
(Authors’ names blinded for peer review)

**Problem Definition:** Governments have adopted various subsidy policies to promote investment in renewable energy sources, such as rooftop solar panels. The German government uses a feed-in-tariff policy that provides a guaranteed stream of payments for each unit of electricity generated by a household. In contrast, the U.S. government uses a tax-rebate policy that reduces the initial investment cost and the household receives the retail price for the generated electricity. In this paper, we study the key economic factors that favor one policy over the other from the perspective of the government. These factors are the heterogeneity in the generating efficiency, and the variability in the electricity price and in the investment cost.

**Academic Relevance:** Unlike the previous literature on the feed-in-tariff and tax-rebate policies, we focus on the effects of variability on a household’s investment timing. We identify the optimal policy for the government to manage the aggregate household investment, accounting for the heterogeneity in efficiency.

**Methodology:** We consider an infinite-horizon, continuous-time model where the government moves first and announces either a feed-in tariff or tax rebate. Then, each household dynamically decides if and when to invest in a unit of solar panel. The objective of the government is to maximize the expected value of a subsidy policy, i.e., the difference between the societal benefit of solar panel investments and the subsidy cost over time.

**Results:** We characterize the timing of the investment decision of the households and the optimal value of the subsidy policies to the government. We also identify which economic factors favor the feed-in-tariff or the tax-rebate policy.

**Policy Implications:** Our results suggest that a government should prefer the feed-in-tariff policy when the electricity price is highly uncertain. Intuitively, feed-in-tariff policy eliminates the price variability; thus, it removes the strategic delay in the investment. The tax-rebate policy should be adopted if the households are heterogeneous in generating efficiency, if the investment cost is highly variable, or if the price and cost uncertainty are positively correlated.

**Key words:** Feed-in Tariff, Tax Rebate, Energy-related Operations, Sustainable Operations, Investment under Uncertainty
1. Introduction

In recent years, governments have adopted various subsidy policies to promote investment in renewable energy sources that generate sustainable electricity without carbon dioxide emissions. As a result of these policies, more than one million rooftop solar panels are installed in both Germany and the U.S. (SEIA 2016, Wirth 2017), yet these governments use two different subsidy programs. In Germany, the investments are facilitated by a feed-in-tariff policy, whereas in the U.S., the most common subsidy program for households is the tax-rebate policy. Under the feed-in-tariff policy, the German government provides a long-term price guarantee for the electricity generated by a household. Under the tax-rebate policy, the U.S. government offers a tax refund for a fraction of the initial investment cost and the household sells the electricity generated by the panels at the retail electricity price (by the net metering policy, see EEI 2016). Feed-in-tariff policy is more commonly used in the world: two thirds of solar panels have received feed-in tariffs in 2016 (IEA 2017) and several energy experts have suggested that the U.S. should also adopt this policy (GTM 2014 and Trabish 2016). In this paper, we characterize the conditions under which a government should prefer the feed-in-tariff or the tax-rebate policy. This research question is of paramount importance for responsible operations because generating electricity from rooftop solar panels is an environmentally and socially responsible way of harnessing energy.

In a simplest model, as the following analysis demonstrates, feed-in-tariff and tax-rebate policies are equivalent because both incur the same cost to the government to promote the same household investments. Consider a representative household that can generate $Q$ units of electricity in perpetuity (for simplicity) after incurring a constant investment cost of $X$. Let the price of electricity be $P$, also constant in perpetuity. Because the price and the cost are constant, a household can decide immediately whether to invest or not. The household invests as long as the net present value (NPV) of the investment is positive. Without any subsidy policies, the NPV of the investment is $\int_0^\infty PQe^{-rt} dt - X = \frac{P}{r}Q - X$, where $r$ is the discount rate. If this NPV is negative, the government can use a subsidy policy to encourage the investment. Under the feed-in-tariff policy, the government purchases the generated electricity at rate $F$ from a household so that the NPV of the household’s investment becomes $\int_0^\infty FQe^{-rt} dt - X = \frac{F}{r}Q - X$. Essentially, the government increases the household’s NPV by $\frac{F-P}{r}Q$ compared to the no subsidy case. Under the tax-rebate policy, the government reduces the investment cost by a fraction $R$, making the NPV of
the investment $\int_0^\infty PQe^{-rt} \, dt - (X - RX) = \frac{F}{r} - X(1 - R)$. Essentially, the government increases the household’s NPV by $RX$ compared to the no subsidy case. As long as the increase in the NPV under the feed-in-tariff and tax-rebate policies are equal, i.e., $\frac{F - P}{r} Q = RX$, the two policies induce the same investment decision from a representative household and levy the same cost on the government. Therefore, this simple analysis suggests that the feed-in-tariff and tax-rebate policies are equivalent.

This simple model ignores two salient economic factors that affect the government’s preference between the feed-in-tariff and tax-rebate policies in practice. First, a government faces a population of households with different levels of generating efficiencies rather than a representative household. The heterogeneity in the electricity generation is due to the differences in roof types and orientation, and shading from neighboring structures (Alizamir et al. 2016). Second, there is variability in residential electricity prices and solar panel investment costs. Specifically, the electricity prices fluctuate due to the adjustments under the so-called utility ratemaking process (RAP 2011). For example, in California, the annual volatility of the residential electricity price is estimated to be 35% (see Section 5.1). The prices tend to increase over time as the cost of electricity generation increases. Furthermore, the investment cost in solar panels is uncertain because of the fluctuations in the price of the commodities used in the panels (Fu et al. 2015). Also, the cost exhibits a downward trend (Fu et al. 2017).

Price and cost dynamics are significant in analyzing feed-in-tariff and tax-rebate policies because, in practice, households are not obligated to make investment decisions immediately after the policies are announced. Instead, under dynamic prices and costs, forward-looking households may prefer to wait to invest. In fact, several energy startups, such as EnergySage, are trying to combat this waiting behavior by matching households with solar developers (EnergySage 2016). The two subsidy policies affect the uncertainties differently: Feed-in-tariff policy eliminates the price variability by offering a guaranteed stream of payments to the households, whereas the tax-rebate policy reduces the exposure to the cost variability. Furthermore, the correlation between the price and cost uncertainties should be taken into account while comparing feed-in-tariff and tax-rebate policies. In the presence of these economic factors, it is not clear if the two subsidy policies are equivalent and which one should be preferred by the government.
To understand the effects of the aforementioned economic factors on the policy preference, we consider a continuous-time, infinite-horizon model where the government moves first and announces either the feed-in-tariff or the tax-rebate policy. Then each household responds by dynamically deciding if and when to invest in a solar panel so as to maximize the value of this investment. Both the electricity price and the investment cost follow stochastic processes and the households are forward-looking meaning that they take the dynamics of prices and costs into account while making investment decisions. Furthermore, the households are heterogeneous in terms of their generating efficiency. Accounting for the investment decisions of all households, the government maximizes the expected value of a subsidy policy. This value equals to the societal benefit of solar panels minus the subsidy cost. The societal benefit includes the environmental benefit of avoided carbon emissions, increased grid resiliency during extreme weather events (NREL 2014), and hedging against the fluctuations in fuel prices (CEC 2008). The benefit is obtained over time from the moment the investments are undertaken by the households. Under the feed-in-tariff policy, the cost is the feed-in tariff payments incurred over time from the moments of households’ investments, whereas the cost of the tax-rebate policy is the tax rebates paid to the households at the times of their investments.

We analyze the preference of the government between the two subsidy policies by focusing on each of the economic factors one-by-one to isolate their effects. First, we study the heterogeneity in the generating efficiency and find that the tax-rebate policy yields a higher value to the government compared to the feed-in-tariff policy. Under each policy, there exists a marginal household which is indifferent between making the investment or not. Intuitively, the feed-in-tariff policy incurs a higher cost in a heterogeneous population because households that generate more electricity than the marginal household receive higher payments. On the other hand, the rebate level is the same for each household and is equal to the payment to the marginal household. We further find that, under price variability, the feed-in-tariff policy outperforms the tax-rebate. This is because the uncertainty in price creates an option value of waiting to invest (see Section 4.3). However, the feed-in tariff eliminates the price variability for the household by offering a guaranteed stream of payments. Similar to the price variability, cost variability also creates an option value to wait. The tax-rebate policy reduces this option value to the household by reducing the exposure to the investment cost. Therefore, under cost variability, the tax-rebate policy
achieves a higher value compared to the feed-in-tariff policy. Finally, we consider these
economic factors simultaneously and examine the conditions under which a subsidy policy
outperforms. One interesting finding is that as the correlation between the price and the
cost uncertainty increases, the tax-rebate policy outperforms the feed-in-tariff policy. This
is because the positive correlation creates a natural hedge between prices and costs under
the tax-rebate policy, reducing the overall variability exposure of a household.

The rest of the paper is organized as follows. Section 2 reviews the related literature.
Section 3 presents our model, which is then analyzed in Section 4. Section 5 studies the
joint effect of the aforementioned economic factors. Section 6 concludes.

2. Literature Review

Our paper is related to the sustainable operations literature (see Drake and Spinler 2013
for a review) because we study the subsidy policies for environmentally sustainable rooftop
solar panels. More specifically, there is a growing literature on energy-related operations
that has investigated the effect of electricity prices on households’ consumption (Ata et al.
2016 and Kök et al. 2016), supply function equilibrium in wholesale electricity markets
(Al-Gwaiz et al. 2016 and Sunar and Birge 2017), joint management of storage and wind
energy (Qi et al. 2015 and Zhou et al. 2017), and investments of utility firms in renewable
and conventional sources (Aflaki and Netessine 2017 and Kök et al. 2017). We focus on the
residential electricity market and on the investments of the households. In this domain,
Hu et al. (2015) characterize the optimal investment level in rooftop solar panels for a
commercial building under the net metering policy that allows households to receive the
retail electricity price for the generation of solar panels. They show that the investment
decisions become more accurate as the data on electricity consumption and generation of
solar panels becomes more granular. Singh and Scheller-Wolf (2017) consider the effects of
the net metering policy and show that an electricity pricing policy with fixed charges allows
a regulator to freely allocate the costs (or benefits) of the solar panels among households,
a utility firm, and a solar manufacturer. We complement these studies by comparing the
combination of the net metering and tax-rebate policies against the feed-in-tariff policy.

The design of subsidy policies has received significant attention in recent years. Several
papers, including, Akkaya et al. (2016), Alizamir et al. (2017), and Gupta et al. (2017),
consider subsidy policies in the agriculture sector. In the context of developing countries,
Levi et al. (2016) show that a uniform subsidy given to a set of cost-heterogenous firms can be almost as effective as a firm-specific subsidy in increasing the customer welfare. Shen et al. (2016) investigate whether the manufacturers or the customers should be given a subsidy to maximize the social welfare. Ata et al. (2012) study a waste-to-energy firm, which collects household waste and turns it into renewable electricity. They compare the effectiveness of a lump-sum investment subsidy against a price premium for generated electricity. The results indicate that the lump-sum subsidy (similar to the tax-rebate policy) is more effective in increasing the electricity generation but the price premium (similar to the feed-in-tariff policy) achieves a higher coverage area for waste collection. Cohen et al. (2015) consider demand uncertainty in the design of consumer subsidies and show that a higher level of uncertainty results in a higher production quantity and lower price; however, the consumer surplus may decrease.

