Design of Advance Market Commitment Contracts

(Authors’ names blinded for peer review)

(1) Problem Definition: Pharmaceutical companies are reluctant to invest in production capacity for vaccines and other products to combat diseases prevalent in developing countries because demand is uncertain and the affected countries have limited ability-to-pay to cover product development and capacity infrastructure costs.

(2) Academic/Practical Relevance: We obtain equilibrium decisions facing manufacturers and global health agencies within the framework of an advance market commitment (AMC) contract via formal models. From a practitioner perspective, we evaluate the base case and several variants of the AMC contract that global health agencies have considered implementing.

(3) Methodology: We formulate and solve a Principal-Agent model using concepts from game theory. Our two-manufacturer model involves a simultaneous capacity game between the manufacturers, embedded in a leader-follower game involving the global health agency and the manufacturers.

(4) Results: We show that the global health agency can choose contract terms such that a single manufacturer with a late-stage product is induced to invest in an amount of capacity that maximizes the number of doses supplied. We also show that among variants of the base contract, minimum capacity requirement contract outperforms the minimum purchase guarantee contract. When there are two manufacturers, we show that demand allocation rules matter, and that it may not be optimal for the global health agency to strictly prioritize one manufacturer, as long as both manufacturers’ products meet minimum functional requirements.

(5) Managerial Implications: Global health agencies have considered proposals to offer purchase guarantees as part of AMC contracts. We show that a better alternative might be to require minimum capacity investments from manufacturers. Similarly, with two manufacturers, it might be beneficial for the health agency to consider allocating demand proportional to each manufacturer’s capacity investment.

Key words: Advance Market Commitments; Incentives; Global Health; Applied Game Theory

1. Introduction

Every year, millions of people in developing countries die from infectious diseases such as malaria, HIV, rotavirus, human papillomavirus, and pneumococcal disease. However, pharmaceutical companies are reluctant to invest in the research and development (R&D) of vaccines for such diseases...
and also to build manufacturing capacity necessary to bring the vaccines to market (Levine et al. 2005). This reluctance is driven in large part by the commercial viability (or a lack thereof) of the vaccine markets in developing countries. Pharmaceutical companies perceive these markets to be unattractive because demand is uncertain and recipient countries have insufficient “ability-to-pay” to cover vaccine development and capacity infrastructure costs (Cernuschi et al. 2011).

Advance Market Commitments (AMCs) are legally–binding contracts that are aimed at making the developing–country markets for vaccines more attractive and commercially viable. Specifically, AMCs create incentives for pharmaceutical companies to develop and manufacture vaccines by committing to pay a pre–specified price, which is typically well above the price that the recipient countries can afford to pay, for a fixed number of doses if the firm successfully develops and brings the vaccine to market (Berndt et al. 2007). The price commitment offered by AMCs is structured in the form of a “top-up” (in addition to the base price paid by the recipient country) for a pre–specified number of doses of a vaccine that meets certain functional requirements. For example, an AMC contract might offer a top-up of $10/dose for the first 100 million doses (i.e., the potential to earn a total of $1 billion in top-ups) to any firm(s) that develops and brings the vaccine to market. The additional funding offered in the form of top–ups helps to offset the product development and manufacturing capacity costs, thereby making the markets for vaccines more viable.

AMCs could be used, in principle, to simulate R&D investments for products that are in the early stages of development (e.g., several candidate vaccines exist for Malaria and Dengue, see Arama and Troye-Blomberg 2014) and/or to encourage investments in manufacturing capacity for products that are in the late stages of development (e.g., vaccines for pneumococcal disease and rotavirus during the conceptualization of AMCs in the mid-2000s, see Levine et al. 2005). The AMC contract implemented to date focuses on a late–stage product, namely the pneumococcal vaccine, and the focus of our paper is also on such settings. The objective is to maximize investments in manufacturing capacity through appropriate AMC contract design (Cernuschi et al. 2011).

In the context of AMCs for late–stage products, a central question of interest to humanitarian healthcare organizations (defined broadly to include global health agencies, charitable foundations and not–for–profit organizations) relates to the choice of the top–up amount and the top–up quantity cap, i.e., the number of doses of the drug or vaccine eligible to receive the top–up. These organizations (referred to as the ‘Principal’ throughout the paper) typically have a limited amount of funds dedicated to AMC contracts and it is important to use the limited funds in the best possible way by offering the right combination of the top–up amount and top–up quantity to maximize the manufacturer’s capacity investment. If the principal sets the top–up quantity cap to be too low (meaning a high per unit top–up amount), the manufacturer may lack the motivation to make substantial investments in capacity since it has to produce and sell only a (relatively) small number
of units to earn the entire contract amount. On the other hand, setting the top-up quantity cap to be too high (meaning a low per unit top-up amount) makes it difficult, if not impossible, for the manufacturer to earn the full AMC contract amount. The lower per-unit top-up amount could significantly drive down the marginal benefit of additional capacity, which in turn may lead to lower investment by the manufacturer. Hence, identifying the optimal combination of the top-up amount and quantity cap is critical to creating the right incentives for the manufacturer to invest in production capacity.

In practice, there may be more than one manufacturer with drugs or vaccines in the advanced stages of development. For example, in the case of the pneumococcal vaccine, there were two firms with late-stage formulations at the time the AMC contract was being developed (Cernuschi et al. 2011). When there are multiple potential manufacturers with late-stage candidate vaccines, the AMC contract design problem involves additional considerations and decisions beyond identifying the optimal combination of the top-up amount and quantity cap. For example, when entering into contracts with more than one manufacturer, the principal needs to think about how best to split the realized demand and the quantity cap between the different manufacturers (potential strategies include strict priority, and allocation proportional to capacity investment). In addition to the optimal contract design, a more fundamental question also emerges in settings involving multiple manufacturers with late-stage candidate vaccines: is it in the best interest of the principal to enter into contracts with multiple manufacturers or would it be better to offer the AMC contract to only one manufacturer? When the principal contracts with multiple manufacturers, the top-up quantity cap for each manufacturer might be lower than the quantity cap offered when there is only one manufacturer. This could lead to a reduction in the capacity investments by the individual manufacturers relative to the case where the principal enters into a contract with one manufacturer. However, the presence of multiple potential manufacturers could also motivate firms to increase their capacity investments in an effort to attract a larger proportion of the total demand. It is not clear apriori which of these two effects dominate and if and how contracting with multiple manufacturers impacts the total capacity investments.

We begin our analysis with the single manufacturer scenario and analyze the AMC contract design problem using a one-period model with stochastic demand. As mentioned earlier, in case of late-stage vaccines/drugs, the key motivation behind AMC contracts is to create incentives for the manufacturer to invest in production capacity. The principal moves first and offers an AMC contract that specifies the top-up amount (denoted by $\delta_r$) and the quantity of vaccines/drugs that would be eligible to receive the top-up (denoted by $\bar{q}$). Given the AMC contract terms, the manufacturer decides how much production capacity ($\kappa$) to install. Then, subject to the capacity constraints, the manufacturer makes production quantity decisions. Finally, revenues are realized
based on the actual demand, production quantity, and the top-up quantity cap (the first \( \bar{q} \) units of demand satisfied receive the top-up amount in addition to the base price paid by the recipient country). We use backwards induction to identify the optimal combination of top-up amount and quantity cap that the principal should offer in order to maximize the manufacturer’s capacity investments. In the two-manufacturer scenario, the AMC contract specifies the top-up amount \( (\delta_r) \), the top-up quantity that each manufacturer is eligible to receive (denoted by \( \bar{q}_1 \) and \( \bar{q}_2 \)), and also how the realized demand will be split between the two manufacturers. Given the AMC contract terms, the two manufacturers make their capacity investment decisions simultaneously.

Our analysis yields several key results and insights regarding the design of AMC contracts for drugs that are in the late stages of development. For settings where there is only one manufacturer with a viable late-stage product, we demonstrate that under the optimal AMC contract, the manufacturer’s installed capacity precisely matches the top-up quantity offered by the principal. A key implication of this result is that through appropriate contract design, AMCs have the potential to induce manufacturers to make the desired capacity investments to produce drugs and vaccines for diseases that are concentrated in low-income countries — investments that pharmaceutical companies are otherwise reluctant to make. For settings where there are two manufacturers with viable late-stage products, we have the following results. (i) When allocating demand, it is not optimal to prioritize one manufacturer over another solely based on the quality or efficacy of the products. A question that frequently comes up in practice is whether the principal should give priority to a product with higher efficacy for procurement or if the principal should treat all the products equally as long as they meet certain functional requirements (Berndt et al. 2007). Our analysis indicates that the latter strategy is better in terms of inducing higher total capacity investments from the manufacturers. (ii) When multiple products meet the functional requirements, it is not optimal for the principal to selectively enter into an AMC contract with just one manufacturer. Our analysis of the two-manufacturers scenario indicates that the principal can induce higher total capacity investments by entering into AMC contracts with both manufacturers instead of offering the contract to only one of them.

1.1. Contributions

Academic contributions. Prior papers within the ‘humanitarian and not-for-profit operations’ literature have examined the impact of sales and purchase subsidies on increasing the consumption of health products. AMC contracts are fundamentally different from the subsidy models studied in the literature and one of our key contributions is to conduct a rigorous analysis of how AMC contracts influence the manufacturers’ capacity investment decisions and the number of people served. Another key contribution of our work is to study the impact of incentives on capacity investment
decisions in settings that involve not–for–profit players. This is a topic that has not received much attention within the existing literature on supply chain contracts for capacity investment.

**Managerial contributions.** Our paper presents a framework to analyze the impact of AMC contracts on the manufacturer’s capacity investments and ultimately, the number of people who benefit from receiving a health product that might otherwise be unavailable in developing countries. The results emanating from this study could be valuable to humanitarian healthcare managers and policy–makers in designing the appropriate AMC contracts to ensure that the available funds are put to the best use by maximizing the number of people served. Our analysis also shines light on how different factors including the production and capacity installation costs and the available budget influence the optimal AMC contract design. More importantly, our work could offer insights regarding how much AMC funding commitment might be required to generate a certain level of capacity investment from the manufacturers, a fundamental question of interest to both global health organizations and policy–makers. For settings where there are multiple manufacturers with viable late–stage products, there are ongoing discussions within the global health community regarding how contracts to the different manufacturers should be structured and whether it is beneficial for the principal to engage all manufacturers or only a subset of them through the contracting process. Our work contributes to this discussion by providing some insights regarding how different demand and top–up quantity allocation strategies influence the manufacturers’ capacity decisions.

2. **Literature Review**

Our paper is related to three streams of literature, which we discuss one by one.

**Humanitarian and not-for-profit operations:** This area of research has received increasing attention from operations management (OM) scholars in recent years and our work contributes to the emerging literature on this topic. Previous works have examined a variety of topics including capacity allocation (e.g., Deo et al. 2013, Jonasson et al. 2017), inventory and funding allocation (e.g., Devalkar et al. 2017, Natarajan and Swaminathan 2017, and Yang et al. 2013), impact of uncertainty and delays in donor funding (e.g., Natarajan and Swaminathan 2014, and Gallien et al. 2017), and incentive design (e.g., Taylor and Xiao 2014, Berenguer et al. 2017). We focus in the rest of our discussion on incentive design because that is the topic of our paper as well. Taylor and Xiao (2014) analyze a donor’s subsidy design problem with the objective of maximizing the number of units of malaria drugs sold through a retailer. The donor has a fixed budget and the central question of interest is whether the donor should use the funding to offer the retailer a sales subsidy, or a purchase subsidy, or both, to maximize sales of malaria drugs. Berenguer et al. (2017) analyze how purchase and sales subsidies impact the consumption of a socially beneficial good when
the product under consideration is sold in a single period through for-profit and not-for-profit vendors. They identify conditions (corresponding to the donor’s budget) under which subsidizing not-for-profit vendors results in more consumption than subsidizing for-profit vendors (and vice versa). Our work differs from the two papers mentioned above along some key dimensions.

First, in both these papers, retailers do not face capacity constraints when making purchase quantity decisions whereas in our case, the manufacturer may produce only up to its capacity, which is also a decision variable. This adds another layer of complexity and inter-dependency in the decision-making process. Second, the AMC incentive works differently. For example, AMC contracts do not offer purchase subsidies and the per-dose top-up is available only until the manufacturer exhausts the top-up quantity cap. In case of Taylor and Xiao (2014) and Berenguer et al. (2017), in contrast, sales subsidies amount to a “top-up” on every unit sold. Put differently, the unit sales revenue in our model is different on items sold before and after the demand catches up with the top-up quantity, which in turn leads to structurally different production quantity decisions than the purchase quantity decisions under a sales subsidy.

