Efficiency and Information Aggregation in Heterogeneous Markets

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Abstract

This paper studies how heterogeneous preferences shape the informational and allocative efficiency of centralized markets with asymmetric information. We start by showing that introducing agent-level heterogeneity (e.g., in terms of trading costs) to the standard rational expectations equilibrium models reduces price informativeness by creating a bias towards the private information of agents with smaller trading costs. We then establish that this reduction in price informativeness in turn manifests itself as an informational externality: in the presence of heterogeneity, agents do not internalize the impact of their trading decisions on the information revealed to others via prices, even in competitive markets. We conclude the paper by investigating the welfare implications of market segmentation in the presence of the heterogeneity-induced informational externality.

Keywords: Price discovery, information aggregation, heterogenous agents.

Preliminary and Incomplete
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1 Introduction

Prices in financial markets have a remarkable ability to aggregate dispersed information and form as a public signal for market participants. In particular, it allows traders to learn from information that is held by other traders, and that would not have been available to them otherwise. However, it is not clear how well do prices aggregate information and what impact different market characteristics have on the aggregation process and on the overall performance of the market. In this paper, we examine how heterogeneity in the agent-level affects the aggregation process, and further analyze what are the implications on welfare.

The literature on information aggregation concentrates on informational efficiency of markets. An efficient market is one in which the price fully reflects all the available information, and such a market also results in an efficient resource allocation (Fama (1970)). A large body of this literature is devoted to models with rational expectations equilibrium (henceforth, REE). Grossman [Grossman (1976), Grossman (1978)] developed the concept of a competitive REE with asymmetric information. Grossman (1976) shows that in a competitive economy with \( n \) traders, each with private information, the equilibrium price aggregates all the economy’s information (in the sense that the price is a sufficient statistic for all the available information in the economy). However, one issue that arises in his model is that prices become "too informationally" efficient. Indeed, when agents observe the price, their privet information becomes redundant, and there is no incentive to acquire information. This is known as the "Grossman-Stiglitz paradox" and is discussed in Grossman and Stiglitz (1980).

One approach to overcome this paradox, is to introduce noise into the price. This can be done by adding a noisy demand/supply shock (Grossman and Stiglitz (1980)), or by introducing noise traders (Kyle (1985)). A different approach was introduced by Vives (2011), upon which we build our model.

Following Vives (2011), we model a finite market with privately informed agents competing in schedules. The underlying shocks (fundamental values) determining agents’ values of the good differ from one another (but may be correlated). This addition of an idiosyncratic component to the information structure insures that prices can be fully revealing only if they are combined with traders’ private signals (for each trader separately). This allows us to avoid the use of noise traders or noise in supply. Our model differs from the one in Vives (2011), in that we allow for heterogeneity in agents trading costs and in the precision of agents’ private information. We will show that this modification results in novel conclusions.

Our first set of results establishes that introducing heterogeneity in agents’ trading costs reduces price informativeness. In particular, when agents’ values are correlated and privet signals are noisy, in markets with more than two agents the price will be fully revealing if and only if all trading costs coincide. One thing to notice is that in our model, prices are partially reveling even though there is no noise in the price and there exists a sufficient statistic for the pooled information. The partial revelation is a result of agents asymmetric endogenous responses. A new result is that the price informativeness to an individual trader, depends not only on his own characteristics, but rather on the characteristics of the other traders. In particular, we show that for an individual agent, the price becomes less revealing as the heterogeneity in the rest of the market increases.
Our second set of results establishes the existence of an informational externality when introducing agent heterogeneity. Agents do not internalize the impact their actions have on other agents via the change in the information they infer from prices. Interestingly, the direction of this externality depends on the agent’s characteristics. This is in contrast to existing literature, as in Morris and Shin (2002), where all agents may over-react to public signal, or as in Vives (2017), where all agents simultaneously either over-react or under-react to private signals. This is a result of the heterogeneity in our model, in terms of agents’ trading costs. Roughly speaking, agents with lower trading costs trade more than agents with high trading costs. This, in turn, results in a price which is biased towards the private information of agents with lower trading costs. When the informational externality is strong (depending on the elasticity of demand), a social planner would want agents with low trading costs to trade less, and agents with higher trading costs to trade more.

Our third result is also novel. We show that in some cases (depending on the distribution of agents heterogeneity), dividing the market into segments, such that traders can only trade within their segment, increases social welfare. This may seem counterintuitive at first. Indeed, by centralizing the market (unifying all segments into one market and allowing all traders to trade with one another) allows for more trading opportunities. But, notice that centralizing the market has two counter effects on price informativeness. On one hand, it brings traders with more information into the market (positive effect). On the other hand, by doing so, it may increase market heterogeneity, which we show damages price informativeness. When the negative informational effect of centralizing the market is stronger than the positive effects of centralization a segmented market is socially better. This shows that the informational externality that we identify in the presence of agent heterogeneity can have first-order policy implications on the interaction between different exchanges.