Our paper considers the design of the feed-in-tariff policy which is reviewed extensively by Mendonça et al. (2009) and Couture et al. (2010). Ritzenhofen et al. (2016) analyze the effects of the feed-in-tariff policy on the wholesale electricity market. In the residential market, Lobel and Perakis (2011) consider a discrete choice model to study the diffusion of solar panels and conclude that the initial subsidy levels should have been higher in Germany to increase the speed of diffusion. Goodarzi et al. (2015) study the pricing decisions of a solar panel manufacturer under the feed-in-tariff policy. The closest paper in this domain to our work is Alizamir et al. (2016) that use a dynamic programming formulation where the government sets the feed-in tariff level in each period, based on which a population of households adopt solar panels. Alizamir et al. (2016) assume that the households are heterogeneous in generating efficiency (as in our case) and forward-looking that they form rational expectations on the future feed-in-tariff levels. The authors characterize the conditions under which the feed-in tariffs should be set so that the profitability of the investment either increases or decreases over time. Different than Alizamir et al. (2016), we abstract away from the diffusion dynamics to focus on the impact of the variability in the electricity price and the investment cost. These variabilities and the irreversibility of the investment create an option value of waiting to invest, resulting in a delay in the investment until the investment has a strictly positive NPV. By using this real options approach, we show an additional benefit of the feed-in-tariff policy that eliminates the price variability and reduces the delay in investments.
The option value of waiting to invest has been characterized by the seminal paper of McDonald and Siegel (1986). Specifically, if a firm faces an irreversible investment project whose value stochastically evolves, it is optimal for the firm to delay the investment until the NPV becomes strictly positive. This is because by investing at any point in time, the firm forgoes the option to invest in the future when the value of the project may be more favorable. Dixit and Pindyck (1994) present various extensions of this basic setting. The real options approach has been used for many applications, including supply chain options (Burnetas and Ritchken 2005) and franchising contracts (Babich and Tang 2016). We use the real options approach as a building block in characterizing the investment decisions of a household. Then, in contrast to the majority of the real options literature, we aggregate the benefit and the cost of a subsidy policy across a population of households, accounting for the heterogeneity in the generating efficiency. Based on the aggregate benefit and the cost, we compute the optimal policy for the government to manage the aggregate household behavior.

Finally, in the energy economics literature, real options have been widely used to evaluate the investment decisions. A detailed review of this literature is given by Kozlova (2017). For example, Fleten et al. (2016) analyze the investments in hydropower plants in Norway, concluding that the investments are undertaken following the real options method. As an extensive numerical study, Ritzenhofen and Spinler (2016) investigate different feed-in-tariff designs on the renewable energy investment decisions. They focus on a single household for an exogenously given feed-in-tariff level, whereas we optimize the feed-in tariff over a population of households.

3. Model

We consider a continuous-time, infinite-horizon model with two decision makers: the government and the households. The government acts first and offers either a feed-in-tariff or a tax-rebate policy to the households in the beginning of the time horizon. Based on the subsidy policy offered, the households dynamically decide if and when to invest in solar panels. The objective of the households is to maximize the value of the investment, which is derived from the value of the electricity sold to the grid (from the time of the investment in perpetuity) minus the investment cost. If the government offers the feed-in-tariff policy as in Germany, a household receives the feed-in rate for the electricity generated by
the panels (Couture et al. 2010). If the government uses the tax-rebate policy as in the U.S., the household receives the retail electricity price (by the net metering policy, see EEI 2016), but the investment cost is reduced by the rebate.

3.1. Heterogeneity in Generating Efficiency
A household can install one unit of solar panels (e.g., 1 kW) at any point in the problem horizon. One unit of investment generates $Q \leq 1$ units of electricity per unit time in perpetuity (for simplicity). We refer to $Q$ as the generating efficiency. The households have different levels of efficiency because of the differences in roof angle/orientation and shading from neighboring structures (Alizamir et al. 2016).

To model the heterogeneity in generating efficiency, $Q$ follows a continuous probability density function (p.d.f.) $\psi(Q)$. This distribution is known to the government and each household draws its own efficiency level $Q$ from $\psi(Q)$. This level is fixed for the duration of the problem. Without loss of generality, we normalize the size of the household population to 1.

![Electricity Price and Investment Cost](image_url)

**Figure 1** Historical Electricity Prices and Investment Costs in California

3.2. Variability in the Electricity Price and the Investment Cost
A household is exposed to two sources of variability while making solar panel investments. First, the residential electricity price is variable as indicated by Figure 1a, which plots the historical electricity prices in California. These prices are determined by a regulatory process where minor changes are made every month and major changes are made every...
few years, according to the utility rate making process (RAP 2011). As shown in Figure 1a, in addition to the uncertainty, residential price process exhibits an upward trend due to the increases in the cost of electricity generation over time. To represent these features and, for analytical convenience, we let the electricity price follow a geometric Brownian motion (GBM):

$$dP(t) = \mu_p P(t)dt + \sigma_p P(t)dZ_p(t),$$

(1)

where $P(t)$ is the price at time $t$, $\mu_p \geq 0$ and $\sigma_p \geq 0$ are the drift and volatility parameters. Here, $Z_p(t)$ is a Wiener process, representing the price shock. We shall discuss the interpretation of process (1) with respect to the risk-neutral valuation method after we introduce the cost process (2) below.

The second source of variability for a household is the investment cost of a solar panel. The cost is uncertain due to the fluctuations in the prices of commodities, such as polysilicone, used in the manufacturing of the panels (Fu et al. 2015). In addition to the uncertainty, the investment cost process exhibits a downward trend as illustrated in Figure 1b. This trend could be due to the improvements in solar panel production technology (Fu et al. 2017). We denote the investment cost as $X(t)$ and, again for analytical convenience, model it as a GBM:

$$dX(t) = \mu_x X(t)dt + \sigma_x X(t)dZ_x(t),$$

(2)

where $\mu_x \leq 0$ and $\sigma_x \geq 0$ are the drift and volatility parameters, respectively, and $Z_x(t)$ is a Wiener process. We let $\rho dt$ be the instantaneous correlation between the cost shock $dZ_x(t)$ and the price shock $dZ_p(t)$.

The common discount rate for the households and the government is given as $r$ and $r > \mu_p$. We assume that the households are forward-looking in the sense that they take the dynamics of electricity prices and investment cost into account.

There are two interpretations of the price and cost processes (1) and (2). The first interpretation follows from a fundamental valuation result (e.g., Harrison and Pliska 1981), which states that as long as markets are arbitrage-free, there exists a pricing measure, such that all traded assets are martingales with respect to this measure and all valuations are performed by taking expectations with respect to this measure. If, in addition, markets are complete, this pricing measure is unique. The pricing measure captures investors’ non-linear utilities, while producing a linear pricing operator (see Cochrane 2001, Chapter 1).
Thus, we do not need to model non-linear risk preferences of investors explicitly. One can think of processes (1) and (2) as being presented under this pricing measure. Assumption $r > \mu_p$ reflects the convenience yield of holding tradable commodities spanning the risks in the $P(t)$ process.

The second interpretation of (1) and (2) is the actuarial one, where these processes are presented under the so-called “physical” measure and decision makers are risk-neutral. The assumption that $r > \mu_p$ ensures the convergence of infinite series of cash flows from investments. Mathematically, the two interpretations are equivalent (Dixit and Pindyck 1994, Chapter 4). In our analysis, we are agnostic to which of the interpretations is used and we refer readers to the literature that explores this point (e.g., Pliska 1997, Björk 1999, and Hull 2009).

We finally note that simple GBM processes, given in (1) and (2), do not reflect all practical aspects of either commodity prices in general or electricity prices in particular. For example, for commodity prices, mean-reversion might be considered (Schwartz and Smith 2000). For electricity prices, as in Geman (2005, Chapter 11) and Deng (2000), jump-diffusion structure can be more accurate (at least for wholesale electricity prices; here we model residential prices). For our purposes of providing economic intuition to policy makers rather than a decision support system for traders, a simpler model is a better tool to analyze the effect of the price and cost variabilities on the households’ investment decisions. Specifically, this model produces closed-form solutions for the investment decisions of households which allow us to study the government’s policy optimization problem and it is easier to interpret the effects of the aforementioned economic factors. Moreover, our insights are readily extendable to more complex models. For example, strong mean reversion reduces the volatility in the price and cost processes. Conversely, adding a jump component to the price and cost processes increase their total volatility. Our insights apply.

3.3. Government’s Objective and Decisions
As noted above, the government credibly commits to either a feed-in-tariff or a tax-rebate policy to promote solar panel investments from the beginning of the problem horizon. Under the feed-in-tariff, the government commits to perpetually provide the feed-in rate $F$ to the households for each unit of electricity produced. Under the tax-rebate policy, the government reduces the investment cost of the households by a fraction $R$ by providing a tax-rebate of $RX(\tau)$ at the investment time $\tau$. 
The objective of the government is to maximize the expected present value of a subsidy program. Specifically, for each unit of electricity produced, a societal benefit at rate $b$ is obtained perpetually from the time of the investment under either subsidy policy. This benefit includes the environmental benefit of the sustainable electricity, increased resiliency of the electricity grid during extreme weather events (NREL 2014), and hedging against the fluctuations in fuel prices (CEC 2008).

Under the feed-in-tariff policy, the cost to the government is the perpetual feed-in tariff payments to the households with rooftop solar panels minus the price of electricity in the market. That is, under the feed-in-tariff policy, the government becomes an intermediary, purchasing the generated electricity from the solar panels at rate $F$ and selling the electricity in the market at price $P(t)$. Under the tax-rebate policy, the cost to the government is the rebate payments $RX(\tau)$ made at the investment time $\tau$.

In our model, the government’s objective function is based on the entire taxpayer population and it does not include the surplus obtained by the households that invest in solar panels. This is because this surplus constitutes a transfer of wealth from the entire population of taxpayers to a few households with panels. Our approach reflects the recent criticism of tax-rebate and net metering policies on this issue (Potts 2015, Singh and Scheller-Wolf 2017). If the surplus of the households with solar panels were to be included in the government’s objective, it would mask the cost to the taxpayers and allow for arbitrarily large wealth transfers as possible solutions, which are not desirable.

In the subsequent analysis, we index important quantities with superscript $i$, where $i = F$ denotes the feed-in-tariff policy and $i = R$ denotes the tax-rebate policy. We use the asterisk notation (*) to denote the optimal quantities. All proofs are presented in Appendix.

4. Analysis and Results

In this section, we analyze whether the feed-in-tariff or the tax-rebate policy yields a higher value from the government’s perspective, based on different economic factors. Specifically, we consider the heterogeneity in generating efficiency as well as price and cost variability. We focus on one economic factor at a time to isolate its impact on the government’s preference. We start with a benchmark case which abstracts away from these economic factors. We investigate the joint effect of the aforementioned economic factors in Section 5.
4.1. Benchmark Setting

In this benchmark setting, we ignore the price and cost variability by letting \( P(t) = P \) and \( X(t) = X \), and assume that the households are homogeneous in generating efficiency, which equals to \( Q \). This setting is similar to the motivating example in the introduction. Here, in addition, we identify the optimal feed-in-tariff and tax-rebate policies and introduce thresholds on efficiency level that are necessary for subsequent discussion.

We first characterize the investment decision of a representative household. Because neither the benefit nor the cost of the investment changes over time under any subsidy policy in this benchmark setting, the household either invests immediately at the beginning of the problem horizon or it never does. The household invests if the net present value (NPV) of the investment is positive.