**Advanced market commitments:** Research on AMCs is relatively recent and it focuses on evaluating the AMC pilot initiative launched in June 2009. The pilot initiative was specifically tailored for pneumococcal vaccines because of the vaccine’s expected health impact and because of an already existing robust pipeline of efficacious vaccines. The pilot-initiative contract offered a base price of $3.50, a top-up price of another $3.50, a program length of ten years, and a total funding of US $1.5 billion (Cernuschi et al. 2011, Barden et al. 2006, Snyder et al. 2011). Note that this amounted to a top-up quantity cap of approximately 428.6 million doses. The AMC research papers provide insights and lessons from the pilot study.

In particular, Cernuschi et al. (2011) suggest that AMC contracts would be effective in establishing a long-term supply commitment from manufacturers, although the success of such contracts is highly dependent on donor funding. They recommend targeting first and second generation suppliers separately because the second generation suppliers require more time to enter the market and may not find it attractive to invest in capacity due to the depletion of AMC funds by first generation suppliers. Note that the top-up amount itself is the same for first and second generation suppliers but the top-up quantity cap might be different (GAVI 2018). Snyder et al. (2011) perform an economic analysis of the pilot AMC and provide numerical examples with a single firm participating in the market. The purpose of their paper is to assess the AMC contract’s performance. They conclude that making the terms of the contract more generous could increase the supply of vaccines for developing countries, resulting in large social gains.

Barden et al. (2006) argue that AMC contracts can accelerate the development of new vaccines because they address two market failures that have discouraged the private sector from investing
in vaccines against neglected diseases. The first is that vaccine purchasers face time-inconsistent incentives, and the second is the failure of markets to mediate a balance between supply and demand for public goods (because of significant externalities). Besides addressing market failures, an added benefit of AMC contracts is that they are also cost-effective since the funds are spent only if the desired vaccine is produced and if there is demand for it. All the papers mentioned above provide insights from the AMC pilot study, but they mostly present anecdotal observations or results based on numerical experiments. Our paper differs from the above literature in that, to the best of our knowledge, it is the first attempt to obtain equilibrium contract parameters under the original AMC contract design and several variants of the original design.

**Supply chain contracts with incentives for capacity investment:** Our paper also adds to the literature on supply chain contracts with incentives aimed at increasing capacity investment. In particular, extensive research exists in the field of semiconductors, consumer electronics and telecommunications industries. Suppliers in these industries share similar market pressures — i.e. highly unpredictable demand, high cost of capacity investment, and declining prices that do not justify capacity investment without incentives. Below, we discuss a few selected papers that explore this line of research.

Erkoc and Wu (2005) study a capacity reservation contract, designed for short life-cycle high-tech products with stochastic demand, and aimed at encouraging the supplier to invest in more capacity. Under this contract, the buyer pays a fee upfront for each unit of capacity reserved. Once the capacity is used, the fee is deducted from the order payment. The authors consider a single-period setting with profit-maximizing players and provide insights on when capacity reservation contracts are beneficial.

Taylor and Plambeck (2007b) study relational contracts. These are informal agreements between the supplier and the buyer that are often used when capacity investment occurs before the new product is fully designed. Taylor and Plambeck (2007b) provide insights into how relational contracts can be designed to provide incentives for capacity investment. Specifically, they show that a relational contract can be sustained in repeated game settings since the value of future business motivates players to abide by the contract terms.

Taylor and Plambeck (2007a) consider a similar setting and compare the performance of relational contracts in which the buyer commits to purchasing a fixed quantity with those in which only a per-unit price is specified. They find that if the capacity cost is sufficiently high, then it is better for the buyer to specify a price-only contract. Otherwise, a price and quantity contract may be more efficient. In contrast to relational contracts, AMC contracts are legally-binding and their terms are typically not renegotiated. In addition, we consider a one-shot contract as opposed to the above-mentioned setting, which is characterized by a repeated game.
Mathur and Shah (2008) consider a scenario in which a supplier needs to build capacity before demand is realized and the contract specifies enforceable penalties for both the supplier and the manufacturer. The manufacturer incurs a penalty for underutilized capacity and the supplier incurs a penalty for each unit under-supplied. They show that the expected profit of the supplier is unimodal and determine its optimal capacity investment. They conclude that the manufacturer can influence the capacity investment of the supplier through appropriate capacity commitments, and that there exists a continuum of contracts that coordinate the supply chain. The AMC, in contrast, is a market-driven mechanism in the sense that a top-up is paid on each unit demanded and supplied up to the top-up quantity, but the principal does not incur any penalty for unused capacity in the event of low demand realizations.

To conclude, the principal-agent framework has been used in many previous OM papers to study incentives for capacity investment. However, the model details and constraints imposed by market realities are different, leading to different dynamics, analyses, and insights. This paper makes a contribution by formulating and solving an economic model that has the potential to help global health agencies design better AMC contracts.

3. Single Manufacturer Models
We formulate two models in the single manufacturer case that differ in terms of the timing of the production quantity decision. In the first model, the manufacturer decides how much to produce after observing demand and in the second, this decision is made before knowing demand. The sequence of events in the single manufacturer setting is shown in Figure 1. We derive the optimal production and capacity investment decisions of the Manufacturer (M) as a function of the contract terms set by the Principal (P), and the contract terms that P will select in equilibrium. The notation used in these models is summarized in Table 1.

<table>
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<tr>
<th>Table 1 Key Notation</th>
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<tr>
<td><strong>Problem Primitives</strong></td>
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<tr>
<td>( D \in [d, \bar{d}] ) = random (continuous) demand; ( d \geq 0, \bar{d} &lt; \infty )</td>
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<tr>
<td>( f(\cdot), F(\cdot) ) = pdf and cdf of demand ( D )</td>
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<tr>
<td>( b ) = Principal’s budget</td>
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<tr>
<td>( r ) = base unit price</td>
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<tr>
<td>( c ) = unit production cost</td>
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<tr>
<td>( c_e ) = unit cost of capacity</td>
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<tr>
<td><strong>Principal’s Decision Variables</strong></td>
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<tr>
<td>( \bar{q} ) = top-up quantity cap</td>
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<tr>
<td>( \delta_r ) = top-up amount on sales up to the top-up cap</td>
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<tr>
<td><strong>Manufacturer’s Decision Variables</strong></td>
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<tr>
<td>( p ) = production quantity</td>
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<td>( \kappa ) = production capacity</td>
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The base unit price $r$, also called tail price, is the maximum amount that the recipient country is willing to pay for the product. The top-up amount and the top-up quantity cap are related by the budget constraint. In particular,

$$ (\delta_r)(\bar{q}) \leq b. \quad (1) $$

Our models are based on some assumptions. First, $M$ must install capacity before observing demand. This is reasonable because capacity installation usually requires a long time and must be undertaken before knowing demand. Second, $r > c$, i.e., if $M$ has capacity, it will produce enough to meet demand, regardless of whether it receives top-up or not for all the units produced. Third, $r < c + c_\kappa$, i.e. the Manufacturer will not invest in capacity if no top-up is offered. However, the Principal can induce the Manufacturer to invest in production capacity by providing top-up $\delta_r$ such that $r + \delta_r > c + c_\kappa$ holds. In this case, if $M$ makes a sale with top-up, it earns enough revenue to make a profit, net of its capacity installation and production costs for that capacity unit. Finally, we assume that all players are risk neutral.

3.1. Model 1: M Produces After Observing Demand

When $M$ observes demand before deciding how much to produce, the production decision is trivial. A profit-maximizing $M$ produces either equal to its production capacity or the demand, whichever is smaller. That is,

$$ p^*(\kappa) = \min\{d, \kappa\}, \quad (2) $$

where $d$ is the realized demand and $\kappa$ is the Manufacturer’s production capacity. To identify $M$’s optimal choice of the production capacity, we need to consider the following two regions of $\kappa$. Subscripts L and R denote the “left” and “right” regions.
Left Region \((\kappa \leq \bar{q})\): In this case, M’s profit-maximization problem, given strategy \(p^*(\kappa)\), can be written as follows.

\[
\max_{0 \leq \kappa \leq \bar{q}} \left[ \pi_L(\kappa) = (r + \delta_r - c)E(\min(D, \kappa)) - c_\kappa \kappa \right] \tag{3}
\]

The first term in the objective function is the expected total revenue from sales and the second term is the cost of capacity installation.

Right Region \((\kappa \geq \bar{q})\): Now, M’s profit-maximization problem is as follows:

\[
\max_{\kappa \geq \bar{q}} \left[ \pi_R(\kappa) = (r - c)E(\min(D, \bar{q})) + (\delta_r)E(\min(D, \bar{q})) - c_\kappa \kappa \right] \tag{4}
\]

The first two terms in the objective function denote the expected total revenue from sales and the top-up, and the third term is the cost of capacity installation. In writing (4), we have used the fact that after observing demand, M receives top-up on \(\min[p^*(\kappa), \bar{q}] = \min[\min\{d, \kappa\}, \bar{q}] = \min\{d, \bar{q}\}\), when \(\kappa \geq \bar{q}\).

Let \(\kappa_i^*, i \in \{L, R\}\) denote the optimal capacity investment in each region. Lemma 1 presents the expressions for the optimal investments in the two regions. For clarity of exposition, we use a function \(g(x)\) to specify optimal investments, which is defined as follows for every \(x \geq 0\).

\[
g(x) = \bar{F}^{-1}\left( \frac{c_\kappa}{r - c + x} \right). \tag{5}
\]

Note that \(g(x)\) is an increasing function of \(x\), and \(g(0) = 0\) when \(r < c + c_\kappa\), which we assume throughout.

**Lemma 1.** It can be verified that \(\pi_L(\kappa)\) and \(\pi_R(\kappa)\) are concave in \(\kappa\), and that the optimal capacity investment in each region is as follows.

\[
\kappa_L^* = \min\{\bar{q}, g(\delta_r)\}, \tag{6}
\]

\[
\kappa_R^* = \bar{q}. \tag{7}
\]

The optimal capacity investments specified in Lemma 1 follow directly from the concavity of \(\pi_L\) and \(\pi_R\). The proof is omitted in the interest of brevity. Note that \(\pi_L(\bar{q}) = \pi_R(\bar{q})\), i.e., the profit functions in the two regions coincide at \(\kappa = \bar{q}\).

Given AMC contract terms \((\delta_r, \bar{q})\), the Manufacturer will decide whether to pick a solution in either the left or the right region, depending on which of these two choices maximizes its expected profit. We define \(\kappa^*(\delta_r, \bar{q}) = \arg\max_{\kappa^*} \{\pi_L(\kappa_L^*), \pi_R(\kappa_R^*)\}\). The optimal capacity investment will be either \(\bar{q}\), or \(g(\delta_r)\), depending on \((\delta_r, \bar{q})\).
The Principal’s Decision Problem

Next, we solve the Principal’s problem of choosing the optimal contract parameters to maximize the Manufacturer’s capacity investment, subject to the budget constraint. Theorem 1 presents some key properties of the optimal contract parameters that the Principal will pick in equilibrium.

**Theorem 1.** The Principal will choose the AMC contract parameters such that \( \kappa^* = \bar{q} \) and \( \delta_r \bar{q} = b \), with \( 0 \leq \bar{q} \leq g(\delta_r) \) and the expected number of beneficiaries will equal \( E[\min(D, \bar{q})] \).

The proof of Theorem 1 is presented in Appendix A. From the theorem, we see that in equilibrium, the manufacturer’s installed capacity exactly matches the top-up quantity cap offered by the Principal. An important implication of this result is that AMCs, if designed in the right away, can be a powerful tool to induce manufacturers to install the desired production capacity necessary to meet developing country needs.

Notice from Theorem 1 that in equilibrium, the budget constraint is tight. Hence, efforts to increase \( \bar{q} \) will result in a lower \( \delta_r \), which may, in turn, lead to a lower \( \bar{q} \) through the constraint \( 0 \leq \bar{q} \leq g(\delta_r) \). These arguments enable us to offer a more precise characterization of the Principal’s optimal contract, as reported in Corollary 1.

**Corollary 1.** The Principal’s optimal contract is given by the solution to the following optimization problem.

\[
\max_{\delta_r} g(\delta_r) \quad \text{subject to: } \delta_r g(\delta_r) = b
\]

In choosing an objective function in Corollary 1, we have used the fact that a maximizer of \( g(\delta_r) \) also maximizes \( E[\min(D, g(\delta_r))] \). Having analyzed Model 1, we now turn our attention to Model 2 where the Manufacturer makes the production quantity decision before observing demand.

