Optimal Assortment, Quantity and Dynamic Pricing for Multiple Vertically-Differentiated Products with Short-Life Cycle

1 Introduction and Literature Review

This work is motivated by the example of an online travel agency (OTA), such as Orbitz.com and Expedia.com, that purchases rooms of different types (e.g. standard, deluxe or premium) from hotels for a given date and sells them to its customers (Lee et al. 2013). The products (i.e. rooms) are perishable, substitutable and have differential quality. The demand is random but price-elastic. For each date, the OTA decides which types of rooms to purchase in a hotel and the corresponding quantity. Further, it varies room prices dynamically throughout the selling horizon to maximize its profit.

Motivated by this example, we study a firm’s optimal assortment, order quantity and dynamic pricing problem for multiple vertically-differentiated and substitutable products with short life-cycle. At the beginning of the selling horizon, the firm determines the composition of the assortment as well as the order quantity for each selected product. The two decisions are inter-linked since a product can be a part of the optimal assortment if and only if the optimal order quantity for it is non-zero. Depending upon whether or not there is a cost to include a product in the assortment, we consider two scenarios. In either scenario, the firm also pays a variable cost proportional to the quantity purchased for each product included in the assortment.

Once the optimal assortment is built, the firm starts selling products to the customers. We assume that the selling horizon consists of a finite number of periods such that at most one customer arrives during a period. The customers are heterogeneous in nature, in that their willingness to pay for quality varies from one customer to another. An arriving customer chooses a product from available alternatives that maximizes her utility, which is equal to the difference between her value or maximum willingness to pay for the product less the price.
We take the value for the product as product quality multiplied by the \textit{type} of the customer. We assume that the types of different customers are independent and drawn from a common Uniform distribution.

Given these assumptions, we formulate a stochastic dynamic program to determine the optimal price vector in a generic period during the selling horizon given the vector of unsold inventories at the beginning of the period.

In the existing literature, two papers closest to this study are Ackay et al. (2010) and Pan and Honhon (2012). Our model is a direct extension of Ackay et al. (2010) and uses the same modeling framework as them. Similar to our work, Ackay et al. (2010) consider multiple vertically-differentiated products with the objective of determining insights on the optimal dynamic pricing policy. However, the assortment as well as initial inventory vector are fixed in their model. Compared to them, we derive additional properties of the optimal dynamic pricing policy as well as develop insights on the optimal assortment and order quantity decisions. On the other hand, Pan and Honhon (2012) determine the optimal assortment for multiple vertically-differentiated products, but prices are not varied dynamically over the selling horizon in their model. Compared to them, our contribution is to illustrate how to determine the optimal assortment and order quantity decisions in the presence of dynamic pricing as well as how these decisions change when prices are varied dynamically compared to when they are static.

2 Results

A summary of key insights from the paper are as follows:

1. We prove that the expected revenue-to-go function in any period during the selling horizon is an anti-multimodular function of the vector of inventories available at the beginning of the period.

2. We also show that the marginal worth of a high-quality product decreases if its own
stock level increases by one unit and the stock level of any low-quality product decreases by one unit. Further, the marginal worth of a product is more sensitive to its own stock level compared to the stock level of a lower-quality substitute.

3. We develop two approaches to determine the optimal assortment and the order quantity vector when there is no cost to include products in the assortment. In the first approach, we show that the problem of finding the optimal assortment and order quantities can be represented as a shortest path problem. Thus, any algorithm to solve the shortest-path problem can also be used to solve our problem. In the second approach, we develop a single-pass algorithm that has polynomial time-complexity.

4. We establish several structural properties regarding the optimal assortment and quantity decisions. For example, we prove that if a product is not included in the optimal assortment when there is no cost for including a product in the assortment, it will also not be included in the optimal assortment when such a cost exists. Using these properties, we develop an efficient algorithm to obtain the optimal assortment and the quantity vector when there is a non-zero cost to include products in the assortment.

5. Using computational experiments, we find that it is optimal to carry more products in the optimal assortment in the dynamic pricing scenario compared to the static pricing scenario. Further, the quantities are larger when prices are varied dynamically.

References