Under the feed-in-tariff policy, the household receives the feed-in rate \( F \) perpetually and incurs the investment cost \( X \) at time \( t = 0 \). The NPV of the investment for a household is

\[
\int_0^\infty F Q e^{-rt} \, dt - X = \frac{F}{r} Q - X.
\]

A household invests if and only if its efficiency exceeds threshold \( Q^F \):

\[
Q \geq Q^F = \frac{r X}{F}.
\]

Under the tax-rebate policy, the household receives the price of electricity \( P \) perpetually and incurs a reduced investment cost \( X(1 - R) \) at time \( t = 0 \). The NPV of the investment is

\[
\int_0^\infty P Q e^{-rt} \, dt - X(1 - R) = \frac{P}{r} Q - X(1 - R),
\]

and a household with efficiency \( Q \) invests if and only if its efficiency exceeds threshold \( Q^R \):

\[
Q \geq Q^R = \frac{r X (1 - R)}{P}.
\]

Accounting for a household’s investment decisions, the government maximizes the value of each subsidy program. For a given feed-in tariff level \( F \), this value is

\[
\pi^F_Q = \int_0^\infty [b - (F - P)] Q e^{-rt} \, dt \times 1_{\{Q \geq Q^F\}} = \left[ \frac{b - (F - P)}{r} \right] Q \times 1_{\{Q \geq Q^F\}},
\]

where the government receives a societal benefit \( b \), incurs a feed-in tariff payment \( F \), and sells the generated electricity at rate \( P \) for \( Q \) units of generation as long as the household
invests. Here, \(1_{\{\cdot\}}\) is the indicator function. The government maximizes the value of the feed-in-tariff policy:

\[
\pi_{Q}^{F^*} = \max_{F \geq P} \pi_{Q}^{F}.
\] (8)

The feed-in tariff \(F\) is constrained to exceed the electricity price \(P\) because, otherwise, it would be a tax on a household, rather than a subsidy.

For a given tax-rebate level \(R\), the value to the government is

\[
\pi_{Q}^{R} = \int_{0}^{\infty} bQe^{-rt} dt - RX \times 1_{\{Q \geq Q^R\}} = \left[ \frac{b}{r}Q - RX \right] \times 1_{\{Q \geq Q^R\}},
\] (9)

where the government receives a societal benefit \(b\) and incurs a tax-rebate payment \(RX\) as long as the household invests. The government solves the following problem:

\[
\pi_{Q}^{R^*} = \max_{0 \leq R \leq 1} \pi_{Q}^{R}.
\] (10)

**Proposition 1.** Consider the benchmark setting where \(P(t) = P, X(t) = X\), and household generating efficiency is homogeneous. For any efficiency \(Q\), \(\pi_{Q}^{F^*} = \pi_{Q}^{R^*}\), i.e., the government is indifferent between the feed-in-tariff and the tax-rebate policies.

Proposition 1 shows that, in the benchmark model, the optimal feed-in-tariff and the optimal tax-rebate policies are equivalent. Intuitively, as discussed in Section 1, the government can match the benefit and the cost of the two subsidy policies exactly by choosing the feed-in tariff and the tax rebate accordingly. We next analyze how heterogeneity, and price and cost variability affect the government’s preference between the two policies.

### 4.2. Effect of the Heterogeneity in Generating Efficiency

In this subsection, we study how the heterogeneity in generating efficiency affects the government’s preference between the two subsidy policies. As in the benchmark setting, we abstract away from the variability in the electricity price and the investment cost. Hence, the investment decision of a particular household with a given efficiency remains the same. That is, a household with efficiency \(Q\) invests in a panel at time \(t = 0\) if \(Q \geq Q^i\) under subsidy policy \(i \in \{F,R\}\), where \(Q^F\) and \(Q^R\) are given in (4) and (6), respectively. If \(Q < Q^i\) for \(i \in \{F,R\}\), the household never invests.

From the government’s perspective, we aggregate the benefits and the costs of a subsidy policy across the household population. Specifically, under the feed-in-tariff policy, the government’s optimization problem is

\[
\max_{F \geq P} \left\{ \pi^{F} = \int_{0}^{1} \pi_{Q}^{F} \psi(Q)dQ = \int_{Q^F}^{1} \left[ \frac{b}{r}Q - \frac{F - P}{r} \right] Q\psi(Q)dQ \right\},
\] (11)
where \( \pi^F_Q \) is given in (7) and \( Q^F \) is defined in (4). Function \( \psi(Q) \) is the distribution function of efficiency in the household population. Under the tax-rebate policy, the government’s problem is

\[
\max_{0 \leq R \leq 1} \left\{ \pi^R = \int_0^1 \pi^R_Q \psi(Q) dQ = \int_{Q_R}^1 \left[ \frac{b}{r} Q - RX \right] \psi(Q) dQ \right\},
\]

where, \( \pi^R_Q \) is given in (9) and \( Q^R \) is defined in (6).

**Proposition 2.** Assume that \( P(t) = P \) and \( X(t) = X \), and the generating efficiency \( Q \) is distributed according to the p.d.f. \( \psi(Q) \). Then, \( \pi^F_* \leq \pi^R_* \), i.e., the government prefers the tax-rebate policy to the feed-in-tariff policy.

Proposition 2 states that the tax-rebate policy yields a higher government value compared to the feed-in-tariff policy when the households are heterogeneous in their generating efficiency. This result holds for any arbitrary efficiency distribution. To see the intuition consider the case where the two policies lead to the same threshold efficiency, i.e., \( Q^F = Q^R \). In this case, the aggregate benefit of the government \( \int_{Q^F}^{1} \frac{bQ}{r} \psi(Q) dQ \) is the same under both subsidy policies. Under both policies, the marginal household with efficiency \( Q = Q^F = Q^R \) levies the same cost on the government. Under the tax-rebate policy, the cost to the government from a household with efficiency \( Q \geq Q^R \) is \( RX \), which is constant in \( Q \). In contrast, under the feed-in-tariff policy, the cost to the government from a household with efficiency \( Q \geq Q^F \) is \( \frac{F - P}{r} Q \), which increases with \( Q \). Thus, when the cost is aggregated across the households, the aggregate cost is higher under the feed-in-tariff policy. This argument holds for any threshold efficiency. Thus, it also holds for the optimal threshold efficiency under the feed-in-tariff policy.

Proposition 2 reveals an important policy insight. A government prefers the tax-rebate policy instead of the feed-in-tariff policy if the households have significantly different levels of generating efficiency. This is the case for the countries such as the U.S., which spans a large geographical area with different levels of solar radiation.

### 4.3. Effect of the Price Variability

In this subsection, we focus on price variability and, to isolate its effect, we let \( P(t) \) follow the GBM defined in (1), consider a fixed investment cost, i.e., \( X(t) = X \), and a homogeneous population with generating efficiency \( Q \).
4.3.1. **Households’ Investment Timing.** Under the feed-in-tariff policy, the investment decision of the representative household remains the same as in the benchmark setting. This is because the feed-in-tariff policy removes any price variability as the household receives the feed-in rate $F$ perpetually. Therefore, the NPV criterion is still optimal and the household invests at time $t = 0$ if $Q \geq Q^F$, where $Q^F$ is given in (4). If $Q < Q^F$ the household never invests.

Under the tax-rebate policy, the household receives the electricity price for the output from the solar panel. Therefore, it is exposed to the price variability. By investing at time $t$ when the electricity price is $P(t)$, the household obtains
\[
E_t \left[ \int_0^\infty P(t + s)Qe^{-rs}ds \right] - X(1 - R) = \frac{P(t)}{r - \mu_p}Q - X(1 - R),
\]
as the value of the investment, where $E_t$ is the expectation conditional on the information at time $t$. The equality in (13) follows from Dixit and Pindyck (1994, p. 72). If the household followed the NPV rule, the household would invest as soon as the price exceeded the following threshold:
\[
P(t) \geq \frac{X(r - \mu_p)}{Q} (1 - R). \tag{14}
\]
However, the NPV rule is not optimal due to the price dynamics. Specifically, by investing at any time $t$, the household forgoes the opportunity to invest in the future at a higher price, should prices increase. The flexibility of the household to choose the investment time creates a waiting option. As McDonald and Siegel (1986) show, by investing, the household loses this option. Thus, the household requires a strictly positive NPV as a compensation for this loss. The household solves an optimal stopping problem. The optimal investment policy has a threshold structure where the investment is made at time $\tau^R_Q$ as soon as the price $P(t)$ exceeds a threshold $P^R_Q$. Formally, $\tau^R_Q = \inf\{t : P(t) \geq P^R_Q\}$. The threshold $P^R_Q$ depends on $Q$ and $R$, and we characterize $P^R_Q$ below.

**Lemma 1.** Assume $P(t)$ follows the process (1) and $X(t) = X$. Consider a household with efficiency $Q$. (i) Under the feed-in-tariff policy, the household invests at time $t = 0$ if and only if $Q \geq Q^F = \frac{rX}{F}$. (ii) Under the tax-rebate policy, the household invests at time $t$ if
\[
P(t) \geq P^R_Q = \frac{\theta}{\theta - 1} \frac{X(r - \mu_p)}{Q} (1 - R), \tag{15}
\]
where
\[
\theta = \frac{1}{2} - \frac{\mu_p}{\sigma_p^2} + \sqrt{\left( \frac{\mu_p}{\sigma_p^2} - \frac{1}{2} \right)^2 + \frac{2r}{\sigma_p^2}} \geq 1. \tag{16}
\]
Compared to the break-even NPV condition (14), the threshold $P_Q^R$ in (15) includes a factor of $\frac{\theta}{\theta-1}$, which is greater than 1. This factor reflects the delay in the investment due to the waiting option of a household.

**Lemma 2.** The delay factor $\frac{\theta}{\theta-1}$ increases in $\sigma_p$ and $\mu_p$.

Lemma 2 reports a standard result (Dixit and Pindyck 1994, p. 144) that the strategic delay in the investment of the representative household increases in the price uncertainty (i.e., higher $\sigma_p$) and in the drift (i.e., higher $\mu_p$). When our model is calibrated by real data on electricity prices (see Section 5.1), we observe that the delay factor is approximately 1.9, indicating that the NPV of the benefit of the investment needs to be almost twice the cost to justify the exercise of the waiting option. Ignoring the waiting option can lead to either suboptimal investment decisions by households or a significant miscalculation of policy outcomes for the government.