### 3.2. Model 2: M Produces Before Observing Demand

For a fixed \( \kappa \), let \( \pi_M(y) \) denote the Manufacturer’s expected profit when it chooses an aspirational production level of \( y \). The aspirational level ignores the capacity constraint. M’s expected profit function can be written as follows:

\[
\pi_M(y) = \begin{cases} 
(\delta_r + r)E[\min(y, D)] - cy & \text{if } y \leq \bar{q}, \\
\delta_r E[\min(\bar{q}, D)] + (r)E[\min(y, D)] - cy & \text{otherwise}.
\end{cases}
\]

In (10), the two cases arise because the top-up is earned over the minimum of \( y, \bar{q} \) and \( D \). Define critical numbers \( y^*_L(\delta_r) \) and \( y^*_R \) such that

\[
y^*_L(\delta_r) = F^{-1} \left( \frac{c}{\delta_r + r} \right), \quad \text{and,}
\]

\[
y^*_R = F^{-1} \left( \frac{c}{r} \right)
\]

(11) and (12)
It is easy to verify from (11) and (12) that \( y_R^* \leq y_L^*(\delta_r) \). To identify M’s optimal production quantity, we need to consider the two regions of \( y \) namely \( y \leq \bar{q} \) and \( y \geq \bar{q} \). The Manufacturer’s profit function is concave in \( y \) in both regions, which leads to the following result regarding M’s optimal production policy (see Appendix B for a formal proof).

**Theorem 2.** The Manufacturer’s optimal production policy depends on \( \bar{q} \) and critical numbers \( y_L^*(\delta_r) \) and \( y_R^* \) as follows:

\[
p^*(\kappa) = \begin{cases} 
\min(y_R^*, \kappa), & \bar{q} \leq y_R^*, \\
\min(\bar{q}, \kappa), & \bar{q} \leq y_R^* \leq y_L^*(\delta_r), \\
\min(y_L^*(\delta_r), \kappa), & \text{otherwise}
\end{cases}
\]

**The Principal’s Decision Problem**

In (13), we observe that the optimal production quantity is different in three regions characterized by the value of \( \bar{q} \). Moreover, \( y_L^* \) depends on \( \delta_r \). Therefore, to determine optimal contract parameters from the Principal’s perspective, we adopt the following strategy. First, we fix \( \delta_r \) (i.e. \( y_L^* \)) and calculate the impact of \( \bar{q} \) on M’s capacity investments. Then, we consider the simultaneous choice of \( \delta_r \) and \( \bar{q} \), which are linked through the budget constraint (1). Finally, we obtain the Principal’s optimal strategy as indicated in Theorem 3. It utilizes a quantity \( \kappa_0 \) that represents the desired capacity investment from the Principal’s perspective for each \( \delta_r \).

\[
\kappa_0 = F^{-1}\left(\frac{c + c_\kappa}{\delta_r + r}\right).
\]

**Theorem 3.** The Principal will choose AMC contract parameters that maximize \( \kappa_0 \), subject to the constraint that \( \kappa_0 \delta_r = b \).

Note the similarity between Theorem 3 and Corollary 1 that follows Theorem 1. A proof of Theorem 3 is presented in Appendix C. The values of the problem primitives will affect the contract terms that the Principal will offer and the capacity and production quantity choices that the Manufacturer will make. We study these effects next.

**Corollary 2.** Cost increases (i.e. higher \( c \) and \( c_\kappa \)) serve to decrease \( \kappa_0 \), which will be partially counterbalanced by a higher \( \delta_r \). Specifically, we can prove that \( \delta_r \) increases as \( c \) increases, but the capacity investment (\( \kappa_0 \)) decreases.

The proof of Corollary 2 is presented in Appendix D. Intuitively, it makes sense that optimal top-up amount increases as the Manufacturer’s costs increase.
3.3. Variants of the AMC contract

In this section, we look at two contract variations that are motivated by potential AMC design proposals discussed in practice (see Levine et al. 2005 and GAVI 2008). In the first variant, the Principal requires the Manufacturer to invest in a minimum amount of capacity, while in the second, the Principal guarantees that it will purchase a certain number of doses if demand is lower than that amount. Each of these contracts increases the Principal’s power by allowing it to select a third parameter, with the result that it can control the Manufacturer’s decisions more precisely. This can erode the Manufacturer’s profit. Therefore, to counterbalance the additional power given to the Principal, we introduce another parameter for the Manufacturer as well – its reservation profit $\bar{\gamma}$. The Principal knows $\bar{\gamma}$ and must choose contract parameters such that the Manufacturer’s expected profit is at least $\bar{\gamma}$, or else the Manufacturer will not participate. We treat the Manufacturer’s expected profit in the original contract (with no purchase guarantee or minimum capacity requirements) as an upper bound on $\bar{\gamma}$ and consider each of the two contracts in a separate section.

3.3.1. Minimum-Capacity Contract

Suppose that the Principal offers a contract in which the Manufacturer earns top-up only if it invests in at least $\hat{\kappa}$ units of capacity. Then, the Principal’s contract parameters are $(\delta_r, \bar{q}, \hat{\kappa})$. In the original contract, the Manufacturer invested in $\kappa^* = g(\delta_r)$. Therefore, if $\hat{\kappa} \leq g(\delta_r)$, then the Manufacturer will continue investing in $g(\delta_r)$ and the addition of $\hat{\kappa}$ will have no impact. Hence, we only look at cases in which $\hat{\kappa} \geq g(\delta_r)$.

When the Principal specifies $\hat{\kappa} \geq g(\delta_r)$, the Manufacturer either invests in $\hat{\kappa}$ or does not participate. This is the case because the Manufacturer’s profit decreases in $\kappa$ for $\kappa > g(\delta_r)$. Hence, the manufacturer will install exactly $\hat{\kappa}$ units of capacity as long as $\pi(\hat{\kappa}) \geq \bar{\gamma}$. In addition to choosing $\hat{\kappa}$, the Principal also needs to decide whether to set $\bar{q}$ less than or greater than $\hat{\kappa}$. Suppose P selects $\bar{q} > \hat{\kappa}$. Because M’s expected profit is decreasing in $\kappa$ for $\kappa > g(\delta_r)$, M’s capacity investment will not exceed $\hat{\kappa}$, implying that the top-ups offered over the $(\bar{q} - \hat{\kappa})$ doses will not be utilized. Therefore, it is not optimal for the Principal to set $\bar{q} > \hat{\kappa}$. Then, the only reasonable choice for the Principal is to set $\bar{q} \leq \hat{\kappa}$, and the Manufacturer responds by installing $\hat{\kappa}$ units of capacity, as long as $\pi(\hat{\kappa}) \geq \bar{\gamma}$.

Combining the above arguments, we have the following corollary regarding the optimal contract parameters when the Principal’s contract includes a minimum-capacity requirement.

**Corollary 3.** Under a minimum capacity requirement, the optimal contract parameters $(\delta^*_r, \bar{q}^*, \hat{\kappa}^*)$ are obtained by solving the following optimization problem:

\[
\max_{\delta_r, \bar{q}, \hat{\kappa}} \hat{\kappa} \quad \text{subject to:} \quad \delta_r \bar{q} = b
\]
\[ \bar{q} \leq \hat{\kappa} \quad (17) \]
\[ \hat{\kappa} \geq g(\delta_r) \quad (18) \]
\[ \pi(\hat{\kappa}) \geq \bar{\gamma} \quad (19) \]

Because the choice of \( \hat{\kappa} \) is constrained by the reservation profit constraint (19), the optimal contract parameters \( (\delta^*_r, \bar{q}^*, \hat{\kappa}^*) \) will be such that the expected profit of the Manufacturer is exactly \( \bar{\gamma} \). From our earlier analysis, we know that \( \pi(g(\delta^*_r)) \) is the expected profit that the Manufacturer can earn in the contract with no minimum capacity requirements. Therefore, for any \( 0 \leq \bar{\gamma} \leq \pi(g(\delta^*_r)) \), a solution such that \( \hat{\kappa} \geq g(\delta^*_r) \) must be feasible since M’s expected profit is unimodal. In section 3.3.3, we numerically compare the performance of the different contracts namely the minimum–capacity contract, the purchase–guarantee contract and a contract with no capacity or purchase volume stipulations. Before doing a comparative analysis of the different contracts, we focus on the purchase–guarantee contract in the next section.

### 3.3.2. Purchase-Guarantee Contract

Suppose the Principal guarantees that it will purchase either \( \hat{D} \) or \( D \) (i.e., realized demand) doses from the Manufacturer, whichever is greater. \( \hat{D} \) is a contract parameter selected by the Principal. It ensures that the Manufacturer faces a minimum demand of \( \hat{D} \), thus limiting its potential losses. The purchase-guarantee contract is then characterized by parameters \( (\delta_r, \bar{q}, \hat{D}) \). The Principal can set \( \bar{q} \) to be less than or greater than \( \hat{D} \). Suppose that \( \bar{q} < \hat{D} \). Then, the Manufacturer will be able to earn top-up on at most \( \bar{q} \) units. Despite a guarantee to sell \( \hat{D} \) units, the Manufacturer will have no incentive to invest in capacity greater than \( \bar{q} \) since \( r - c - c_\kappa < 0 \), i.e., the cost of investing in additional capacity is not recovered if no top-up is earned on those units. Therefore, the Manufacturer will choose \( \kappa^* = \min\{\bar{q}, g(\delta_r)\} \) and providing a purchase guarantee does not lead to an increase in the Manufacturer’s capacity investments. Hence, when offering a purchase guarantee contract, the Principal should set \( \bar{q} \geq \hat{D} \).

In addition to choosing \( \bar{q} \), the Principal also needs to pick a value of \( \hat{D} \), which could either be less than or greater than \( g(\delta_r) \).

- When \( \hat{D} < g(\delta_r) \), the Manufacturer will invest in \( g(\delta_r) \) (provided \( \bar{q} \geq g(\delta_r) \)) because that is the optimal capacity investment of the contract without a purchase guarantee.
- When \( \hat{D} \geq g(\delta_r) \), the Manufacturer will invest in \( \hat{D} \) because it will receive the top-up on \( \hat{D} \) units. However, the Manufacturer will not invest in more than \( \hat{D} \). This is due to the fact that M’s expected profit is unimodal and decreasing in \( \kappa \) for \( \kappa > g(\delta_r) \).

Based on the above discussion, we see that the Principal would prefer \( \hat{D} \geq g(\delta_r) \) since this results in a higher capacity investment. Additionally, the Principal will set \( \bar{q} = \hat{D} \) since any top–up in excess of \( \hat{D} \) will not be utilized (the Manufacturer will not invest in more than \( \hat{D} \)). Given these
contract parameters, the Manufacturer will install \( \hat{D} \) units of capacity as long as \( \pi(\hat{D}) \geq \bar{\gamma} \). Notice that in this setting, the profit of the Manufacturer is no longer random, and it is given by

\[
\pi(\hat{D}) = (r + \delta_r - c - c_\kappa) \hat{D}.
\] (20)

The Principal’s objective is to maximize the Manufacturer’s capacity investment subject to the reservation profit constraint \( \pi(\hat{D}) \geq \bar{\gamma} \), which in this case translates to the condition \( \delta_r \geq \frac{\hat{\gamma}}{\hat{D}} + c_\kappa + c - r \). Under the purchase–guarantee contract, the Principal’s budget constraint is given by \( \delta_r \hat{D} + r E[(\hat{D} - D)^+] \leq b \). Combining the above statements, we have the following corollary regarding the optimal contract parameters when the Principal’s contract includes a purchase guarantee.

**Corollary 4.** Under a purchase-guarantee agreement, the optimal contract parameters \((\delta^*_r, \bar{q}^*, \hat{D}^*)\) are obtained by solving the following optimization problem:

\[
\max_{\delta_r, \hat{D}} \hat{D}
\]

subject to:

\[
\delta_r \hat{D} + r E[(\hat{D} - D)^+] \leq b
\] (22)

\[
\hat{D} \geq g(\delta_r)
\] (23)

\[
\delta_r \geq \frac{\bar{\gamma}}{\hat{D}} + c_\kappa + c - r
\] (24)

The optimal decision for the Principal is to set \( \delta_r \) as low as possible, and \( \hat{D} \) as high as possible. Because M’s expected profit is unimodal, a solution such that \( \hat{D} \geq g(\delta_r) \) must be feasible in this setting.

