Worst-Case Analysis of Process Flexibility Designs with Price Differentials

1 Introduction

In today’s competitive business environment where consumers’ demand have become increasingly uncertain and volatile, the ability to match supply and demand is arguably key to a firm’s success. Unfortunately, capacity investment decisions typically frozen far in advance before the uncertain demand is realized, misalignments between supply and demand are usually inevitable and often times bear severe economic consequences. In order to maintain a competitive advantage, firms have employed an operations strategy known as process flexibility, defined as a firm’s ability to “build different types of products in the same manufacturing plant or on the same production line at the same time” (Jordan and Graves 1995, p. 577), to better respond to demand uncertainty. Process flexibility has been proven to be a successful operational strategy in manufacturing industries to hedge against demand uncertainty, and a seminal work by Jordan and Graves (1995) demonstrates that even just a little flexibility, if configured in the right way, can be extremely effective in mitigating supply-demand mismatch. Among all the designs with limited flexibility, the chaining strategy, and in particular the long chain design proposed by Jordan and Graves (1995) is perhaps one of the most influential strategies both studied in the literature and used in practice. They show that there exist sparse networks with limited flexibility that perform almost as well as a full flexibility, and this limited flexibility accrues the greatest benefits when configured to chain products and plants together to the greatest extent possible. Motivated by the empirical findings in Jordan and Graves (1995), the effectiveness of the long chain and designs with limited flexibility has been investigated theoretically in many recent works, such as Chou et al. (2010), Bassamboo et al. (2010), Chou et al. (2011), Simchi-Levi and Wei (2012), Simchi-Levi and Wei (2015), Wang and Zhang (2015), Chen et al. (2015) and Désir et al. (2016). Most if not all of the above works investigate the performance of flexibility designs by choosing a metric that is based on the maximum sales in units that can be achieved in a given design. In particular, for any realized demand instance, one solves a max flow problem to determine the optimal way to allocate available capacities to fulfill the demand as much as possible. However, the maximum sales metric does not take into account any potential price differentials that may appear among the products, which could lead to sub-
stantial profit losses when the maximum sales metric is used to guide flexibility designs whereas the products have considerably large price differences. Our objective in this paper is to examine the worst-case performance of flexibility designs when there exist price differentials among the products and the evaluation metric that aims to maximize profit is of interest.

2 The Model

We consider a general manufacturing system with $m$ plants and $n$ products, where each plant has a fixed capacity and each product has a given fixed price. A flexibility design is represented the arc set of a bipartite graph defined on the product nodes and plant nodes. The product demands are stochastic, and for a given a demand instance, the maximum profit achieved by a flexibility design can be determined by solving a linear program. We consider the worst-case profit among all possible demand instances in a given uncertainty set to evaluate the worst-case performance and hence measures the robustness of a flexibility design. In worst-case analysis, symmetric uncertainty sets are frequently used to model symmetric demand variations, where a set $U$ is said to be symmetric if for any $d \in U$, $d^\sigma \in U$ for any permutation $\sigma$ of the index set $\{1, \ldots, n\}$. A generalization of symmetric uncertainty sets is the class of symmetric perturbation uncertainty sets, where a set $U$ is called a symmetric perturbation uncertainty set if $E := \{x - \mu | x \in U\}$ is symmetric for some fixed $\mu$. Intuitively, one can interpret symmetric perturbation uncertainty sets as having product demands estimated to be $\mu$, and the estimation error (perturbation) has the same fluctuation across products around $\mu$. In our model, products are grouped into different categories where products in the same category have identical price. In view of the price heterogeneity across different product categories, we consider a class of uncertainty sets, the so-called part-wise symmetric perturbation uncertainty sets, which assume that the perturbation set is symmetric within each product category.

3 Main results

Our main results and contributions in this paper are summarized below. To study the worst-case performance of flexibility designs when there exist price differentials among the products, we introduce the Profit Plant Cover Index (PPCI) and prove that the PPCIs can be used to characterize a broad class of worst-case performance measures for general unbalanced and asymmetric production systems, which in turn leads to a partial ordering that enables us to compare the performance
of different flexibility designs. To the best of our knowledge, we are among the first to examine the performance of flexibility designs by adopting evaluation metrics based on maximizing total profits rather than sales in units.

Applying the above results, we prove that a class of long chain designs, the alternate long chain, is optimal among all long chain designs where there are two product categories and the number of high price products is equal to that of low price products. One implication of this result that may provide useful guidelines to practitioners is that, when designing the production systems, the most effective design is to equip each plant to be capable of producing both high profit and low profit products so that high profit products never have to directly compete for shared resources.

Finally, we propose a heuristic based on PPCI for constructing effective flexibility designs when there exist price differentials among the products and one aims to maximize the total profits. Our computational study suggests that the sparse design generated by our PPCI-Heuristic can capture most of the benefit of full flexibility from the expected profit point of view, and achieves satisfactory performance in the worst case for small to moderate level of demand uncertainty.

References