4.3.2. The Government’s Decisions. The government receives the following value from a household with efficiency $Q$, when the feed-in tariff is $F$:

$$\pi^F_Q = E \left[ \int_0^\infty \left[ b - (F - P(t)) \right] Q e^{-rt} dt \right] \times 1_{\{Q \geq Q^*\}} = \left[ \frac{b - F}{r} + \frac{P(0)}{r - \mu_p} \right] Q \times 1_{\{Q \geq Q^*\}}. \quad (17)$$

This is similar to the government’s value given in (7) in the benchmark case because a household’s investment decision does not change under price uncertainty for the feed-in-tariff policy. Here and later in the paper we reuse notation $\pi^F_Q$, $\pi^R_Q$, $\tau^F_Q$, and $\tau^R_Q$ because their meanings are immutable across models and it should be clear from the context to which version of the model these quantities apply. Given (17), the government solves the following optimization problem:

$$\pi^{F^*}_Q = \max_{F \geq P(0)} \pi^F_Q. \quad (18)$$

Under the tax-rebate policy, price variability affects a household’s investment timing and, thus, the government’s value. This value $\pi^R_Q$ depends on the optimal investment time of the household $\tau^R_Q$, defined above for a given efficiency $Q$ and rebate $R$. Specifically,

$$\pi^R_Q = E \left[ \int_{\tau^R_Q}^\infty bQe^{-rt} dt - e^{-r\tau^R_Q} RX \right] = \left( \frac{b}{r} - Q - RX \right) E[e^{-r\tau^R_Q}] = \left( \frac{b}{r} - Q - RX \right) \left\{ \left[ \frac{P(0)}{P^R_Q} \right]^\theta \right\} \wedge 1, \quad (19)$$
where the government obtains the societal benefit starting from time $\tau_Q^R$ and it incurs the subsidy cost at time $\tau_Q^R$. In (19), the last equality is derived in Lemma 8 in the appendix, we define $a \land b = \min(a, b)$ and threshold $P_Q^R$ is given in (15). Here, one can think of $E[e^{-r \tau_Q^R}]$ as the price of an Arrow-Debrue security that pays $1$ at time $\tau_Q^R$. If $P(0) \geq P_Q^R$, the household invests immediately $(\tau_Q^R = 0)$ and the security pays out $1$ at time $t = 0$ so that its price is also $1$. Otherwise, if $P(0) < P_Q^R$, the price of the security is less than $1$.

The government optimizes its value from a household by determining the rebate:

$$\pi_Q^{R*} = \max_{0 \leq R \leq 1} \pi_Q^R.$$  \hfill (20)

Before characterizing the optimal feed-in-tariff and tax-rebate policies, we assume

$$\frac{P(0)}{r - \mu_p} \leq X$$  \hfill (21)

so that the investment is not favorable without a subsidy policy for any household. Specifically, following (14), the NPV of the investment without any subsidy is $\frac{P(0)}{r - \mu_p} Q - X$, which is negative for all $Q$ if (21) holds. This assumption is consistent with the electricity price and the investment cost data presented in Section 5.1. We impose this assumption in the remainder of this section.

**Lemma 3.** Assume $P(t)$ follows the process (1) and $X(t) = X$. Consider a household with efficiency $Q$. Suppose (21) holds. Let $F^*$ and $R^*$ be the optimal feed-in-tariff and tax-rebate levels with the corresponding government values $\pi_Q^{F*}$ and $\pi_Q^{R*}$, respectively. (i) The optimal feed-in-tariff policy is given as follows.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$F^*$</th>
<th>$\pi_Q^{F*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \in \left[0, \frac{X}{r + \frac{P(0)}{r - \mu_p}}\right]$</td>
<td>$P(0)$</td>
<td>$0$</td>
</tr>
<tr>
<td>$Q &gt; \frac{X}{r + \frac{P(0)}{r - \mu_p}}$</td>
<td>$\frac{r X}{Q} \left(b + \frac{P(0)}{r - \mu_p}\right) Q - X$</td>
<td>$\pi_Q^{F*}$</td>
</tr>
</tbody>
</table>

(ii) The optimal tax-rebate policy is given as follows.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$R^*$</th>
<th>$\pi_Q^{R*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \leq \frac{X}{\theta}$</td>
<td>$0$</td>
<td>$\frac{bQ}{r} \left[1 - \frac{P(0) Q}{X - \frac{\mu_p}{\theta}} \theta \right]$</td>
</tr>
<tr>
<td>$Q \in \left(\frac{X}{\theta}, \frac{b + \left(\frac{1}{\theta} - \frac{P(0)}{r - \mu_p}\right) X}{r + \frac{P(0)}{r - \mu_p}}\right]$</td>
<td>$\frac{1}{(\theta - 1) X} \left(b Q \frac{\theta - 1}{\theta} - X\right)$</td>
<td>$\frac{1}{(\theta - 1) X - \frac{b Q}{r - \mu_p}} \left[1 - \frac{P(0) Q}{X - \frac{\mu_p}{\theta}} \theta \right]$</td>
</tr>
<tr>
<td>$Q &gt; \frac{X}{\theta + \left(\frac{1}{\theta} - \frac{P(0)}{r - \mu_p}\right) X}$</td>
<td>$1 - \frac{P(0) Q}{r - \mu_p} \frac{\theta - 1}{\theta}$</td>
<td>$\frac{bQ}{r} + \frac{P(0) Q}{r - \mu_p} \theta - X$</td>
</tr>
</tbody>
</table>
Lemma 3 describes the optimal feed-in-tariff and tax-rebate policies based on the generating efficiency, $Q$, of a household. If $Q$ is small, the government does not offer a subsidy policy. Although the value to the government is zero under the feed-in-tariff policy, the government obtains a positive value under the tax-rebate policy. This is because, under the tax-rebate policy, the household may eventually invest as the electricity price increases. However, under the feed-in-tariff policy, the household either invests immediately or it never invests. Thus, the tax-rebate policy outperforms the feed-in-tariff policy when $Q$ is sufficiently small. Otherwise, feed-in-tariff policy generally results in a higher value to the government as shown below.

Proposition 3. Assume $P(t)$ follows the process (1) and $X(t) = X$. Consider a household with efficiency $Q$. Suppose (21) holds.

(i) If $Q \in \left[0, \frac{X}{\theta + \frac{P(0)}{r - \mu_p}}\right]$, then $\pi_F^{Q^*} \leq \pi_R^{Q^*}$, i.e., the government prefers the tax-rebate to the feed-in-tariff policy.

(ii) If $Q \in \left[\frac{X}{\theta + \frac{P(0)}{r - \mu_p}}, 1\right]$ and

$$\frac{b}{\theta} \geq \frac{P(0)}{r - \mu_p},$$

then $\pi_F^{Q^*} \geq \pi_R^{Q^*}$, i.e., the government prefers the feed-in-tariff policy to the tax-rebate policy.

Proposition 3 part (i) shows that for low values of generating efficiency $Q$, tax-rebate policy yields a higher value to the government as explained above. On the other hand, part (ii) indicates that the feed-in-tariff policy outperforms when $Q$ and the societal benefit $b$, as given in (22), are sufficiently high. These conditions ensure that the government finds it optimal to offer a strictly positive feed-in tariff to motivate the investment in the beginning of the time horizon. For a practically relevant range of problem parameters, these conditions hold and the government prefers the feed-in-tariff policy. Intuitively, the tariff removes the price variability and, thus, the strategic waiting, by offering a guaranteed stream of payments to the household. Consequently, feed-in tariff can promote the investment of the household at a smaller cost to the government, yielding a higher value to the government, compared with the tax rebate.

Proposition 3 does not describe the preference for the subsidy policy for medium values of generating efficiency $Q$. To study this region, we conduct a numerical study. The result is
shown in Figure 2. The parameters are calibrated by using historical electricity prices (see Section 5.1). The dashed vertical lines in this figure mark the boundaries of the $Q$ regions from Proposition 3, so that the interval between the vertical lines is the indeterminate region. Figure 2 illustrates that the tax-rebate policy outperforms the feed-in-tariff policy only if the generating efficiency is small. In this case, there is a chance that the tax-rebate results in the household’s investment eventually and it yields a small, but positive value to the government, whereas the feed-in-tariff policy is not offered by the government at all. However, for sufficiently high $Q$ values, which are more realistic, feed-in-tariff policy yields a higher value. The main conclusion in this subsection is that a government should offer the feed-in-tariff policy, instead of the tax-rebate policy, when there is a significant level of price uncertainty and, thus, strategic waiting by households.

4.4. Effect of the Cost Variability

The cost of a solar panel is variable due to the fluctuations in the prices of the commodities used in the manufacturing of the panels. We model this variability by letting $X(t)$ follow the GBM defined in (2). To isolate the effect of cost variability, we consider fixed prices, i.e., $P(t) = P$, and a representative household as in the benchmark model.

Similar to the price variability case considered in Section 4.3, the NPV rule is not optimal when the investment cost is variable. This is because the household has the option to time its investment decision. Neither the feed-in-tariff nor the tax-rebate policy fully
eliminates the cost variability and a household exhibits a strategic waiting behavior under both policies. However, the tax-rebate policy reduces the exposure of the household to the cost uncertainty. We present the optimal investment thresholds for a household in Lemma 4.

**Lemma 4.** Assume $X(t)$ follows the process (2) and $P(t) = P$. Consider a household with efficiency $Q$. (i) Under the feed-in-tariff policy, the household invests at time $t$ if

$$X(t) \leq X_Q^F = \frac{FQ}{r} \frac{\eta}{\eta+1},$$

where

$$\eta = \sqrt{\left(\frac{\mu_x}{\sigma_x^2} - \frac{1}{2}\right)^2 + \frac{2r}{\sigma_x^2} + \left(\frac{\mu_x}{\sigma_x^2} - \frac{1}{2}\right)} > 0.$$ (24)

(ii) Under the tax-rebate policy, the household invests at time $t$ if

$$X(t) \leq X_Q^R = \frac{PQ}{r(1-R)} \frac{\eta}{\eta+1}.$$ (25)

In Lemma 4, similar to the price variability case discussed in Section 4.3, a household exhibits a strategic delay in the investment, reflected in the factor $\frac{\eta}{\eta+1}$ in thresholds $X_Q^F$ and $X_Q^R$ given by (23) and (25).

Given the investment decision of the household from Lemma 4, the government’s value under the feed-in-tariff policy is

$$\pi_Q^F = E \left[ \int_{\tau_Q^F}^{\infty} [b - (F - P)] Q e^{-rt} dt \right] = \left( \frac{b - F + P}{r} \right) Q E \left[ e^{-r\tau_Q^F} \right] = \left( \frac{b - F + P}{r} \right) Q \left\{ \left[ \frac{X_Q^F}{X(0)} \right]^{\eta} \right\} \wedge 1,$$ (26)

where $\tau_Q^F = \inf \{t : X(t) \leq X_Q^F\}$ with threshold $X_Q^F$ is defined in (23), and the value of $E \left[ e^{-r\tau_Q^F} \right]$ is derived from Lemma 4. Recall that we share notation $\pi_Q^F, \pi_Q^R, \tau_Q^F$, and $\tau_Q^R$ across multiple models. The government’s optimization problem is

$$\pi_Q^{F*} = \max_{F \geq P} \pi_Q^F.$$ (27)

Under the tax-rebate policy, the government’s value is

$$\pi_Q^R = E \left[ \int_{\tau_Q^R}^{\infty} b Q e^{-rt} dt - e^{-r\tau_Q^R} RX(\tau_Q^R) \right] = \left( \frac{b}{r} Q - RX_Q^R \right) E \left[ e^{-r\tau_Q^R} \right] = \left( \frac{b}{r} Q - RX_Q^R \right) \left\{ \left[ \frac{X_Q^R}{X(0)} \right]^{\eta} \right\} \wedge 1,$$ (28)
where $\tau_Q^R = \inf \{ t : X(t) \leq X_Q^R \}$ with threshold $X_Q^R$ is defined in (25), and for the first equality we use $X(\tau_Q^R) = X_Q^R$. The value of $E\left[e^{-r\tau_Q^R}\right]$ is derived from Lemma 4. The government’s optimization problem is

$$\pi_Q^{R^*} = \max_{0 \leq R \leq 1} \pi_Q^R. \quad (29)$$

The solutions of the government’s optimization problems (27) and (29) are presented in the following lemma.