**3.3.3. Efficiency Comparison** Under purchase guarantee and the minimum–capacity contract, the Principal includes an additional parameter, either \( \hat{D} \) or \( \hat{\kappa} \), in the original contract, and hence it can induce the Manufacturer to increase its capacity investment, thus satisfying a larger demand. While serving more people is certainly important, humanitarian organizations are also often interested in understanding how the different contracts vary in terms of their efficiency. Consistent with practice, we define contract efficiency as the cost to the Principal of each unit of demand satisfied, i.e., \( C_{P,j} = \frac{\text{Cost to P}}{\text{demand satisfied}} \) where \( j \) refers to the \( j \)-th contract. We use the convention that \( j = 1 \) represents the original contract (i.e., no stipulations related to minimum capacity and no purchase guarantee), \( j = 2 \) corresponds to the minimum-capacity contract, and \( j = 3 \) represents the purchase-guarantee contract. We calculate and compare \( C_{P,j} \) for each \( j \).

- **Original Contract** \((j = 1)\): the Principal pays \( \delta^*_{r,1} \) per unit supplied. Therefore, in this case \( C_{P,1} = \delta^*_{r,1} \).
- **Minimum Capacity Requirement** \((j = 2)\): Here \( C_{P,2} = \frac{\delta^*_{r,2} E[\min\{\hat{\kappa}^*_{z}, D\}]}{E[\min\{\hat{\kappa}^*_{z}, D\}]} \).
- **Purchase Guarantee** \((j = 3)\): Here, \( C_{P,3} = \frac{\delta^*_{r,3} \hat{D}^* + r E[(\hat{D}^* - D)^+]}{E[\min\{\hat{D}^*, D\}]} \).
Note that the optimal contract parameter values may be different in the three contracts. To compare the capacity investment and cost per dose under the different contracts, we conduct some numerical experiments. The parametric values used in the numerical study are reported in Table 2, and we selected these values based on Snyder et al. (2011).

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Parameter values used in the numerical study</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D$</td>
<td>$\sim$ Lognormal(204.41) (in millions)</td>
</tr>
<tr>
<td>$b$</td>
<td>$=$ $1500$ (in millions)</td>
</tr>
<tr>
<td>$r$</td>
<td>$=$ $3.5$</td>
</tr>
<tr>
<td>$c$</td>
<td>$=$ $3.5$</td>
</tr>
<tr>
<td>$c_\kappa$</td>
<td>$=$ $4.0$</td>
</tr>
</tbody>
</table>

We first solve the original contract and find that $\delta^*_1 = 7.59$, $g(\delta^*_1) = 197.64$, $\pi(g(\delta^*_1)) = 611.82$, and $C_{P.1} = 7.59$. For the other two contracts, we calculate the per-dose cost and the optimal capacity investment for several values of the ratio $\bar{\gamma}/$611.82 ranging from 0 to 1 in increments of 0.1. The results are plotted in Figure 2. In the figure, the plot on the left shows the per-dose cost under the different contracts for different values of the reservation profit, while the plot on the right shows the optimal capacity investments under the different contracts. Notice that the reservation profit is not a parameter for the original contract and therefore, we plot the cost per dose and capacity investment only at $\bar{\gamma}/$611.82 = 1 in case of the original contract.

![Cost per dose supplied and optimal capacity investment vs reservation profit](image)

From Figure 2, we see that the Principal’s cost per dose is smaller and capacity investment is higher under the minimum-capacity contract when compared to the purchase-guarantee contract. In the purchase-guarantee contract, the Principal takes all of the risk, while the Manufacturer faces no demand uncertainty. The cost per dose in this case is higher than the the minimum-capacity
contract because when demand is low, the Principal purchases $\hat{D}$ units, but the effective number of doses used to satisfy demand, which is $E[\min\{\hat{D}, D\}]$, is less than $\hat{D}$. Moreover, because of the additional cost $rE[(\hat{D} - D)^+]$ in the budget constraint, the Principal may have to offer either a lower top-up amount or a lower quantity cap or both, and this could discourage the Manufacturer from investing in high capacity. As a result, the capacity investment under a purchase-guarantee contract is lower when compared to the minimum-capacity contract. In light of this discussion, we see that the minimum-capacity contract might be more beneficial to the Principal than a contract that involves purchase guarantees.

The comparison between the original contract and the minimum-capacity contract is more nuanced. For reservation profit up to $\approx 90\%$ of $\pi(g(\delta^*_r))$, i.e., when the Manufacturer’s reservation profit is smaller than approximately $\$550$, the Principal would clearly prefer the minimum-capacity contract over the original contract since it induces a higher capacity investment and has a lower cost per dose. However, when the Manufacturer’s reservation price is more than $\$550$, offering a minimum-capacity contract leads to higher capacity investment ($\kappa^*_2 = 221.99 > 197.64 = \kappa^*_1$) but it comes at the expense of a higher per-dose cost to the Principal. In this case, the Principal might prefer one contract over the other, depending on the relative importance of inducing higher capacity investment vs. choosing a more efficient contract.

The results discussed in this section offer some key insights that could be valuable to humanitarian organizations exploring the use of AMC contracts. In particular, there are ongoing discussions within the global health community regarding minimum-capacity requirements and purchase guarantees within the context of AMCs. The idea of requiring manufacturers to install a certain minimum level of capacity has received broad support from humanitarian health organizations since it has the potential to ensure sustainable long-term supply (Cernuschi et al. 2011, GAVI 2008). However, there has been push back against purchase guarantees since they are generally perceived to be interfering with the markets dynamics inside the recipient country (see e.g., Levine et al. 2005). Furthermore, as discussed earlier, purchase guarantees are less effective at inducing capacity investments and they have higher per-dose costs compared to minimum-capacity contracts. These results and the above-mentioned practical considerations suggest that adding minimum-capacity requirements to AMC contracts may be a better alternative to offering purchase guarantees to manufacturers. In addition, the minimum–capacity requirements could significantly enhance the effectiveness of AMC contracts, as long as the manufacturer’s reservation profit is not too high. Having analyzed the single–manufacturer setting, we now turn our attention to a two–manufacturer scenario.
4. Two Manufacturers Models

Suppose there are two profit-maximizing manufacturers ($M_1$ and $M_2$) who have late-stage products that meet all functional requirements. Since our objective is to use our models to provide insights to the Principal regarding contract design, we assume that the two manufacturers have identical production and investment costs ($c$ and $c_\kappa$) and earn the same per–unit revenue ($r$). They face demand $D$ from the same recipient country – assumed to be a single country for simpler exposition. Mirroring what would normally happen in practice, we assume that the Principal first observes demand (receives orders from the recipient country), and then allocates this demand between the two manufacturers. When the demand from the recipient country exceeds the combined capacity investment of the two manufacturers, i.e., $D \geq \kappa_1 + \kappa_2$, the demand allocation process is straightforward. In this case, each manufacturer receives a portion of the demand equal to its capacity. This is the optimal allocation from the perspective of the principal as well as both manufacturers.

When $D < \kappa_1 + \kappa_2$, the question of how to split the demand between the two manufacturers is more nuanced and we consider the following three demand–allocation models labeled ‘Model A’, ‘Model B’ and ‘Model C’ respectively.

- **Model A**: Demand allocation to Manufacturer $i$ ($i=1,2$) is proportional to its capacity investment, i.e., $D_A^i = \frac{\kappa_A^i}{\kappa_1^i + \kappa_2^i} D$.

- **Model B**: Under this model, one of the manufacturers (indexed by $i$) receives priority for demand allocation. Specifically, demand allocated to $M_i$ equals $\min\{D, \kappa_i\}$. Manufacturer $M_{3-i}$ is allocated the left-over demand, if any. Effectively, $D_{3-i}^B = (D - \kappa_i^B)^+$.

- **Model C**: Each Manufacturer has probability $p$ of receiving priority for demand allocation. In our analysis, we focus on the case of $p=1/2$, i.e., each manufacturer is equally likely to be given priority for demand allocation.

Demand allocation models A and B are motivated by potential strategies discussed within the humanitarian health community to deal with competition involving multiple manufacturers in the context of AMCs (see Levine et al. 2005 and Cernuschi et al. 2011). Model C is more appropriate for settings where the two manufacturers have late–stage products but the products may not have undergone final testing and regulatory approval. Hence, while both of them are likely to meet the Principal’s functional requirements, the product characteristics may not be publicly available in their entirety. In the absence of full information regarding the competitor’s product, it is reasonable for each manufacturer to assign a probability that the Principal would prioritize their product for demand allocation.

The sequence of events in the two–manufacturers setting is as follows. The Principal moves first, choosing contract parameters ($s, \delta, \bar{q}_1, \bar{q}_2$), where $s \in \{A, B, C\}$ refers to demand–allocation model under consideration. Consistent with practice, we assume that $M_1$ and $M_2$ receive the same
Manufacturers invest in capacities $\kappa_1$ and $\kappa_2$

Demand $D$ materializes

Principal allocates $D$ (3 models)

Manufacturers produce $p_1^*$ and $p_2^*$

**Demand splitting models**
- Model A: Proportional to $\kappa_i$
- Model B: Strict Priority to one of the manufacturers
- Model C: Equiprobable high priority to one of the manufacturers

Vaccine is supplied, manufacturers realize profits

Figure 3  Sequence of events with two manufacturers.

After observing the contract, the manufacturers make their capacity investment decisions simultaneously. The capacity investment decisions are made before demand $D$ is realized, i.e., at the time of capacity investments, the manufacturers do not know the actual demand but the underlying demand distribution and the Principal’s demand–allocation rule ($s = A, B$ or $C$) are common knowledge.

After the capacity investments are made, demand materializes, demand allocations are made by the Principal, and finally, the manufacturers make their production quantity decisions. The sequence of events explained above is illustrated in Figure 3. Since the manufacturer makes the production quantity decision after observing demand, it is straightforward to see that Manufacturer $i$’s optimal production quantity will be

$$p_i^* = \min\{\kappa_i^*, d_i^*\}, \quad \forall i = 1, 2, \forall s \in \{A, B, C\}. \tag{25}$$

We use expression (25) to write the manufacturers’ profit functions and then work backwards to identify their optimal capacity decisions, and the Principal’s choice of contract parameters.
4.1. Model A: Demand Allocation Proportional To Capacity Investment

In this model, when \( D \geq \kappa_1 + \kappa_2 \), the demand allocated to Manufacturer \( i \) is \( D_i^A = \kappa_i^A \). When \( D \leq \kappa_1 + \kappa_2 \), the Principal will split the demand in the following way:

\[
D_i^A = \left( \frac{\kappa_i^A}{\kappa_i^A + \kappa^A_{3-i}} \right) D, \quad \forall i = 1, 2
\]

Then, we can write \( M_i \)’s expected profit as

\[
\pi_i^A(\kappa_i^A, \kappa^A_{3-i}) = -c_\kappa \kappa_i^A + (r - c) \left[ \kappa_i^A F(\kappa_i^A + \kappa^A_{3-i}) + \frac{\kappa_i^A}{\kappa_i^A + \kappa^A_{3-i}} \int_{0}^{\kappa_i^A + \kappa^A_{3-i}} x dF(x) \right]
\]

\[
+ \delta_i^A \left[ \min(\kappa_i^A, \bar{q}_i^A) \bar{F}(\kappa_i^A + \kappa^A_{3-i}) + \int_{0}^{\kappa_i^A + \kappa^A_{3-i}} \min \left( \frac{\kappa_i^A}{\kappa_i^A + \kappa^A_{3-i}} x, \bar{q}_i^A \right) dF(x) \right]
\]

The first term in (27) is the capacity investment cost, the second term represents \( M_i \)'s base revenue, and the third term is the revenue earned from top-ups. To identify \( M_i \)'s optimal capacity investment, we analyze the following two regions.

