EMA risk in China’s farming supply chains, based on supply chain and quality data across five different industries. Second, by explicitly modeling exogenous quality uncertainty (common in agricultural production) and distinguishing it from endogenous adulteration decisions, we caution that quality investments need to be accompanied by improvement in testing capabilities. This is because a lower (exogenous) chance of producing low-quality output could inadvertently induce suppliers to endogenously adulterate the realized low-quality units, if the detection of adulteration relies on having a sufficient amount of adulterants in the total output. In such situations, the ability to differentiate fake high-quality products from truly high-quality ones is essential to mitigate risk of EMA.

Our results have important practical implications and also open up a fruitful avenue for future research. We recommend that companies address risks resulting from supply chain dispersion by mitigating the potential underlying issues in a dispersed supply chain, for example, by enabling better traceability and risk sharing between farms and manufacturers. Some possible strategies include creating farming cooperatives that can allow for better traceability and knowledge transfer in terms of best practices, or developing fairer contracts and support systems such as protective prices and guaranteed distribution channels for farms (especially smallholder farms who face significant financial pressures). Future analytical and empirical research should be done to better understand the effectiveness of these remedies in mitigating EMA risk under different market and socio-economic environments. For policy makers and regulators, we underscore the importance of collecting data and verifying a food manufacturer’s sourcing supply chain to more proactively manage EMA risk in food products. Current
practices primarily focus on sampling at the product level and inspection at the manufacturer level, with little attention to the upstream parts of the supply chains where agricultural supply is produced. As countries around the world are scaling up their food defense efforts, with prominent examples such as the recent enactment of the Food Safety Modernization Act in the U.S. and the new Food Safety Law in China, our results offer timely and actionable insights on the relatively overlooked supply chain perspective in the defense of food safety.

References