**Lemma 5.** Assume $P(t) = P$ and $X(t)$ follows the process (2). Consider a household with efficiency $Q$. (i) Under the feed-in-tariff policy, if $b \leq \frac{P}{\eta}$, $F^* = P$ and the optimal value to the government is $\pi_Q^{F^*} = \frac{bQ}{r} \left[ \frac{PQ}{rX(0)(\eta+1)} \right]^\eta$; otherwise, the optimal policy and government’s value are given as follows.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$F^*$</th>
<th>$\pi_Q^{F^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \in \left[ 0, \frac{rX(0)}{(b+P)(\frac{\eta}{\eta+1})^2} \right]$</td>
<td>$(b+P)\frac{\eta}{\eta+1}X(0)(\eta+1)\frac{Q}{rX(0)}^\eta \left[ \frac{(b+P)Q}{rX(0)} \right]^\eta+1$</td>
<td></td>
</tr>
<tr>
<td>$Q \in \left( \frac{rX(0)}{(b+P)(\frac{\eta}{\eta+1})^2}, \frac{rX(0)}{P\frac{\eta}{\eta+1}} \right]$</td>
<td>$\frac{rX(0)}{Q} X(0)\frac{\eta+1}{\eta} \frac{b+P}{r} Q - X(0)\frac{\eta+1}{\eta}$</td>
<td></td>
</tr>
<tr>
<td>$Q &gt; \frac{rX(0)}{P\frac{\eta}{\eta+1}}$</td>
<td>$P$</td>
<td></td>
</tr>
</tbody>
</table>

(ii) Under the tax-rebate policy, if $b \leq \frac{P}{\eta+1}$, $R^* = 0$ and the optimal value to the government is $\pi_Q^{R^*} = \frac{bQ}{r} \left[ \frac{PQ}{rX(0)(\eta+1)} \right]^\eta$; otherwise, the optimal policy and government’s value are given as follows.

<table>
<thead>
<tr>
<th>Condition</th>
<th>$R^*$</th>
<th>$\pi_Q^{R^*}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Q \in \left[ 0, \frac{rX(0)}{(b+P)(\frac{\eta}{\eta+1})^2} \right]$</td>
<td>$\frac{b-P}{\eta+1} \frac{\eta}{\eta+1} \frac{Q}{rX(0)}\left[X(0)\frac{\eta}{\eta+1}\right]^\eta+1$</td>
<td></td>
</tr>
<tr>
<td>$Q \in \left( \frac{rX(0)}{(b+P)(\frac{\eta}{\eta+1})^2}, \frac{rX(0)}{P\frac{\eta}{\eta+1}} \right]$</td>
<td>$1 - \frac{PQ}{rX(0)} \frac{\eta}{\eta+1} \frac{bQ}{r} + \frac{PQ}{r} \frac{\eta}{\eta+1} - X(0)$</td>
<td></td>
</tr>
<tr>
<td>$Q &gt; \frac{rX(0)}{P\frac{\eta}{\eta+1}}$</td>
<td>$0$</td>
<td>$\frac{bQ}{r}$</td>
</tr>
</tbody>
</table>

Based on the optimal values of the subsidy policies in Lemma 5, the tax-rebate policy generates a higher value to the government compared with the feed-in-tariff policy. This is because the tax-rebate policy reduces the exposure of the household to the cost variability and, thus, the strategic waiting behavior. Proposition 4 formalizes this result.

**Proposition 4.** Assume $P(t) = P$ and $X(t)$ follows the process (2). Consider a household with efficiency $Q$. For any $Q$, $\pi_Q^{F^*} \leq \pi_Q^{R^*}$, i.e., the government prefers the tax-rebate to the feed-in-tariff policy.
5. Joint Effects of the Economic Factors

In this section, we study the joint effects of all economic factors on the government’s preference between the feed-in-tariff and tax-rebate policies. In particular, we jointly consider the heterogeneity in generating efficiency and the variability in the electricity price and the investment cost. In this setting, the co-movements of the price and cost uncertainty are important. This is because, under the tax-rebate policy, a household invests based on the price to cost ratio \( Y(t) \):

\[
Y(t) = \frac{P(t)}{X(t)}.
\] (30)

Recall that \( \rho \) is the correlation between the price and cost processes with respective volatilities \( \sigma_p \) and \( \sigma_x \), given in (1) and (2). We define

\[
\sigma^2 = \sigma_p^2 + \sigma_x^2 - 2\rho \sigma_p \sigma_x.
\] (31)

Using these definitions, we characterize the investment decisions of a household below.

**Lemma 6.** Assume \( P(t) \) and \( X(t) \) follow processes (1) and (2), respectively. Consider a household with efficiency \( Q \). (i) Under the feed-in-tariff policy, the household invests at time \( t \) if \( X(t) \leq X_F^Q \), where \( X_F^Q \) is given in (23). (ii) Under the tax-rebate policy, the household invests at time \( t \) if

\[
Y(t) \geq Y_R^Q = \frac{\gamma - \frac{r - \mu_p}{Q}}{\gamma - 1} (1 - R),
\] (32)

where

\[
\gamma = \sqrt{\left(\frac{\mu_p - \mu_x}{\sigma^2} - \frac{1}{2}\right)^2 + \frac{2(r - \mu_x)}{\sigma^2} + \left(\frac{1}{2} - \frac{\mu_p - \mu_x}{\sigma^2}\right)} > 1,
\] (33)

and \( \sigma \) is defined in (31).

Lemma 6(i) shows that, under the feed-in-tariff policy, the household’s investment decision remains the same as in the cost variability case in Section 4.4. This is because the feed-in-tariff eliminates the price variability. In contrast, under the tax-rebate policy, a household is subject to both price and cost variabilities and the factor \( \frac{\gamma}{\gamma - 1} \) in (32) reflects the delay in the investment due to the waiting option of the household.

**Lemma 7.** The delay factor \( \frac{\gamma}{\gamma - 1} \) increases in \( \mu_p \) and \( \sigma \), decreases in \( \mu_x \) and \( \rho \).
The results in Lemma 7 are similar to those in Lemma 2, which studies the delay factor under price variability. The new insight here is that, under the tax-rebate policy, a household exhibits a shorter strategic wait under positive correlation. Intuitively, under positive correlation, the price and cost variabilities present a natural hedge to each other as they tend to either increase or decrease together. Because the household invests according to the ratio of the price to the cost, the positive correlation reduces the overall volatility under the tax-rebate policy. In contrast, the strategic wait of a household under the feed-in-tariff policy is not affected by the correlation. This is an important difference between the two subsidy policies, which affects the value of the policies to the government as we discuss below.

Next, given the investment decisions of a household from Lemma 6, we characterize the value of the feed-in-tariff policy to the government. Our approach involves two steps. First, we focus on the value of a single household with efficiency $Q$. Second, we aggregate the value over all the households in the population. Let $\tau^F_Q$ be the optimal investment time of a household with efficiency $Q$ under $F$. That is, $\tau^F_Q = \inf\{ t : X(t) \leq X^F_Q \}$, where $X^F_Q$ is given in (23). Then, the value of this household to the government is

$$
\pi^F_Q = E \left[ \int_0^\infty \left[ b - (F - P(t)) \right] Q e^{-rt} dt \right] = \frac{b - F}{r} Q E \left[ e^{-r\tau^F_Q} \right] + \frac{Q}{r - \mu_p} E \left[ e^{-r\tau^F_Q} P(\tau^F_Q) \right]
$$

$$
= \frac{b - F}{r} Q \left\{ \left[ \frac{X^F_Q}{X(0)} \right]^{\eta} \Wedge 1 \right\} + \frac{Q}{r - \mu_p} P(0) \left\{ \left[ \frac{X^F_Q}{X(0)} \right]^{\nu} \Wedge 1 \right\},
$$

where $\eta$ is given in (24) and

$$
\nu = \sqrt{\left( \frac{\mu_x + \rho \sigma_x \sigma_p}{\sigma_x^2} - \frac{1}{2} \right)^2 + \frac{2(r - \mu_p)}{\sigma_x^2} + \left( \frac{\mu_x + \rho \sigma_x \sigma_p}{\sigma_x^2} - \frac{1}{2} \right)} > 0.
$$

Recall that we share notation $\pi^F_Q$, $\pi^R_Q$, $\tau^F_Q$, and $\tau^R_Q$ across multiple models. Similar to the government’s value in (19), the final two expectations in (34) are the prices of the Arrow-Debrue securities that pay $1$ and $\$P(\tau^F_Q)$, respectively, at time $\tau^F_Q$. These prices are given in (35) and are derived in Lemma 8. Given the value from a single household, the government maximizes the aggregate value across the population:

$$
\max_{F \geq P(0)} \left\{ \Pi^F = \int_0^1 \pi^F_Q \psi(Q) dQ \right\},
$$

where $\psi(Q)$ is the distribution of the generating efficiency.
Under the tax-rebate policy, let $\tau_Q^R$ be the optimal investment time of a household with efficiency $Q$ under rebate $R$. That is, $\tau_Q^R = \inf\{t : Y(t) \geq Y_Q^R\}$, where $Y(t)$ and $Y_Q^R$ are given in (30) and (32), respectively. The government’s value from the investment of this household is

$$
\pi_Q^R = \mathbb{E} \left[ \int_{\tau_Q^R}^{\infty} b Q e^{-rt} dt - e^{-r\tau_Q^R} RX(\tau_Q^R) \right] = \frac{b}{r} Q \left[ Y(0) \frac{Y_R^Q}{Y_Q^R} \right]^\gamma \wedge 1 - RX(0) \left( Y(0) \frac{Y_R^Q}{Y_Q^R} \right)^\gamma \wedge 1,
$$

where $\gamma$ is given in (33) and

$$
\zeta = \sqrt{\left( \frac{\mu_p - \mu_x + \sigma_x^2}{\sigma^2} - \rho \sigma_x \sigma_p \right)^2 + \frac{2r}{\sigma^2} + \left( 1 - \frac{\mu_p - \mu_x + \sigma_x^2}{\sigma^2} - \rho \sigma_x \sigma_p \right)}.
$$

Lemma 8 computes the value of the final two expectations in (38) as given in (39). Aggregating across households yields the optimization problem of the government:

$$
\max_{0 \leq R \leq 1} \left\{ \Pi^R = \int_{0}^{1} \pi_Q^R \psi(Q) dQ \right\}.
$$

The optimization problems (37) and (41) are difficult to analyze because the government’s value for a single household given in (35) and (39) are not necessarily concave. However, we observe that these objective functions are unimodal in extensive numerical studies. In the next subsection, we present a numerical study, where we calibrate our model using historical electricity price and investment cost data in the U.S.

5.1. Numerical Analysis of Joint Effects

This numerical analysis serves two purposes. First, recall that, in Section 4, we consider each economic factor in isolation to identify its effect on the government’s preference between the feed-in-tariff and tax-rebate policies. Here, we analyze how these effects change when heterogeneity in efficiency as well as variability in the electricity price and the investment cost are considered simultaneously. Second, we investigate the effect of the correlation $\rho$ on the government’s preference between the two subsidy policies.