**Left Region** \( (\kappa_i^A \leq \bar{q}_i^A) \): In this region, \( M_i \) solves \( \max_{0 \leq \kappa_i^A \leq \bar{q}_i^A} [\pi_i^A(\kappa_i^A, \kappa^A_{3-i})] \). For a fixed \( \kappa^A_{3-i} \), the solution to the above optimization problem yields \( M_i \)'s best response correspondence in the region \( \kappa_i^A \leq \bar{q}_i^A \), and is obtained by taking the minimum of \( \bar{q}_i^A \) and the solution to the following implicit equation:

\[
-c_\kappa + (r - c + \delta_i^A) \left[ \bar{F}(\kappa_i^A + \kappa^A_{3-i}) + \frac{\kappa^A_{3-i}}{(\kappa_i^A + \kappa^A_{3-i})^2} \int_{0}^{\kappa_i^A + \kappa^A_{3-i}} x dF(x) \right] = 0
\]

**Right Region** \( (\kappa_i^A \geq \bar{q}_i^A) \): In this region, \( M_i \) solves \( \max_{\kappa_i^A \geq \bar{q}_i^A} [\pi_i^A(\kappa_i^A, \kappa^A_{3-i})] \). The best response correspondence in this region is obtained by taking the maximum of \( \bar{q}_i^A \) and the solution to the following implicit equation:

\[
-c_\kappa + (r - c) \left[ \bar{F}(\kappa_i^A + \kappa^A_{3-i}) + \frac{\kappa^A_{3-i}}{(\kappa_i^A + \kappa^A_{3-i})^2} \int_{0}^{\kappa_i^A + \kappa^A_{3-i}} x dF(x) \right] +
\]

\[
\delta_i^A \frac{\kappa^A_{3-i}}{(\kappa_i^A + \kappa^A_{3-i})^2} \int_{0}^{\bar{q}_i^A} \bar{F}(\kappa_i^A + \kappa^A_{3-i}) x dF(x) = 0
\]

Because the manufacturers are identical, their best-response correspondences will be the same. Therefore, we can find the equilibrium capacity investment for each manufacturer by setting \( \kappa_i^A(\kappa^A_{3-i}) = \kappa^A_{3-i}(\kappa_i^A) \). Let \( \kappa^A_{L} \) and \( \kappa^A_{R} \) be the solution to equations (28) and (29) respectively with \( \kappa_i^A(\kappa^A_{3-i}) \) set equal to \( \kappa^A_{3-i}(\kappa_i^A) \), i.e., \( \kappa^A_{L} \) and \( \kappa^A_{R} \) are obtained by solving the following equations:

\[
-c_\kappa + (r - c + \delta_i^A) \left[ \bar{F}(2\kappa^A_{L}) + \frac{1}{4\kappa^A_{L}} \int_{x=0}^{2\kappa^A_{L}} x dF(x) \right] = 0
\]
and

\[-c_\kappa + (r - c) \left( \hat{F}(2\kappa^*_A) + \frac{1}{4\kappa^*_R} \int_{x=0}^{2\kappa^*_A} x dF(x) \right) + \delta^*_A \frac{1}{4\kappa^*_R} \int_{x=0}^{2\kappa^*_i} x dF(x) = 0 \]  

(31)

Using equations (30) and (31), we identify the equilibrium capacity investment by Manufacturer \( i \). The results are presented in Lemma 2.

**Lemma 2.** Under Model A, the optimal capacity investment of \( M_i \) is

\[ \kappa^*_i = \begin{cases} 
\kappa^*_R, & \text{if } \bar{q}^i \leq \min\{\kappa^*_L, \kappa^*_R\} \\
\bar{q}^i, & \text{if } \min\{\kappa^*_L, \kappa^*_R\} < \bar{q}^i \leq \max\{\kappa^*_L, \kappa^*_R\} \\
\kappa^*_L, & \text{if } \bar{q}^i > \max\{\kappa^*_L, \kappa^*_R\}
\end{cases} \]  

(32)

A sketch of the proof of Lemma 2 is provided in Appendix E.

**The Principal’s Decision Problem**

Under Model A, the Principal’s objective is to choose the contract parameters \((\delta^*_A, \bar{q}^1_A, \bar{q}^2_A)\) so as to maximize the number of people receiving the health product. Since the manufacturers are symmetric in terms of their costs and the Principal has no ex-ante preference for either manufacturer’s product, it is (intuitively) optimal for the Principal to offer the same contract terms to each manufacturer (i.e., \( \bar{q}^1_A = \bar{q}^2_A = \bar{q}^1 \)). The following theorem presents some key properties of the contract parameters that the Principal will choose in equilibrium.

**Theorem 4.** The Principal will choose the AMC contract parameters such that \( \kappa^*_i = \kappa^*_R \) with \( 0 \leq \bar{q}^1 \leq \kappa^*_R(\delta^*_r) \) and satisfying \( 2\delta^*_r \bar{q}^1 = b \).

The proof of Theorem 4 is provided in Appendix F. Using the same arguments that we presented following Theorem 1, we have the following result concerning the Principal’s optimal contract.

**Corollary 5.** Under Model A, the Principal’s optimal contract is given by the solution to the following optimization problem.

\[
\max_{\delta^*_r} 2\kappa^*_L(\delta^*_r) \quad \text{subject to: } 2\delta^*_r \kappa^*_L(\delta^*_r) = b
\]

(33) (34)

**4.2. Model B: Strict Priority to One Manufacturer**

Suppose that both manufacturers have a product that meets functional requirements, but \( M_i \)’s product is deemed superior to \( M_{3-i} \)’s product by the principal based on some criterion (e.g., efficacy). Then, the Principal might prioritize \( M_i \) by allocating as much demand as possible (specifically, \( \min\{D, \kappa^B_i\} \)) and offering the leftover demand, if any, to \( M_{3-i} \). Without loss of generality, we assume that the Principal gives priority to \( M_1 \) for demand allocation. Then, under this model,

\[
\begin{align*}
D^B_1 &= \min\{D, \kappa^B_i\} \\
D^B_2 &= (D - \kappa^B_i)^+
\end{align*}
\]  

(35)
where $D^B_i$ represents the demand allocated to Manufacturer $i$. Under Model B, $M_1$’s profit is independent of the capacity investment of $M_2$, but $M_2$’s profit depends on $\kappa_1^B$ since higher $\kappa_1^B$ will result in a lower demand for $M_2$. Given this difference, we consider each $M_i$’s capacity investment decision separately. First, we look at the profit function of $M_1$, which is given below.

\[
\pi_1^B(\kappa_1^B) = -c_n\kappa_1^B + (r - c) \left[ \kappa_1^B F(\kappa_1^B + \kappa_2^B) + (r - c) \int_{x = 0}^{\kappa_1^B + \kappa_2^B} xdF(x) \right] + \delta_r \min(\kappa_1^B, \bar{q}_1^B) F(\kappa_1^B + \kappa_2^B) + \delta_r \int_{x = 0}^{\kappa_1^B + \kappa_2^B} \min(\kappa_1^B, x, \bar{q}_1^B) dF(x).
\]

(36)

We consider the following two regions in order to find $M_1$’s optimal capacity investment.

**Left Region** ($\kappa_1^B \leq \bar{q}_1^B$): In this region, $M_1$ maximizes its expected profit subject to the constraint that $0 \leq \kappa_1^B \leq \bar{q}_1^B$. Let $\kappa_{1,L}(\delta_r^B)$ be defined as follows.

\[
\kappa_{1,L}(\delta_r^B) = F^{-1} \left( \frac{c_n}{r - c + \delta_r^B} \right).
\]

(37)

Then, the optimal solution to $M_1$’s problem in the left region is $\min\{\bar{q}_1^B, \kappa_{1,L}(\delta_r^B)\}$. Notice that $\kappa_{1,L}(\delta_r^B)$ is identical to $g(\delta_r^B)$ that we defined earlier in the single manufacturer setting.

**Right Region** ($\kappa_1^B \geq \bar{q}_1^B$): The solution to $M_1$’s problem in the right region is $\max\{\bar{q}_1^B, \kappa_{1,R}^B\}$, where

\[
\kappa_{1,R} = F^{-1} \left( \frac{c_n}{r - c} \right).
\]

(38)

Note that $\kappa_{1,R}^B = 0$ under our model assumption $r - c < c_n$. Next, we look at $M_2$’s decision. Its expected profit depends on $\kappa_1^B$ and is given by

\[
\pi_2^B(\kappa_1^B, \kappa_2^B) = -c_n\kappa_2^B + (r - c) \left[ \kappa_2^B F(\kappa_1^B + \kappa_2^B) + (r - c) \int_{x = \kappa_1^B}^{\kappa_1^B + \kappa_2^B} (x - \kappa_1^B) dF(x) \right] + \delta_r \min(\kappa_2^B, \bar{q}_2^B) F(\kappa_1^B + \kappa_2^B) + \delta_r \int_{x = \kappa_1^B}^{\kappa_1^B + \kappa_2^B} \min(x - \kappa_1^B, \bar{q}_2^B) dF(x).
\]

(39)

Similar to our analysis for $M_1$, we consider the following two regions of $\kappa_2^B$ to identify $M_2$’s optimal capacity investment.

**Left Region** ($\kappa_2^B \leq \bar{q}_2^B$): In this region, $M_2$ maximizes its expected profit subject to the constraint that $0 \leq \kappa_2^B \leq \bar{q}_2^B$. Then, $M_2$’s best response correspondence in the left region is given by $\kappa_{2,L}(\kappa_1^B) = \min\{\bar{q}_2^B, \kappa_{1,L}(\delta_r^B) - \kappa_1^B\}$.

**Right Region** ($\kappa_2^B \geq \bar{q}_2^B$): In this region, $M_2$’s best response correspondence is given by $\kappa_{2,R}(\kappa_1^B) = \max\{\bar{q}_2^B, \kappa_{1,R} - \kappa_1^B\} = \bar{q}_2^B$ since $\kappa_{1,R}^B = 0$.

Based on the above analysis, we can derive the manufacturers’ equilibrium capacity investments. The results are presented in Lemma 3.
Lemma 3. Under Model B, the optimal capacity investment of $M_1$ is
\[ \kappa_1^* = \begin{cases} q_1^B, & \text{if } 0 \leq q_1^B \leq \kappa_{1L}^B \\ \kappa_{1L}^B, & \text{otherwise} \end{cases} \] (40)

while the optimal capacity investment of $M_2$ is
\[ \kappa_2^* = \begin{cases} q_2^B, & \text{if } 0 \leq q_2^B \leq \kappa_{1L}^B - \kappa_1^* \\ \kappa_{1L}^B - \kappa_1^*, & \text{otherwise} \end{cases} \] (41)

The proof of Lemma 3 is similar to that of Lemma 2. Under Model B, Manufacturer 1 receives priority for demand allocation and as a result, $M_1$’s profits are not impacted by how much capacity $M_2$ installs. Hence, we would intuitively expect $M_1$’s capacity investment to be the same as in the single manufacturer case and Lemma 3 confirms that it is indeed the case. For Manufacturer 2, the lemma offers a precise characterization of how $M_2$’s decisions are impacted by $M_1$’s capacity investments. From the lemma, we see that $M_2$’s capacity investment is invariant to $\kappa_1^*$ when the top–up quantity cap is less than a certain threshold but decreases linearly in $M_1$’s investment, otherwise. Thus, efforts to boost the capacity investment of $M_1$ could be countered by an equivalent reduction in $M_2$’s capacity investment. This has important implications for the Principal’s contract design problem, which we discuss next.

The Principal’s Decision Problem

Under Model B, the preferential treatment received by $M_1$ during demand allocation leads to noticeable differences in the capacity investment strategies of $M_1$ and $M_2$. Since the two manufacturers react differently to the Principal’s contract terms, it is reasonable to expect that the Principal would offer different contract terms to the two manufacturers. Specifically, $q_1^B$ may not be necessarily the same as $\bar{q}_2^B$. The following theorem establishes this intuition rigorously, and presents some key properties related to the optimal contract terms offered by the Principal.

Theorem 5. Under Model B, the Principal will choose the AMC contract parameters such that $\kappa_1^B = q_1^B$ with $0 \leq q_1^B \leq \kappa_{1L}^B$ and $\kappa_2^B = q_2^B = \kappa_{1L}^B - \bar{q}_2^B$. Moreover, $\delta_r^B(\bar{q}_1^B + \bar{q}_2^B) = b$ holds.

The proof of Theorem 5 is provided in Appendix F. In the context of late–stage AMCs, the Principal’s objective is to maximize the total capacity investment of the two manufacturers, which in this setting is $\bar{q}_1^B + \kappa_{1L}^B - \bar{q}_1^B = \kappa_{1L}^B$ (see statement of Theorem 5). This leads to the following corollary regarding the Principal’s optimal contract.

Corollary 6. Under Model B, the Principal’s optimal contract is given by the solution to the following optimization problem:
\[ \max_{\bar{q}_1^B} \kappa_{1L}^B(\delta_r^B) \] (42)
\[ \text{Subject to: } \delta_r^B\kappa_{1L}^B(\delta_r^B) = b \] (43)
We point out that the optimal solution to the Principal’s contract design problem may not be unique since the total capacity investment will remain the same (\(= \kappa_{1L}^B(\delta_{r}^B)\)) regardless of the specific value of \(q_1^B\) and \(q_2^B\), as long as they satisfy the conditions of Theorem 5. Notice that the set of optimal solutions includes \(q_1^B = \kappa_{1L}^B(\delta_{r}^B)\) and \(q_2^B = 0\), contract terms that are practically appealing in settings where Manufacturer 1 is given priority with respect to demand allocation.

4.3. Model C: Equiprobable Priority

Under this model, the Principal allocates demand in the following way.

\[
D_i^C = \begin{cases} 
\min\{D, \kappa_i^C\} & \text{with probability 0.5} \\
(D - \kappa_{i-1}^C)^+ & \text{with probability 0.5}
\end{cases}
\]  

(44)

\(M_i\)'s profit function is then given by

\[
\pi_i^C(\kappa_i^C, \kappa_{i-1}^C) = -c_\kappa \kappa_i^C + (r-c)\kappa_i^C \bar{F}(\kappa_i^C + \kappa_{i-1}^C) + \delta_c \min(\kappa_i^C, \bar{q}_i^C) \bar{F}(\kappa_i^C + \kappa_{i-1}^C)
\]

\[
+ \frac{1}{2} \left[ (r - c) \int_{x=0}^{\kappa_i^C + \kappa_{i-1}^C} \min(x, \kappa_i^C) dF(x) + \delta_c \int_{x=0}^{\kappa_i^C + \kappa_{i-1}^C} \min(x, \kappa_i^C, \bar{q}_i^C) dF(x) \right]
\]

\[
+ \frac{1}{2} \left[ (r - c) \int_{x=\kappa_i^C}^{\kappa_i^C + \kappa_{i-1}^C} (x - \kappa_{i-1}^C) dF(x) + \delta_c \int_{x=\kappa_i^C}^{\kappa_i^C + \kappa_{i-1}^C} \min(x - \kappa_{i-1}^C, \bar{q}_i^C) dF(x) \right].
\]

(45)

Similar to our approach in sections 4.1 and 4.2, we consider the following two regions of \(\kappa_i^C\) to identify \(M_i\)'s optimal capacity investment.

**Left Region** (\(\kappa_i^C \leq \bar{q}_i^C\)): In this region, for a fixed \(\kappa_{i-1}^C\), \(M_i\)'s best response correspondence is given by the minimum of \(\bar{q}_i^C\) and the solution to the following implicit equation.

\[
-c_\kappa + (r-c+\delta_c) \left[ 1 - \frac{1}{2} F(\kappa_i^C + \kappa_{i-1}^C) - \frac{1}{2} F(\kappa_i^C) \right] = 0
\]

(46)

**Right Region** (\(\kappa_i^C \geq \bar{q}_i^C\)): In this region, \(M_i\)'s best response correspondence is given by the maximum of \(\bar{q}_i^C\) and the solution to the following implicit equation.

\[
-c_\kappa + (r-c) \left[ 1 - \frac{1}{2} F(2\kappa_i^C + \kappa_{i-1}^C) - \frac{1}{2} F(\kappa_i^C) \right] = 0
\]

(47)

Because the manufacturers are symmetric in terms of their costs and they are equally likely to receive priority for demand allocation, it is intuitively clear that the manufacturers will make the same capacity investments in equilibrium. Let \(\kappa_{lC}^* (\delta_C)\) and \(\kappa_{rC}^*\) be the solutions to equations (46) and (47) respectively with \(\kappa_1^C\) set equal to \(\kappa_2^C\). Then, we must have

\[
-c_\kappa + (r-c+\delta_C) \left[ 1 - \frac{1}{2} F(2\kappa_l^C) - \frac{1}{2} F(\kappa_l^C) \right] = 0
\]

(48)

and

\[
-c_\kappa + (r-c) \left[ 1 - \frac{1}{2} F(2\kappa_r^C) - \frac{1}{2} F(\kappa_r^C) \right] = 0
\]

(49)
Notice that $\kappa^*_{\text{CR}}$ cannot be greater than 0 for (49) to hold, based on our model assumptions. Then, we have the following result regarding the manufacturers’ equilibrium capacity investments.

**Lemma 4.** Under Model C, the optimal capacity investment of $M_i$ is

$$\kappa^*_i = \begin{cases} \bar{q}_i^C, & \text{if } 0 \leq \bar{q}_i^C \leq \kappa^*_L(\delta^C) \\ \kappa^*_L(\delta^C), & \text{otherwise} \end{cases}$$  \hspace{1cm} (50)

The proof of Lemma 4 is similar to that of Lemma 4 and is therefore omitted. Next, we look at the Principal’s decision problem.

**The Principal’s Decision Problem:**
Under Model C, the Principal is equally likely to give priority to either manufacturer and the two manufacturers are symmetric in terms of their costs. Hence, it is (intuitively) optimal for the Principal to offer the same contract terms to each manufacturer (i.e., $\bar{q}_1^C = \bar{q}_2^C = \bar{q}^C$). The following theorem presents key properties of the the contract parameters that the Principal will choose in equilibrium.

**Theorem 6.** Under Model C, the Principal will choose the AMC contract parameters such that

$$\kappa^*_i = \bar{q}_i^C, \text{ with } 0 \leq \bar{q}_i^C \leq \kappa^*_L(\delta^C), \text{ for } i = 1, 2. \text{ Moreover, the contract parameters satisfy the constraint } 2\delta^C \bar{q}_i^C = b.$$  \hspace{1cm} (52)

The proof of Theorem 6 follows similar steps as that of Theorem 4 and is therefore omitted. The optimal contract parameters under Model C are specified in the following corollary.

**Corollary 7.** Under Model C, the Principal’s optimal contract is given by the solution to the following optimization problem:

$$\max_{\delta^C} 2\kappa^*_L(\delta^C)$$  \hspace{1cm} (51)

Subject to: $2\delta^C \kappa^*_L(\delta^C) = b$  \hspace{1cm} (52)

### 5. Comparisons between Single- and Two-Manufacturers Models
In the previous two sections, we analyzed the single– and two–manufacturers settings separately. The purpose of this section is to conduct a comparative analysis of the single– and two–manufacturers settings with the ultimate objective of answering the following question: when there are two manufacturers with late–stage products, is the Principal better off entering into a contract with both manufacturers or would it be better to contract with just one of them?

When there are two manufacturers, we know from section 4 that the manufacturers’ capacity choices depend on the demand allocation rule. Hence, to do a comparative analysis of the single–
and two–manufacturers settings, we need to separately compare the manufacturers’ capacity choices under each demand allocation rule with the capacity investment in the single manufacturer setting. As a first step, we compare Model B (strict priority to Manufacturer 1) in the two–manufacturers setting with the single–manufacturer case, and we have the following result.

**Lemma 5.** Regardless of how the Principal sets $\bar{q}_1$ and $\bar{q}_2$, the total optimal capacity investment in Model B, where $M_1$ is always prioritized, is equal to the optimal capacity investment in the single Manufacturer setting. That is, $\kappa_1^B + \kappa_2^B = g(\delta^*_r)$

This result follows directly from Corollary 6. From Lemma 5, we see that when $M_1$ is prioritized, the combined capacity investment by both manufacturers remains at the same level as the optimal capacity investment in the single manufacturer setting. This is because under strict priority, $M_1$’s capacity investments discourage $M_2$’s investments and as a result, the Principal realizes no benefit from the presence of an additional potential manufacturer. Hence, it may not be in the best interests of the Principal to prioritize one manufacturer’s product over the other, as long as both products meet the minimum functional requirements.

Since Model B leads to the same capacity investment as the single manufacturer case, going forward, we focus only on Models A and C in our comparison of the single– and two–manufacturers settings. In Lemma 6, we show that both these models induce the two manufacturers to install more combined capacity than a single manufacturer for the case of uniformly distributed demand. The proof of Lemma 6 is provided in Appendix H.

**Lemma 6.** If the product demand follows a Uniform distribution, the optimal capacity investments in Models A and C are identical, and they are (at least weakly) larger than the optimal capacity investment of the single Manufacturer. That is, $2\kappa^*_L(\delta^*_r) = 2\kappa^*_L \geq g(\delta^*_r)$.

Analytically comparing Models A and C with the single–manufacturer setting is difficult for a general demand distribution and hence, we resort to a numerical study. Specifically, we consider three demand distributions namely Log-Normal, Triangular and Exponential, and we calculate the optimal capacity investment in Models A and C, and the single-manufacturer model. For all distributions, we vary the mean demand from 50 to 450. In the case of Log-Normal distribution, we let $\sigma = 0.2$, while in the case of the Triangular distribution, we set the lower limit $a = 0$ and the upper limit $b = 500$. All other problem primitives are chosen to be the same as in Table 2. Figure 4 reports the results for all three demand distributions. The mean demand is on the x-axis, while the y-axis reports the optimal capacity investment ratios for Models A (blue line) and C (red line). Capacity investment ratio is defined as the combined capacity of the two manufacturers divided by the optimal capacity investment of a single Manufacturer under identical problem primitives, but model-specific optimal contract parameters.
From Figure 4, we make the following observations — when demand follows either a lognormal or a triangular distribution, the total capacity investment in Model C is greater than the total capacity investment in Model A, for the most part. However, when demand follows an exponential distribution, Model A always leads to larger capacity investments. For all demand distributions, the capacity investment ratio is always greater than 1, implying that both Models A and C induce larger total capacity investments than that of a single manufacturer.

Overall, a key takeaway from this analysis is that the Principal could benefit (in some cases, to a significant extent) from the presence of an additional firm with a late-stage product. However, those benefits are completely eroded when one manufacturer receives strict priority for demand allocation and hence, the Principal should avoid offering contracts involving prioritization in settings where both manufacturers’ products meet the necessary functional requirements.

6. Concluding Remarks

In this paper, we formulate and solve a principal-agent model to analyze the impact of AMC contracts on the capacity investment decisions of manufacturers with late-stage products. We consider several variants of the AMC contract in the single-manufacturer setting and examine how demand allocation rules impact the capacity investments in the two-manufacturer setting.

One of the key findings of our work is that minimum-capacity requirements can significantly enhance the effectiveness of AMC contracts and are superior to purchase-guarantees both in terms of the cost-per dose and capacity investments. Hence, minimum-capacity contracts emerge as better alternatives relative to purchase-guarantee contracts. This finding assumes increased significance against the backdrop of a resistance from humanitarian organizations to offer purchase guarantees in light of the perception that it is not a market-based approach.
Our analysis also highlights the importance of demand allocation rules in multi–manufacturer settings. For example, our results suggest that the Principal should not prioritize one manufacturer for demand allocation, as long as both manufacturers’ products meet the functional requirements. More importantly, our analysis of the two–manufacturer setting offers a framework to study the impact of a variety of priority schemes ranging from deterministic to probabilistic prioritization of manufacturers for demand allocation. Understanding the impact of such priority schemes is vital especially in settings where the complete array of product characteristics are not publicly known. In such settings, the manufacturers have to make capacity investments based on their estimates of the likelihood of being prioritized by the Principal.

In this paper, we focus on AMC contracts for late–stage products but clearly, more work is required to analyze the impact of the different market–based mechanisms targeted at the different product life–cycle stages including research and development and clinical trial phases. Examples of such mechanisms include patent buyouts (once the product is successfully developed), best entry tournaments (with prizes awarded to firms that make the most progress by a given date), limiting patent protection in poor countries (patent protection applies only to developed countries) etc. Understanding how the different mechanisms interact with each other could be critical in realizing the full benefits of health innovations and such endeavors are avenues for potential future research.

References


Appendix A: Proof of Theorem 1

For convenience, we divide the Principal’s choice of contract terms \((\delta_r, \bar{q})\) into two cases: (i) \(0 < \bar{q} < g(\delta_r)\), and (ii) \(g(\delta_r) \leq \bar{q}\).

1. In Case (i), the Manufacturer will install \(\kappa^* = \kappa^*_L = \kappa^*_R = \bar{q}\). In this case, if the Principal can ensure that M makes non-negative profit, it is able to incentivize the Manufacturer to have an installed capacity of exactly \(\bar{q}\).

2. In Case (ii), because \(g(\delta_r) \geq 0\), M will either choose the left Region and capacity \(g(\delta_r)\), or the right Region and capacity \(\bar{q}\). Moreover, we can argue that \(\pi_L(g(\delta_r)) = \pi_R(\bar{q})\), and the Manufacturer in this case will choose an installed capacity of \(\kappa^* = g(\delta_r)\).

Before we present any further arguments, we argue that \(\pi_L(g(\delta_r))\) and \(\pi_R(0)\) are always non-negative. This can be verified easily from the expressions for the objective functions in (3) and (4). Intuitively, this means that if in a region, M picks the unconstrained optimal solution, then its expected profit will be non-negative. This makes sense because without constraints on capacity, \(\kappa = 0\) is a feasible solution and it results in a zero profit. Unconstrained optimal profit cannot therefore be lower. That is, the Individual Rationality (IR) constraint arises only in Case (i).