In our numerical analysis, all problem parameters are calculated as annual values whenever relevant. We first estimate the annual drift and volatility parameters of the GBMs for the electricity price and the investment cost given in (1) and (2), respectively. In doing
so, we adopt the actuarial interpretation of these processes as being presented under the physical measure. We use the investment cost data from the period of February 2007 – December 2015 for the state of California\(^1\), the largest solar energy market in the U.S. The drift in the cost is \(-9\%\) and the volatility is 13\%. For the electricity price data, we use the Electricity Data Browser of the Energy Information Administration\(^2\) that reports electricity prices at the state level in the U.S. We set the drift and volatility as 4.5\% and 6\%, respectively, in accordance with the average price across the states. Based on our data, neither cost nor price returns are correlated with the market return (proxied by the return on the S&P500 index) in a statistically and economically significant way. Furthermore, the correlation between the price and cost processes is also not statistically significant and estimated to be \(-0.11\). To compute the initial values of the cost and price processes, we note that the average installation size is 5.07kW and the cost was $12,950/kW in the beginning of the dataset. Hence, we take \(X(0)\) as $65,656 (\(= 12,950 \times 5.07\)). To determine the value of \(P(0)\) we first compute the annual generation of an average solar panel installation. According to NREL (2003), in California, 1kW of a solar panel generates approximately 1,900 kWh of electricity per year. Hence, the average installation generates \(5.07 \times 1,900 = 9,633\) kWh of electricity per year and given that the residential electricity price in California was $0.1418 per kWh in February 2007, we set \(P(0) = 1,366(= 9,633 \times 0.1418)\).

We assume that the interest rate is 10\% per annum and the generating efficiency follows a Normal distribution with mean \(\mu_Q\) and standard deviation \(\sigma_Q\). In the base case, we let \(\mu_Q = 0.5\) and \(\sigma_Q = 0.15\). To estimate the societal benefit parameter \(b\), we first calculate the avoided annual carbon emissions of an average solar panel installation. We find this value as 10 metric tons by multiplying the annual generation of a panel with the emissions intensity of a coal-fired power plant, given as 2.07 pounds per kWH by EIA (2016). Then, we take the social cost of carbon\(^3\) as $800 per ton (Ackerman and Stanton 2012) and set \(b = $8,000 = (10 \times 800)\) per year.

Below, we report the difference between the optimal value to the government under the tax-rebate and feed-in-tariff policies, i.e., \(\Pi^{\tau^*} - \Pi^{F^*}\). A positive difference indicates


\(^2\)[https://www.eia.gov/electricity/data/browser/](https://www.eia.gov/electricity/data/browser/)

\(^3\)Precisely estimating the social cost of carbon emissions is highly challenging and beyond the scope of our study. The estimates vary significantly (Ackerman and Stanton 2012) and we consider a relatively high estimate to ensure that both subsidy policies are used by the government and various societal benefits of solar panels are accounted for.
that the tax-rebate policy outperforms the feed-in-tariff policy. We consider three levels of correlation between the electricity price and the investment cost as $\rho = -1, 0, \text{ and } 1$.

![Graphs showing the effect of generating efficiency on subsidy policies](image)

**Figure 3** Effect of Heterogeneity in Generating Efficiency on the Value of Subsidy Policies

Figure 3 demonstrates the effect of the heterogeneity in the generating efficiency on the government’s preference between the feed-in-tariff and tax-rebate policies. Figure 3a shows that the tax-rebate policy outperforms the feed-in-tariff policy when $\mu_Q$ is high for all levels of correlation $\rho$. This reveals that the feed-in-tariff policy should be favored mostly in regions with low mean efficiency, i.e., low solar radiation (e.g., Germany). Similarly, Figure 3b shows that as the standard deviation of the efficiency distribution increases, i.e., the population becomes more heterogeneous in efficiency, the relative performance of the tax-rebate policy improves. This is consistent with Proposition 2, which establishes that the heterogeneity favors the tax-rebate policy.

Figure 3 further illustrates the effect of the correlation $\rho$ between the electricity price and the investment cost. For any value of $\mu_Q$ or $\sigma_Q$, as $\rho$ increases, $\Pi^R - \Pi^F$ also increases, indicating that under a more positive correlation the government favors the tax-rebate policy more. This result is due to the two-fold effect of the correlation on the households’ investments and the government’s value under each policy.

Under the feed-in-tariff policy, Lemma 6 shows that a household’s investment is not affected by the correlation $\rho$. Consider the government’s value (34). Although the government is risk-neutral, its objective function depends on $\rho$. The reason is that the government’s cost, $F - P(t)$, depends on $P(t)$ and this cost multiplies a function of $\tau_Q^F$, which
depends on $X(t)$. The government’s value decreases in $\rho$ as can be seen from the second term in (35). Intuitively, under positive correlation, when the investment cost goes down and the households invest, the electricity price also tends to go down. Consequently, the government sells the electricity that it purchased from the households at a lower price, decreasing the value of the feed-in-tariff policy.

Under the tax-rebate policy, the effect of the correlation is even more involved because the correlation $\rho$ explicitly affects both a household’s decisions and the government’s value. First, as Lemma 7 shows, a household invests sooner under positive correlation due to the natural hedge between the electricity price and the investment cost. That is, when $\rho$ is positive, the price and the cost change in the same direction, resulting in a lower overall volatility for the household. This is beneficial for the government because it reduces the strategic waiting. However, the government’s value from a single household still decreases in $\rho$ as can be seen from (39). Overall, Figure 3 shows that, compared to the feed-in-tariff policy, the attractiveness of the tax-rebate policy to the government increases as the correlation $\rho$ increases.

![Figure 4](image_url)  
**Figure 4**  
Effect of Price and Cost Uncertainty on the Value of Subsidy Policies

Figure 4a demonstrates the effect of the price variability and reveals an important insight. Recall that, when considered in isolation, price variability favors the feed-in-tariff policy because feed-in tariff eliminates the price variability for the households. Figure 4a confirms that $\Pi^{R^*} - \Pi^{F^*}$ becomes more negative (less positive) as $\sigma_p$ increases, indicating that the relative performance of the feed-in-tariff policy increases. Interestingly, this result does
not hold when $\rho = 1$ and $\sigma_p$ increases from 0.02 to 0.04. That is, when all economic factors are simultaneously considered, a higher price variability does not always favor the feed-in-tariff policy more. This result can be explained by considering the effects of $\sigma_p$ on the households’ investments and the government’s value. Under the feed-in-tariff policy, households’ investments do not depend on $\sigma_p$, but the government’s value does and, from (35), a higher price variability reduces the government’s value. Under the tax-rebate policy, when $\rho = 1$, the natural hedge between the electricity price and the investment cost, discussed above, moderates the overall uncertainty for a household, reducing the strategic waiting. The government’s value, given in (39), still decreases in $\sigma_p$. As Figure 4a illustrates, when $\sigma_p$ increases from 0.02 to 0.04 and $\rho = 1$, the natural hedge effect outweighs the other factors, so that a higher price variability increases the relative value of the tax-rebate policy for the government.

Figure 4b illustrates that the tax-rebate policy performs better as the cost variability increases when all economic factors are jointly considered. This is consistent with Proposition 4 that studies cost variability in isolation. As in Figure 3, in Figure 4, a higher level of correlation leads to a higher performance gap between the two subsidy policies, in favor of the tax rebate. This is due to the natural hedge effect when $\rho > 0$.

We close this section by illustrating the effect of the societal benefit $b$. In Figure 5, $\Pi^{R^*} - \Pi^{F^*}$ becomes more positive as $b$ increases, indicating that the tax-rebate policy outperforms the feed-in-tariff policy. Hence, the tax-rebate policy should be adopted in a region where the solar panels create a higher societal benefit. This can be the case if the electricity generation fleet is carbon-intensive.

![Figure 5](image-url)  
*Figure 5  Effect of Environmental Benefit on the Value of Subsidy Policies*
6. Conclusion
In this paper, we characterize the optimal timing of households' investments in rooftop solar panels under the feed-in-tariff and tax-rebate policies. After solving the households' problem, we aggregate the government's value of a subsidy policy across the distribution of generating efficiencies, find the optimal feed-in-tariff and tax-rebate levels, and compare the two policies. This allows us to identify how practical factors, such as heterogeneity in generating efficiency and variability in the electricity price and the investment cost, affect the government's preference between the two policies.

<table>
<thead>
<tr>
<th>Heterogeneity in Gen. Efficiency</th>
<th>Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Price</td>
</tr>
<tr>
<td>Tax-Rebate</td>
<td>Feed-in-Tariff</td>
</tr>
</tbody>
</table>

Table 1 Government’s Preference Between the Feed-in-Tariff and Tax-Rebate Policies

Our analysis produces practical policy implications, summarized in Table 1. First, for regions with highly volatile electricity prices, the government needs to adopt the feed-in-tariff policy. On the other hand, the tax-rebate policy is favored by the government in the cases of high heterogeneity in generating efficiency, variability in the investment cost, and positive correlation between the electricity price and the investment cost. Heterogeneity in generating efficiency is prevalent for households that reside in areas with significantly different levels of solar radiation, such as the U.S., whereas the cost variability is associated with a high level of uncertainty in the prices of the commodities used in solar panels. Finally, a positive correlation between the electricity price and the investment cost creates a natural hedge, reducing the overall volatility for the households under the tax-rebate policy, but not under the feed-in-tariff policy.

Our goal was to inform policy makers about the importance of generating heterogeneity and the effects of price and cost variability under each policy. Therefore, we strived to create the simplest model, that affords closed-form expressions and allows us to conduct further aggregate market analysis, and optimize government’s value from each policy. Thus, we had to make several simplifying assumptions. As discussed in Section 3, electricity and commodity price processes in practice may feature mean reversion and jump-diffusion structures. These do not affect our main insights. In fact, one can extrapolate our results,
because mean-reversion reduces variability and jumps increase it. We assumed that generating capacity and government policies are perpetual. This is clearly a simplification. But as long as the capacity lasts long enough, ours is a reasonable approximation. Further, assuming that political forces to revise government policies apply equally to feed-in-tariff and tax-rebate policies, ignoring these forces does not bias conclusions towards one policy.

**Appendix: Proofs and technical lemmas**

**Proof of Proposition 1.** Solving optimization problems (8) and (10), we derive the optimal feed-in-tariff and tax-rebate policies as follows.

<table>
<thead>
<tr>
<th>Condition</th>
<th>( F^* )</th>
<th>( R^* )</th>
<th>( \pi^F_Q = \pi^R_Q )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q \in [0, \frac{rX}{r+P}] )</td>
<td>( P )</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Q \in (\frac{rX}{r+P}, \frac{rX}{P}] )</td>
<td>( \frac{rX}{Q} )</td>
<td>( 1 - \frac{P}{r}Q )</td>
<td>( \frac{b+P}{r}Q - X )</td>
</tr>
<tr>
<td>( Q \in (\frac{rX}{P}, 1] )</td>
<td>( P )</td>
<td>0</td>
<td>( \frac{1}{2}Q )</td>
</tr>
</tbody>
</table>

Because \( \pi^F_Q = \pi^R_Q \) in all cases, the two subsidy policies are equivalent.

**Proof of Proposition 2.** Let \( F^* \) be the optimal feed-in tariff level with the corresponding threshold efficiency for investment \( Q^{F^*} \). By the definition of the threshold efficiencies in (4) and (6), there exists \( R \), computed from \( Q^R = Q^{F^*} \): \[
\frac{rX}{F^*} = Q^{F^*} = Q^R = \frac{rX(1-R)}{P},
\]
as \( R = 1 - P/F^* \). Then, for any \( Q \geq Q^{F^*} \):

\[
\frac{F^* - P}{r} Q \geq \frac{F^* - P}{r} Q^{F^*} = \frac{F^* - P}{F^*} rX = \frac{F^* - P}{F^*} X = RX.
\]

Therefore,

\[
\pi^{F^*} = \int_{Q^{F^*}}^{1} \left[ \frac{b}{r} - \frac{F^* - P}{r} \right] Q^\psi(Q) dQ \leq \int_{Q^R}^{1} \left[ \frac{b}{r} Q - RX \right] \psi(Q) dQ = \pi^R.
\]

Because \( R \) is a feasible tax rebate, \( \pi^R \geq \pi^F \geq \pi^{F^*} \).

**Proof of Lemma 1.** Part (i) follows directly from the analysis in (3) and (4).

Proof of Part (ii) comes from the standard approach (Dixit and Pindyck 1994), with slight modifications to help us to reuse results in the subsequent proofs. First, the household’s investment policy has a threshold structure such that a household with efficiency \( Q \) invests if \( P(t) \geq P_Q^B \), where \( P_Q^B \) is the threshold value (to be computed). Second, instead of solving a free-boundary problem directly to compute the value of the household and the value of the threshold, we transform the problem to a more convenient form as follows.

Assuming that the current time is \( t \) and the investment has already been made, a household receives \( P(t + s)Q \) with \( s \in [0, \infty) \) in perpetuity, which corresponds to value

\[
E_t \left[ \int_0^{\infty} e^{-rs} P(t+s)Q ds \right] = \frac{P(t)Q}{r - \mu_p},
\]

where the expectation is taken under the pricing measure, given the information up to time \( t \). Define the stopping time \( \tau_Q^B = \inf \{ t : P(t) \geq P_Q^B \} \). Under the pricing measure, the value of the household is

\[
W_Q^R = E \left[ e^{-r \tau_Q^B} \left( E_{\tau_Q^B} \left[ \int_0^\infty e^{-rs} P(\tau_Q^B + s)Q ds \right] - X(1-R) \right) \right] = E \left[ e^{-r \tau_Q^B} \left\{ \frac{P(\tau_Q^B)Q - X(1-R)}{r - \mu_p} \right\} \right],
\]

(46)
where equality follows from (45). We use the fact that \( P(\tau_Q^R) = P_Q^R \) to write
\[
W_Q^R = E \left[ e^{-r\tau_Q^R} \left\{ \frac{P_Q^R}{r - \mu_p} Q - X(1 - R) \right\} \right] = E \left[ e^{-r\tau_Q^R} \left\{ \frac{P_Q^R}{r - \mu_p} Q - X(1 - R) \right\} \right].
\] (47)
Next, to compute \( E \left[ e^{-r\tau_Q^R} \right] \) we interpret it as the value \( V(P) \) of an Arrow-Debreu security that pays \$1 at time \( \tau_Q^R \), given that the current price of electricity is \( P \). Now, following the standard approach (Dixit and Pindyck 1994 or McDonald and Siegel 1986), this value satisfies the ordinary differential equation (ODE):
\[
\frac{1}{2} \sigma_p^2 \frac{d^2V}{dP^2} + \mu_p \frac{dV}{dP} - rV = 0
\] (48)
for \( P \leq P_Q^R \), with the boundary conditions: \( V(0) = 0 \) and \( V(P_Q^R) = 1 \). The solution approach for such ODEs is standard (Dixit and Pindyck 1994). It suffices to verify that
\[
V(P) = \left( \frac{P}{P_Q^R} \right)^\theta
\] (49)
for \( P \leq P_Q^R \) and \( V(P) = 1 \) for \( P > P_Q^R \), with \( \theta \) given in (16), satisfies (48) and the boundary conditions.

Finally, we need to compute the value of the threshold \( P_Q^R \). A household chooses the investment threshold \( P_Q^R \) to maximize the household’s value \( W_Q^R \) given in (47). This is equivalent to applying a smooth-pasting condition. Solving optimization \( \max_{P_Q^R} W_Q^R \), we derive value for \( P_Q^R \) given in (15).

**Proof of Lemma 2.** Quantity \( \theta \) in (16) is a solution of the characteristic equation
\[
\frac{1}{2} \sigma_p^2 \theta^2 + \left( \mu_p - \frac{1}{2} \sigma_p^2 \right) \theta - r = 0
\] (50)
for ODE (48). Using implicit differentiation in (50), we show that \( \frac{d\theta}{d\tau_Q^R} = -\frac{\sigma_p^2}{\theta - 1} \sigma_p^2 < 0 \) so that \( \theta \) decreases in \( \sigma_p \). Consequently, \( \frac{\theta}{\theta - 1} \) increases in \( \sigma_p \). Furthermore, \( \frac{d\theta}{d\mu_p} = -\frac{\sigma_p^2}{\theta - 1} \sigma_p^2 < 0 \), thus \( \frac{\theta}{\theta - 1} \) increases in \( \mu_p \).

**Proof of Lemma 3.** (i) Under the feed-in-tariff policy, the government’s optimization problem is (18). This function is maximized when \( F^* = \frac{x}{Q} \) so that \( \pi_{Q^*} = \left( \frac{x}{Q} + \frac{P(0)}{r_{Q^*}} \right) Q - X \), provided it is positive. Requirement that the optimal profit is non-negative produces the conditions on \( Q \). Note that \( F^* \geq P(0) \) by assumption (21).

(ii) Under the tax-rebate policy, the government’s optimization problem is (20). Value \( P_Q^R \) is given in (15) and it decreases in \( R \) with \( P_Q^R = 0 \). There are two cases: \( \frac{P(0)}{P_Q^R} > 1 \) and \( \frac{P(0)}{P_Q^R} \leq 1 \).

Case 1: \( \frac{P(0)}{P_Q^R} > 1 \). It follows that \( \frac{P(0)}{P_Q^R} > 1 \) for all \( R \). Thus, \( R^* \) should be 0. The condition \( \frac{P(0)}{P_Q^R} > 1 \) for \( R = 0 \) is equivalent to \( Q > \frac{X}{\theta + 1} \). However, by assumption (21), this set is empty because \( \frac{X}{\theta + 1} > 1 \).

Case 2: \( \frac{P(0)}{P_Q^R} \leq 1 \) or equivalently \( Q \leq \frac{X}{\theta + 1} + \frac{X}{\theta} \). There exists \( R^1 = 1 - \frac{P(0)Q}{X(\theta + 1)} \theta - 1 \), such that \( \frac{P(0)}{P_Q^R} = 1 \) and \( R^1 \geq 0 \) if and only if \( Q \leq \frac{X}{\theta + 1} \). The condition \( R^1 \leq 1 \) is satisfied because \( \frac{P(0)Q}{X(\theta + 1)} \theta - 1 \geq 0 \). Therefore, the objective function (20) is defined in two pieces: \( R \in [0, R^1] \) and \( R \in [R^1, 1] \).

For all \( R \in [R^1, 1] \), the objective function in (20) is \( \frac{X}{\theta} - RX \) and it decreases in \( R \). Therefore, on this interval, \( R^* = R^1 \). The corresponding objective value is
\[
\pi_{Q^*} = \frac{X}{\theta} + \frac{P(0)Q}{X(\theta + 1)} - X.
\]

For all \( R \in [0, R^1] \), the objective function in (20) is \( \frac{X}{\theta} - RX \) \( \frac{P(0)Q}{P_Q^R} \theta - 1 \). This function is unimodal. The derivative of this function at \( R = R^1 \) is positive if \( Q \geq \frac{X}{\theta + 1} \), then, in this case, \( R^* = R^1 \). The derivative of the objective function is negative at \( R = 0 \) if \( Q \leq \frac{X}{\theta + 1} \). Then, in this case, \( R^* = 0 \) with the corresponding objective value of
\[
\pi_{Q^*} = \frac{X}{\theta} \left( 1 + \frac{P(0)Q}{X(\theta + 1)} - \theta - 1 \right) - X.
\]
For \( Q \in \left( \frac{X}{\theta + 1} \right) \left( \frac{X}{\theta + 1} \right) \), the first order conditions produce \( R^2 = \frac{1}{X(\theta_1 - 1)} \left( \frac{X}{\theta_1} - X \right) \). Hence, \( R^* = R^2 \) and the corresponding objective value is
\[
\frac{1}{X(\theta_1 - 1)} \left( \frac{P(0)Q}{X_1} - \theta - 1 \right)^2. \]
Proof of Proposition 3. \(\text{(i)}\) For all \(Q \in \left[0, \frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}\right], \pi_Q^{R^*} \geq \pi_Q^{F^*}\) as \(\pi_Q^{F^*} = 0\) and \(\pi_Q^{R^*} \geq 0\) by Lemma 3. 

(ii) For \(Q \in \left[\frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}, 1\right], \) because \(\frac{X}{r + \frac{r(1 - \eta)}{\mu_p}} < \frac{X}{r + \frac{r(1 - \eta)}{\mu_p}},\) Lemma 3 yields
\[
\pi_Q^{R^*} = \left[\frac{P(0)Q}{r - \mu_p} - X\right]^{\frac{\theta - 1}{\theta - 1}} \leq \frac{1}{\theta - 1} \left(X - \frac{bQ}{r}\right). \tag{51}
\]
The value of \(\pi_Q^{R^*}\) is defined on two intervals: \(Q \in \left(\frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}, \frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}\right)\) and \(Q \in \left(\frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}, 1\right].\) First, consider \(Q \in \left(\frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}, \frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}\right].\) The assumption \((22)\) that \(\theta \frac{b}{r} > \frac{P(0)}{r - \mu_p} \frac{\theta - 1}{\theta}\) equivalent to \(\theta \frac{b}{r} > \frac{P(0)}{r - \mu_p} \frac{\theta - 1}{\theta}.\) Therefore, \(Q \geq \frac{X}{\theta} + \frac{bQ}{r}\) and, by Lemma 3,
\[
\pi_Q^{R^*} = \left[\frac{P(0)Q}{r - \mu_p} - X\right]^{\frac{\theta - 1}{\theta - 1}} \leq \frac{1}{\theta - 1} \left(X - \frac{bQ}{r}\right). \tag{52}
\]
The inequality is constructed by replacing term \(\left[\frac{P(0)}{r - \mu_p}\right]^{\theta}\) evaluated at \(R = \frac{1}{(\theta - 1)X} \left(\theta \frac{bQ}{r} - X\right)\) with 1. Note that \(\pi_Q^{F^*} = \frac{bQ}{r} + \frac{P(0)Q}{r - \mu_p} - X \geq \frac{bQ}{r} - \left(X - \frac{bQ}{r}\right)\) if and only if \(\frac{bQ}{r} + \frac{P(0)Q}{r - \mu_p} - X \geq 0,\) which is true on this interval.

Second, consider \(Q \in \left(\frac{X}{r + \frac{r(1 - \eta)}{\mu_p}}, 1\right].\) From Lemma 3
\[
\pi_Q^{R^*} = \frac{bQ}{r} + \frac{P(0)Q}{r - \mu_p} - \frac{\theta - 1}{\theta}X \leq \frac{bQ}{r} + \frac{P(0)Q}{r - \mu_p} - X = \pi_Q^{F^*}. \tag{53}
\]

Proof of Lemma 4. The proof is similar to that of Lemma 1.