Which of these cases will the Principal prefer? The Principal, we assume, wishes to maximize the number of citizens of recipient countries who receive a unit dose or a health product. That is, the Principal chooses \((\delta_r, \bar{q})\) to maximize \(E(\min(D, \kappa))\) by influencing M’s choice of \(\kappa\). Because \(E(\min(D, \kappa))\) is an increasing function of \(\kappa\) and \(D\) is independent of the Principal’s decision variables, we hereafter assume that the Principal’s goal is to maximize \(\kappa\). The Principal also faces budget constraints, which depend on the chosen case. We now list the two cases below in terms of the constraints faced by the Principal.

1. Case (i):
   - IR Constraint: \(\pi_L(\bar{q}) = \pi_R(\bar{q}) \geq 0\)
   - Budget: \(\delta_r \bar{q} \leq b\)

2. Case (ii):
   - IR Constraint: none
   - Budget: \(\delta_r g(\delta_r) \leq b\)

Consider Case (ii). In this case, \(\bar{q}\) has no effect on \(\kappa^*\) and the budget constraint, so long as the corresponding capacity investments lie in the appropriate ranges. Therefore, we may assume, without loss of generality, that the Principal will set \(\bar{q} = g(\delta_r)\). This simplifies subsequent analysis because Cases (ii) is subsumed by Case (i).

Next, we argue that the budget constraint in Case (i) must be tight. This follows from the fact that the Principal can increase \(\kappa^*\) by increasing \(\bar{q}\) until the budget constraint prevents that from happening. These arguments complete the proof of the statement of Theorem 1.

\[\square\]

Appendix B: Proof of Theorem 2

In (10), upon considering each region one by one, and taking derivatives with respect to \(y\), we find that \(\pi_M\) is concave in \(y\), and that furthermore, \(y^* = y^*_M\) if \(y \leq \bar{q}\) and \(y^* = y^*_L(\delta_r)\), otherwise. Using this solution to (10), and applying the capacity constraint, we obtain the optimal production function \(p^*(\kappa)\) provided in the statement of Theorem 2. Hence proved. \[\square\]
Appendix C: Proof of Theorem 3

From Theorem 2, we see that in each region, the optimal production quantity is the minimum of two quantities, one of which is $\kappa$. This gives rise to two cases in the first region of $\bar{q}$, and 2 cases in the other regions, i.e. a total of 7 cases. We shall consider these cases one by one.

**Case 1: $\bar{q} \leq y_R^*$**

**Case 1a: $\kappa \leq \bar{q} \leq y_R^*$**

$$\pi_M(\kappa, p^*(\kappa)) = (\delta_r + r)E[\min(\kappa, D)] - (c + c_a)\kappa. \tag{53}$$

The objective function in (53) comes from the fact that $M$ produces exactly $\kappa$, which is smaller than $\bar{q}$. Hence, it earns top-up on $E[\min(\kappa, D)]$.

**Case 1b: $\bar{q} \leq \kappa \leq y_R^*$**

In this case, the Manufacturer still produces $\kappa$, but earns top-up on $E[\min(\bar{q}, D)]$ because the top-up cap is smaller than the production quantity. This gives rise to the following objective function.

$$\pi_M(\kappa, p^*(\kappa)) = (\delta_r)E[\min(\bar{q}, D)] + (r)E[\min(\kappa, D)] - (c + c_a)\kappa. \tag{54}$$

**Case 1c: $\bar{q} \leq y_R^* \leq \kappa$**

In this last case, the Manufacturer produces $y_R^*$ and earns top-up on $\bar{q}$. Therefore, its profit function can be written as follows:

$$\pi_M(\kappa, p^*(\kappa)) = \delta_r E[\min(\bar{q}, D)] + (r)E[\min(y_R^*, D)] - cy_R^* - c_\kappa \kappa. \tag{55}$$

In (55), the Manufacturer’s expected profit function is decreasing in $\kappa$. Therefore, it will not choose $\kappa$ greater than $y_R^*$, and this case is subsumed by Case 1b, leaving only two valid cases.

From (53) and (54), it is straightforward to verify that

$$\kappa^*_1 = \bar{q}^{-1} \left( \frac{c + c_a}{\delta_r + r} \right). \tag{56}$$

$$\geq \kappa^*_1 = \bar{q}^{-1} \left( \frac{c + c_a}{r} \right). \tag{57}$$

Because $c + c_a > r$, $\kappa^*_1 = 0$. However, we continue to analyze a more general case in which $\kappa^*_1$ may not be zero. Combining the above inequality with the defining inequalities of Cases 1a and 1b, we find that either $\kappa^*_1 \leq \bar{q}$, which implies that Case 1b does not arise, or $\bar{q} \leq \kappa^*_1$, which implies that Case 1a does not arise.
however, $\kappa_{1b}^* \leq \bar{q} \leq \kappa_{1a}^*$, the objective function is increasing in the Case 1a and decreasing in Case 1b (because $\pi_M$ is concave in each region). Therefore, $\kappa^*$ must equal $\bar{q}$.

Summarizing our findings so far, we have that

$$\kappa_1^* = \begin{cases} \kappa_{1b}^* & \text{if } \bar{q} \leq \kappa_{1b}^*, \\ \kappa_{1a}^* & \text{if } \kappa_{1a}^* \leq \bar{q} \\ \bar{q} & \text{otherwise} \end{cases}$$ (58)

Case 2: $y_R^* \leq \bar{q} \leq y_L^*(\delta_r)$

Case 2a: $\kappa \leq \bar{q}$

$$\pi_M(\kappa, p^*(\kappa)) = (\delta_r + r)E[\min(\kappa, D)] - (c + c_a)\kappa.$$ (59)

The objective function in (53) comes from the fact that M produces exactly $\kappa$, which is smaller than $\bar{q}$. Hence, it earns top-up on $E[\min(\kappa, D)]$.

Case 2b: $\bar{q} \leq \kappa$

In this case, the Manufacturer produces $\bar{q}$, and earns top-up on $E[\min(\bar{q}, D)]$ because the top-up cap is smaller than the production quantity. This gives rise to the following objective function.

$$\pi_M(\kappa, p^*(\kappa)) = (\delta_r + r)E[\min(\bar{q}, D)] - c\bar{q} - c\kappa.$$ (60)

It is straightforward to observe that $\pi_M$ is decreasing in $\kappa$. Therefore, the Manufacturer will choose $\kappa = \bar{q}$ in this case.

From (59) and (60), we find

$$\kappa_{2a}^* = F^{-1}\left(\frac{\bar{q}}{\delta_r + r}\right)$$
and, $\kappa_{2b}^* = \bar{q}$. (61)

Summarizing our findings so far, we have that

$$\kappa_2^* = \begin{cases} \kappa_{2a}^* & \text{if } \kappa_{2a}^* \leq \bar{q} \\ \bar{q} & \text{otherwise} \end{cases}.$$ (63)

Case 3: $y_L^*(\delta_r) \leq \bar{q}$

Case 3a: $\kappa \leq y_L^*(\delta_r) \leq \bar{q}$

$$\pi_M(\kappa, p^*(\kappa)) = (\delta_r + r)E[\min(\kappa, D)] - (c + c_a)\kappa.$$ (64)
The objective function in (53) comes from the fact that M produces exactly $\kappa$, which is smaller than $\bar{q}$. Hence, it earns top-up on $E[min(\kappa, D)]$.

**Case 3b: $y^*_L(\delta_\epsilon) \leq \kappa$**

In this case, the Manufacturer produces $y^*_L(\delta_\epsilon)$, and earns top-up on $E[min(y^*_L(\delta_\epsilon), D)]$ because the top-up cap is greater than the production quantity. This gives rise to the following objective function.

$$\pi_M(\kappa, p^*(\kappa)) = (\delta_\epsilon + r)E[min(y^*_L(\delta_\epsilon), D)] - cy^*_L(\delta_\epsilon) - c_\kappa \kappa.$$ (65)

In (65), the Manufacturer’s expected profit function is decreasing in $\kappa$. Therefore, it will not choose $\kappa$ greater than $y^*_L(\delta_\epsilon)$.

Similarly, in Case 3a, the unconstrained optimum is

$$\kappa^*_3a = \bar{F}^{-1} \left( \frac{c + c_\kappa}{\delta_\epsilon + r} \right).$$ (66)

Combining the two cases, we obtain that when $y^*_L(\delta_\epsilon) \leq \bar{q}$

$$\kappa^*_3 = \begin{cases} 
\kappa^*_3a & \text{if } \kappa^*_3a \leq y^*_L(\delta_\epsilon) \\
\kappa^*_3a & \text{if } y^*_L(\delta_\epsilon) \leq \kappa^*_3a. 
\end{cases}$$ (67)

Note that $\kappa^*_1a = \kappa^*_2a = \kappa^*_3a$. Let us denote this quantity by $\kappa_0$.

**The Principal’s Decision Problem:**

The Principal chooses $\delta_\epsilon$, which affects $\kappa_0$ and $y^*_L(\delta_\epsilon)$, and $\bar{q}$, which determines the Manufacturer’s response, given $\kappa_0$. Note that $\kappa_0$ is a function of problem primitives ($\bar{F}$, $c$, $r$ and $c_\kappa$), which are not decision variables, and $\delta_\epsilon$, which is chosen by the Principal. Therefore, going forward, the Principal’s choice of $\delta_\epsilon$ is treated as being equivalent to choosing $\kappa_0$. Also, note that $\kappa_0$ is increasing in $\delta_\epsilon$. Next, we consider the three cases one by one.

**Case 1:** Suppose P chooses $\bar{q} \leq y^*_R$. The relative size of $y^*_R$ and $\kappa_0$ depends on the value of $\delta_\epsilon$ relative to $c_\kappa$. In particular, $\kappa_0 \leq y^*_R$ if $\delta_\epsilon \leq \frac{c_\kappa}{c_\kappa}$ and $\kappa_0 \geq y^*_R$, otherwise. Consider the first instance. We now get two further cases. Either $\bar{q} \leq \kappa_0 \leq y^*_R$, or $\kappa_0 \leq \bar{q} \leq y^*_R$. In the first instance $\kappa^* = \bar{q}$ and $p^*(\kappa^*) = \bar{q}$. In the second instance, $\kappa^* = \kappa_0$ and $p^*(\kappa^*) = \kappa_0$. Note that in the second instance, the incentive offered over top-up quantity in excess of $\kappa_0$ is not utilized. Therefore, P has no reason to offer $\bar{q}$ higher than $\kappa_0$.

Next, suppose $\kappa_0 \geq y^*_R$. In this case, $\kappa^* = \bar{q}$ and $p^*(\kappa^*) = \bar{q}$. The two cases can be combined into the following strategy for the Principal.
Either $\tilde{q} = \kappa_0 \Rightarrow \kappa^* = \kappa_0, p^*(\kappa^*) = \kappa_0,$ \hspace{1cm} (68)

or $\tilde{q} \leq \kappa_0 \Rightarrow \kappa^* = \tilde{q}, p^*(\kappa^*) = \tilde{q}$. \hspace{1cm} (69)

In order to maximize the availability of the product, the Principal will choose $\kappa_0$ as large as possible and $\tilde{q}$ just equal to $\kappa_0$, so long as budget permits. That is, the Principal will maximize $\kappa_0$ subject to $\kappa_0 \delta_\epsilon = b$.

Case 2: Next, suppose $P$ chooses $\tilde{q}$ such that $y^*_R \leq \tilde{q} \leq y^*_L(\delta_\epsilon)$. Before analyzing this case, we point out that $\kappa_0 < y^*_L(\delta_\epsilon)$ under the parameter choices we have made so far. Therefore, we consider only two cases, either $\kappa_0 \leq \tilde{q}$ or $\tilde{q} \leq \kappa_0$, resulting in the following two strategies for the Principal.

Either $\kappa_0 \leq \tilde{q} \Rightarrow \kappa^* = \kappa_0, p^*(\kappa^*) = \kappa_0$, \hspace{1cm} (70)

or $\tilde{q} \leq \kappa_0 \Rightarrow \kappa^* = \tilde{q}, p^*(\kappa^*) = \tilde{q}$. \hspace{1cm} (71)

Note that in the first instance, the offered top-up over quantities in excess of $\kappa_0$ is not utilized. Therefore, in the first instance, the Principal will offer $\tilde{q} = \kappa_0$, and we have a situation that mirrors Case 1.

The arguments at the end of the previous case still apply and the Principal will choose $\kappa_0$ as large as possible and $\tilde{q}$ just equal to $\kappa_0$, so long as budget permits. That is, the Principal will maximize $\kappa_0$ subject to $\kappa_0 \delta_\epsilon = b$.