\((\text{i})\) Under the feed-in-tariff policy, the household’s investment policy has a threshold structure such that a household invests if \(X(t) \leq X_Q^F,\) where \(X_Q^F\) is the threshold value (to be computed). Assuming that the current time is \(t\) and the investment has already been made, a household receives \(FQ\) in perpetuity, which corresponds to value \(\int_0^\infty e^{-rt} FQ \, ds = \frac{FQ}{r}.\) Define the stopping time \(\tau_Q^F = \inf\{t \geq X(t) \leq X_Q^F\}.\) Under the pricing measure, the value of the household is
\[
W_Q = E\left[e^{-r\tau_Q^F} \left\{\frac{FQ}{r} - X \left(\tau_Q^F\right)\right\}\right] = E\left[e^{-r\tau_Q^F} \left\{\frac{FQ}{r} - X_Q^F\right\}\right], \tag{54}
\]
where equality follows from \(X(\tau_Q^F) = X_Q^F.\) To compute \(E\left[e^{-r\tau_Q^F}\right],\) we interpret it as the value \(V(X)\) of an Arrow-Debreu security that pays \(1\) at time \(\tau_Q^F,\) given the current cost is \(X.\) This value satisfies the following ODE:
\[
\frac{1}{2} \sigma^2 X^2 \frac{d^2V}{dx^2} + \mu \sigma X \frac{dV}{dx} - rV = 0, \tag{55}
\]
for \(X \geq X_Q^F,\) with the boundary conditions that \(V \to 0\) as \(X \to \infty\) and \(V(X_Q^F) = 1.\) The solution of this ODE with these boundary conditions is
\[
V(X) = \left(\frac{X_Q^F}{X}\right)^{\eta}, \tag{56}
\]
if \(\frac{X_Q^F}{X} \leq 1\) and \(V(X) = 1\) if \(\frac{X_Q^F}{X} > 1,\) where \(\eta\) is given in \((24).\) The last step of the proof is to compute the investment threshold \(X_Q^F\) that maximizes the household’s value \(W_Q^F.\) This optimal \(X_Q^F\) is presented in \((23).\)

(ii) Under the tax-rebate policy, the cost of the household is \(C(t) = X(t)(1 - R),\) which follows the same process \((2)\) as \(X(t).\) The benefit to a household comes from the constant price \(P\) from the time of investment into perpetuity. Thus, the households problem has the same structure as in part \((i)\) with \(C\) in place of \(X\) and \(P\) in place of \(F.\) Adopting the proof from part \((i)\) results follow.
Proof of Lemma 5. (i) Under the feed-in-tariff policy, the government’s optimization problem is in (27). Recall that threshold $X^*_Q$ is given in (23) and it increases in $F$. There are two cases: $\left[ \frac{X^*_Q}{X(0)} \right]^\eta > 1$ and $\left[ \frac{X^*_Q}{X(0)} \right]^\eta \leq 1$.

If $\left[ \frac{X^*_Q}{X(0)} \right]^\eta > 1$, then $F^* = P$. The condition $\left[ \frac{X^*_Q}{X(0)} \right]^\eta > 1$ for $F = P$ is equivalent to $Q > \frac{rX(0)}{P+\pi}$. If $\left[ \frac{X^*_Q}{X(0)} \right]^\eta \leq 1$, there are two candidate solutions: either $F^1$ or $F^2$, where $F^1 = \{F : X^*_Q = X(0)\}$ and $F^2$ is the maximizer of $f_2(R) = \left( \frac{b}{r} - \frac{P+\pi}{r}\right) Q \left[ \frac{X^*_Q}{X(0)} \right]^\eta$. Note that $F^2 \leq F^1$ so that $F^* = F^2$ if and only if $Q \leq \frac{rX(0)}{(b+P)(\frac{2}{\eta}+1)}$. Finally, $f_2'(P) < 0$ if $b \leq \frac{P}{\eta}$. Therefore, in this case, $F^* = P$.

(ii) Under the tax-rebate policy, using the results of Lemma 4, the government’s optimization problem is in (29) and the analysis is similar to the feed-in-tariff policy above. Specifically, there are two cases $\left[ \frac{X^*_Q}{X(0)} \right]^\eta > 1$ and $\left[ \frac{X^*_Q}{X(0)} \right]^\eta \leq 1$.

Case 1: $0 \leq Q \leq \frac{rX(0)}{(b+P)(\frac{2}{\eta}+1)}$. This condition and Lemma 5, yield
\[
\frac{\pi^*_Q}{\pi^*_Q} = \left( \frac{b+P}{b+P+\pi} \right)^{\eta+1} \geq \left( \frac{1}{\eta+1} \right)^{\eta+1} \geq 1,
\]
where the first inequality is due to $b > \frac{P}{\eta}$ and the second follows from the Bernoulli’s inequality.

Case 2: $\frac{rX(0)}{(b+P)(\frac{2}{\eta}+1)} \leq Q \leq \frac{rX(0)}{(b+P)(\frac{2}{\eta}+1)}$. $\pi^*_Q$ is constant, but $\pi^*_Q$ increases. Thus, $\pi^*_Q \geq \pi^*_Q$.

Case 3: $\frac{rX(0)}{(b+P)(\frac{2}{\eta}+1)} \leq Q \leq \frac{rX(0)}{(b+P)(\frac{2}{\eta}+1)}$. By Lemma 5, $\pi^*_Q - \pi^*_Q = \frac{X(0)}{p} - \frac{PQ}{r(\eta+1)} \geq 0$, where the inequality follows from $Q \leq \frac{rX(0)}{P+\pi}$.

Case 4: $Q \geq \frac{rX(0)}{P+\pi}$. From Lemma 5, it follows that $\pi^*_Q = \pi^*_Q$.

Proof of Proposition 4. Based on the optimal policies given in Lemma 5, there are three intervals on $b$: $b \leq \frac{P}{\eta+1}, \frac{P}{\eta+1} < b \leq \frac{P}{\eta},$ and $b > \frac{P}{\eta}$. In the first interval, $\pi^*_Q = \pi^*_Q = \frac{bQ}{r} \left[ \frac{PQ}{rX(0)\eta+1} \right]^\eta$. In the second interval, $F^*$ remains the same, but $R^*$ depends on $Q$. Nevertheless, $R = 0$ is still feasible. Thus, $\pi^*_Q = \pi^*_Q \leq \pi^*_Q$.

Proof of Lemma 6. (i) Under the feed-in-tariff policy, the price dynamics does not affect the household’s investment decisions. Therefore, this part is identical to part (i) in Lemma 4.

(ii) First, we introduce cost process $C(t) = X(t)(1 - R)$ and observe that it follows equation (2). Assuming that the current time is $t$ and the household made an investment, the value of this investment in perpetuity is $V(t) = E_i \int_0^\infty e^{-r(s)} P(t+s)Q ds = \frac{P(t)Q}{r-\mu_p}$. This value follow process (1). Define new variable $Y_i(t) = \frac{V(t)}{C(t)}$. Now the direct application of results from McDonald and Siegel (1986) proves this lemma.

Proof of Lemma 7. This proof is omitted for brevity because it is similar to the proof of Lemma 2.

Lemma 8. Consider a model with price process (1), cost process (2), and $Y(t) = \frac{P(t)}{X(t)}$. 

(i) Let $\tau_1 = \inf \{ t : P(t) \geq P_Q^R \}$, where $P_Q^R$ is given in (15). Then,

$$E[e^{-r\tau_1}] = \left[ \frac{P(0)}{P_Q^R} \right]^{\theta} \wedge 1,$$

where $\theta$ is defined in (16).

(ii) Let $\tau_2 = \inf \{ t : X(t) \leq X_Q^F \}$, where $X_Q^F$ is defined in (23). Then,

$$E[e^{-r\tau_2}] = \left[ \frac{X_Q^F}{X(0)} \right]^{\eta} \wedge 1,$$

where $\eta$ is defined in (24), and

$$E[e^{-r\tau_2}P(\tau_2)] = P(0) \left\{ \left[ \frac{X_Q^F}{X(0)} \right]^{\nu} \wedge 1 \right\},$$

where $\nu$ is defined in (36).

(iii) Let $\tau_3 = \inf \{ t : Y(t) \geq Y_Q^R \}$, where $Y_Q^R$ is defined in (32). Then,

$$E[e^{-r\tau_3}] = \left[ \frac{Y(0)}{Y_Q^R} \right]^{\zeta} \wedge 1,$$

where $\zeta$ is defined in (40), and

$$E[e^{-r\tau_3}X(\tau_3)] = X(0) \left\{ \left[ \frac{Y(0)}{Y_Q^R} \right]^{\gamma} \wedge 1 \right\},$$

where $\gamma$ is defined in (33).

**Proof of Lemma 8.** (i) See the proof of Lemma 1.

(ii) We derived value of $E[e^{-r\tau_2}]$ in the proof of Lemma 4.

Using the standard approach (Dixit and Pindyck 1994), value $V(X, P) = E[e^{-r\tau_2}P(\tau_2)]$ satisfies the following partial differential equation (PDE):

$$\frac{1}{2} \sigma^2_x X^2 \frac{\partial^2 V}{\partial X^2} + \frac{1}{2} \sigma^2_P^2 \frac{\partial^2 V}{\partial P^2} + \rho \sigma_x \sigma_P X P \frac{\partial^2 V}{\partial X \partial P} + \mu_x P \frac{\partial V}{\partial X} + \mu_P \frac{\partial V}{\partial P} - rV = 0,$$

for $X \geq X_Q^F$, with the boundary conditions $V(X_Q^F, P) = P$, $\lim_{X \to \infty} V(X, P) = 0$. We verify by substitution that expression (60) solves this PDE with these boundary conditions.

(iii) Define function $V(Y) = E[e^{-r\tau_3}]$. Quantity $Y(t)$ follows the process

$$dY(t) = (\mu_y - \mu_x + \sigma_x^2 - \rho \sigma_x \sigma_y) Y(t) dt + \sigma_y Y(t) dZ_y(t) - \sigma_y Y(t) dZ_x(t).$$

Using the standard approach (Dixit and Pindyck 1994), function $V(Y)$ satisfies ODE:

$$\frac{1}{2} \sigma^2_Y Y^2 \frac{d^2 V}{dY^2} + (\mu_y - \mu_x + \sigma_x^2 - \rho \sigma_x \sigma_y) Y \frac{dV}{dY} - rV = 0,$$

for $Y \leq Y_Q^R$, with the boundary conditions $V(0) = 0$ and $V(Y_Q^R) = 1$. Direct substitution verifies that expression (61) satisfies this ODE and the boundary conditions.

Redefine function $V(P, X) = E \left[ e^{-r\tau_3}X(\tau_3) \right]$. Using the standard approach (Dixit and Pindyck 1994), this function satisfies PDE:

$$\frac{1}{2} \sigma^2_P^2 \frac{\partial^2 V}{\partial P^2} + \frac{1}{2} \sigma^2_x X^2 \frac{\partial^2 V}{\partial X^2} + \rho \sigma_x \sigma_P X P \frac{\partial^2 V}{\partial P \partial X} + \mu_P \frac{\partial V}{\partial P} + \mu_x X \frac{\partial V}{\partial X} - rV = 0,$$

for $\frac{P}{X} \leq Y_Q^R$, with the boundary conditions $V(0, X) = 0$ and $V(P, X)|_{P = Y_Q^R} = X$. Direct substitution proves that (62) satisfies both the PDE and the boundary conditions. ■
References