Case 3: Next, suppose $P$ chooses $\tilde{q}$ such that $y^*_L(\delta_\epsilon) \leq \tilde{q}$. Because $\kappa_0 < y^*_L(\delta_\epsilon)$, only the first out of three cases for $\kappa_3^*$ is possible. Therefore, we have $\kappa^* = \kappa_0$ and $p^*(\kappa^*) = \kappa_0$. The offered top-up over units in excess of $\kappa_0$ is not utilized. Therefore, the Principal has no reason to offer a top-up cap greater than $y^*_L(\delta_\epsilon)$. The Principal’s strategy in this case is therefore to set $\tilde{q} = y^*_L(\delta_\epsilon)$.

Upon comparing Cases 2 and 3, we find that the Principal ends up offering a higher top-up cap in Case 3, which is not utilized by the Manufacturer, leading to a lower value of $\kappa_0$. Therefore, we argue that Case 2 dominates Case 3 from the Principal’s perspective. Combining Cases 1 and 2, we obtain the statement of Theorem 3.

$\square$

Appendix D: Proof of Corollary 2

The details are presented next. Using the definition of $\kappa_0$, upon performing implicit differentiation and treating $\delta_\epsilon$ as a function of $c$, we have

$$\frac{\partial \kappa_0}{\partial c} = \frac{- f(\kappa_0)}{\delta_\epsilon + r} + f(\kappa_0) \left( \frac{c + c_\kappa}{(\delta_\epsilon^2 + r)^2} \right) \frac{\partial \delta_\epsilon}{\partial c}. \hspace{1cm} (72)$$

From the budget constraint, we have
\[
\frac{\partial \kappa_0}{\partial c} \delta_r + \kappa_0 \frac{\partial \delta_r}{\partial c} = 0. \tag{73}
\]

Upon substituting from (72) into (73), we obtain
\[
\frac{\partial \delta_r}{\partial c} = \left( f(\kappa_0) \delta_r \delta_r + r \right) \left[ \frac{\kappa_0 + f(\kappa_0)(c + c_r)}{(\delta_r + r)^2} \right] \geq 0. \tag{74}
\]

Similarly, after substituting from above into (73) and simplifying, we obtain
\[
\frac{\partial \kappa_0}{\partial c} = \left( f(\kappa_0) \delta_r \delta_r + r \right) \left[ \frac{-\kappa_0 - f(\kappa_0)(c + c_r)}{(\delta_r + r)^2} \right] \leq 0. \tag{75}
\]

The effect of changes in other parameters can be studied in a similar fashion.

**Appendix E: Proof of Lemma 2**

We omit the superscript A from this proof for brevity. First, we show that in both the left and right regions, \( M_i \)'s profit function is concave in \( \kappa_i \). This follows from the following expressions, which are obtained by taking the derivative of expression (27) with respect to \( \kappa_i \) in the regions \( \kappa_i \leq \bar{q}_i \) and \( \kappa_i \geq \bar{q}_i \) respectively.

\[
\frac{\partial^2 \pi_i}{\partial \kappa_i^2} \bigg|_{\kappa_i \leq \bar{q}_i} = -2(r - c + \delta_r) f(2\kappa_i) \leq 0 \tag{78}
\]

and

\[
\frac{\partial^2 \pi_i}{\partial \kappa_i^2} \bigg|_{\kappa_i \geq \bar{q}_i} = -2(r - c) f(2\kappa_i) \leq 0 \tag{79}
\]

Since the \( M_i \)'s profit function is concave in the two regions, it follows from the definition of \( \kappa^*_L \) and \( \kappa^*_R \) that \( \kappa^*_i = \kappa^*_R \) if \( \bar{q}_i \geq \min\{\kappa^*_L, \kappa^*_R\} \) and \( \kappa^*_i = \kappa^*_L \) if \( \bar{q}_i \leq \max\{\kappa^*_L, \kappa^*_R\} \). To show that \( \kappa^*_i = \bar{q}_i \) when \( \min\{\kappa^*_L, \kappa^*_R\} \leq \bar{q}_i \leq \max\{\kappa^*_L, \kappa^*_R\} \), we use the fact that \( M_i \)'s profit function is continuous at \( \kappa_i = \bar{q}_i \) and the following property.

\[
\lim_{\kappa_i \to \bar{q}_i^-} \frac{\partial \pi_i}{\partial \kappa_i} = \lim_{\kappa_i \to \bar{q}_i^+} \frac{\partial \pi_i}{\partial \kappa_i} + \delta_r F(2\kappa_i) \tag{80}
\]

Equation (80) implies that when \( \bar{q}_i \geq \min\{\kappa^*_L, \kappa^*_R\} \), \( \kappa^*_L \geq \kappa^*_R \) holds. This combined with the fact that \( M_i \)'s profit function is concave in the left and right regions implies that \( \kappa^*_i = \bar{q}_i \) when \( \min\{\kappa^*_L, \kappa^*_R\} \leq \bar{q}_i \leq \max\{\kappa^*_L, \kappa^*_R\} \). Hence the result. The three cases of \( \bar{q}_i \) presented in Lemma 2 are illustrated in Figure 5. The blue line in the figure is the expected profit in the left region, while the red line is the expected profit in the right region.
Appendix F: Proof of Theorem 4

For ease of notation, we omit the superscript $A$ in this proof. For any given $\delta$, let $T$ denote the point where \( \bar{q} \) and $\kappa_R^*$ intersect. For \( \bar{q} > T = \kappa_{L^*}^* \), $\kappa_R^*(\bar{q}) \geq \kappa_{L^*}^*$. This follows from equations (30) and (31). Furthermore, \( \bar{q} \geq \kappa_R^*(\bar{q}) \) as well. This follows from expression (80). Hence, for \( \bar{q} > T \), $\kappa^* = \kappa_{L^*}^*$.

Now let us consider \( \bar{q} \leq T \). In this case, by definition, \( \bar{q} < \kappa_{L^*}^* \). We will show that \( \bar{q} < \kappa_R^*(\bar{q}) \) as well in this range of \( \bar{q} \). To see how, let us suppose that \( \bar{q} > \kappa_R^*(\bar{q}) \) instead. Then, if we replace \( \bar{q} \) with $\kappa_R^*(\bar{q})$ in expression (31), the left hand–side would be negative. However, that would be at odds with expression (30) since $\kappa_R^* \geq \kappa_R^*(\bar{q})$ when \( \bar{q} \leq T \). Therefore, it must be that \( \bar{q} < \kappa_R^*(\bar{q}) \). This implies that $\kappa^* = \kappa_R^*(\bar{q})$ when \( \bar{q} \leq T \).

Also notice that $\kappa_R^*(\bar{q}) = \kappa_{L^*}^*$ at \( \bar{q} = T \). Hence, to find the manufacturer's optimal capacity investment, it suffices to consider \( 0 \leq \bar{q} \leq \kappa_{L^*}^* \). Hence the result.

Appendix G: Proof of Theorem 5

We omit the superscript $B$ in this proof. Recall that since the manufacturers invest in different amounts of capacity, the Principal need not necessarily set $\bar{q}_1 = \bar{q}_2$. Therefore, we start by considering the choice of $\bar{q}_1$, since the decision of $M_1$ is independent of that of $M_2$.

1. Case (i): $0 \leq \bar{q}_1 \leq \kappa_{1,L}$, Then, $M_1$ installs $\bar{q}_1$.
2. Case (ii): $\bar{q}_1 \geq \kappa_{1,L}$, Then, $M_1$ installs $\kappa_{1,L}$.

Cases (ii) is subsumed by case (i) since offering $\bar{q}_1 > \kappa_{1,L}$ does not alter $M_1$’s capacity investment. Therefore, the Principal will choose $\bar{q}_1$ such that $0 \leq \bar{q}_1 \leq \kappa_{1,L}$ holds and $M_1$ will react by investing in $\kappa_1^* = \bar{q}_1$. Next, we look at the Principal’s choice of $\bar{q}_2$. Notice that there are only two cases, since $\kappa_{2,R} = \kappa_{1,R} - \bar{q}_1 \leq 0$.

1. Case (i): $0 \leq \bar{q}_2 \leq \kappa_{1,L} - \bar{q}_1$. Then, $M_2$ installs $\bar{q}_2$.
2. Case (ii): $\bar{q}_2 \geq \kappa_{1,L} - \bar{q}_1$. Then, $M_2$ installs $\kappa_{1,L} - \bar{q}_1$.

Notice that in the first case, the total capacity investment of the two manufacturers would be $\kappa_1^* + \kappa_2^* = \bar{q}_1 + \bar{q}_2 \leq \bar{q}_1 + \kappa_{1,L} - \bar{q}_1 = \kappa_{1,L}$. In the second case, the total capacity investment would be $\kappa_1^* + \kappa_2^* = \bar{q}_1 + \kappa_{1,L} - \bar{q}_1 = \kappa_{1,L}$. Therefore, the Principal will prefer the second case. However, setting $\bar{q}_2 > \kappa_{1,L} - \bar{q}_1$ will not increase the capacity investment of $M_2$, and it would result in a waste of top-up units. Therefore, the Principal will choose $\bar{q}_2 = \kappa_{1,L} - \bar{q}_1$, thus proving Theorem 5.
Appendix H: Proof of Lemma 6

Suppose that $D \sim U[d, \bar{d}]$. Let $g(\delta_r)$ be the capacity investment in the single Manufacturer setting, where

$$g(\delta_r) = \left(1 - \frac{c_\kappa}{r + c - \delta_r}\right)(d - d) + d$$  

and let $2\kappa^*_L = 2\kappa^*_L = 2\kappa^*_L$ be the capacity investment of $M_1$ and $M_2$ in the two Manufacturers setting, where

$$2\kappa^*_L(\delta_r) = \frac{4}{3}\left(1 - \frac{c_\kappa}{r + c - \delta_r}\right)(d - d) + 2d$$  

Our goal is to show that $2\kappa^*_L(\delta_r^*) \geq g(\delta_r^*)$. For simplicity, we use $P_1$ to denote the optimization problem in the single Manufacturer setting (see Corollary 1), and $P_2$ to denote the optimization problem in the two Manufacturers setting (see Corollary 3). Let $\delta_{r,1}$ be the solution to $P_1$. Then, $g(\delta_{r,1})$ is the maximum capacity investment in the single Manufacturer setting. Suppose now that there exist $\delta_{r,2}$ in the two Manufacturers setting such that

$$2\kappa^*_L(\delta_{r,2}) = g(\delta_{r,1}^*)$$  

That is, we can find a $\delta_{r,2}$ that achieves the solution of $P_1$. We now need to check that $\delta_{r,2}$ is a feasible solution of $P_2$. Expanding the equation above we get

$$\left(1 - \frac{c_\kappa}{r + c - \delta_{r,1}^*}\right)(d - d) + d = \frac{4}{3}\left(1 - \frac{c_\kappa}{r + c - \delta_{r,2}^*}\right)(d - d) + 2d$$  

Suppose that $\delta_{r,2} > \delta_{r,1}^*$. Then, $\left(1 - \frac{c_\kappa}{r + c - \delta_{r,2}^*}\right) < \left(1 - \frac{c_\kappa}{r + c - \delta_{r,1}^*}\right)$, which implies that

$$\left(1 - \frac{c_\kappa}{r + c - \delta_{r,1}^*}\right) + d < \left(1 - \frac{c_\kappa}{r + c - \delta_{r,2}^*}\right) + d < \frac{4}{3}\left(1 - \frac{c_\kappa}{r + c - \delta_{r,2}^*}\right) + 2d$$  

which violates the equality sign of Equation (84). Therefore, we must have that $\delta_{r,2} \leq \delta_{r,1}^*$. Since $\delta_{r,1}^*$ is the optimal solution to $P_1$, it must satisfy the constraint that $(\delta_{r,1}^*)g(\delta_{r,1}^*) = b$. Using Equation (83), we have that

$$2(\delta_{r,2})\kappa_L(\delta_{r,2}) = (\delta_{r,2})g(\delta_{r,2}^*) \leq (\delta_{r,1}^*)g(\delta_{r,1}^*) = b$$  

Thus, $\delta_{r,2}$ is a feasible solution of $P_2$. Let $\delta_{r,2}^*$ be the optimal solution to $P_2$. Then, it must be the case that

$$2\kappa_L(\delta_{r,2}^*) \geq 2\kappa_L(\delta_{r,2}) = g(\delta_{r,1}^*)$$  

thus proving the statement of Lemma 6.