Finally, we stress the distinction between informational efficiency and allocative efficiency. Most existing papers focus on price informativeness without considering the welfare implications. However, as we argue in the paper, the two do not necessarily coincide. In particular, we illustrate that the allocation can be constrained efficient even if the price may not reveal all the information. This is because in many of these environments, the planner cannot aggregate all the information either. So, what matters is the wedge between the planner’s and the equilibrium allocations.

Related Literature Hellwig (1980) was first to suggest that the way prices aggregate information should depend on agents preferences. More specifically, “the weight with which agent i’s information affects the price should depend on the strength of agent i’s reaction to this information, which in turn should depend on his preferences.” He generalizes the model in Grossman (1978) and adds a noise term to the supply to avoid the Grossman-Stiglitz paradox we mentioned above. Hellwig (1980) shows that the price is generally not fully revealing and depends on agents preferences. When the noise term goes to zero (but not exactly zero) it becomes fully revealing. This coincides with our first result, but from there on all our results are new.

Our paper is most closely related to Rostek and Weretka (2012) and to Dávila and Parlatore (2017). Rostek and Weretka (2012) consider a similar model to ours, but instead of heterogeneous trading costs, they allow for heterogeneous correlations in each pair of traders’ values. They are interested in
the effect of market size on the aggregation process and do not construct a welfare analysis. Dávila and Parlatore (2017) are interested in the affect of trading costs but in a model with traders who trade based on private information or due to hedging demand.

The externalities we identify in this paper are in nature similar to Vives (2017). However, as we will show, they manifest themselves very differently in our model because of the heterogeneity in agents trading costs.

Our results on market fragmentation are most closely related to the recent works of Malamud and Rostek (2017) and Babus and Kondor (2017).\footnote{Also see Peivandi and Vohra (2017).} Even though they also study various market structures, Malamud and Rostek (2017) focus on market power and price impact, whereas our focus is on the interplay between heterogeneity and the quality of price discovery.

Some of the more recent papers in this literature are Lambert, Ostrovsky, and Panov (2017) who build their model on Kyle (1985), Hassan and Mertens (2017) who are interested in market efficiency when traders make possibly small mistakes when forming their expectations, and Du and Zhu (2017).

Outline of the Paper  The rest of the paper is organized as follows. Section 2 presents the model and the solution concept. In Section 3, we show that the price aggregates agents’ information fully if and only if the market exhibits no heterogeneity. Section 4 contains our main results, where we identify the informational externality that arises when agents are heterogenous. Our results in Section 5 explore the implications of these externalities for market segmentation. All proofs and some additional technical details are provided in the Appendix.

\section{Model}

Consider an economy consisting of \( n \) traders, denoted by \{1, 2, \ldots, \( n \)\}, who trade a divisible asset. Traders face quadratic and potentially heterogenous trading costs and are uncertain about their valuations of the asset prior to trading. In particular, trader \( i \)'s utility is given by

\[ u_i(x_i) = \theta_i x_i - \frac{1}{2} \lambda_i x_i^2, \]

where \( x_i \) denotes the units of the asset obtained by \( i \) and \( \theta_i \), which we refer to as \( i \)'s valuation, is a random variable that is drawn from the standard normal distribution. We allow for interdependence in traders’ valuations for the asset by assuming that \( \text{corr}(\theta_i, \theta_j) = \rho \) for all pairs of agents \( i \neq j \), where \( \rho \in [0, 1) \). This formulation thus nests the cases with independent (\( \rho = 0 \)) and common (\( \rho \to 1 \)) valuations as special cases.

We refer to parameter \( \lambda_i \) in (1) as \( i \)'s trading cost. This cost, which may be heterogenous across agents, can arise due to transaction taxes, inventory costs for holding the asset until consumption, or other costs incurred as a consequence of trade. We treat the collection of parameters \( (\lambda_1, \ldots, \lambda_n) \) as a primitive of the model and assume that it is common knowledge in the economy.

In addition to the \( n \) traders, we assume there is a representative uninformed trader with utility

\[ u(y) = \alpha y - \frac{\beta y^2}{2} \]

\[ \text{2 Model} \]
where $y$ is the total units of the asset obtained by the uninformed trader and $\beta > 0$ captures the extent of price elasticity of demand.

Prior to trading, each trader $i$ observes a noisy private signal $s_i = \theta_i + \epsilon_i$ about her valuation, where $\epsilon_i \sim N(0, \sigma_i^2)$ are mutually independent. Variance $\sigma_i^2$ thus parametrizes $i$’s uncertainty about her valuation. Also note that, given the interdependencies in traders’ valuations when $\rho \neq 0$, trader $i$’s private signal is informative about the valuations of all other traders in the economy.

Trade occurs via a one-shot uniform-price double auction mechanism, according to which each trader $i$ submits a downward-sloping schedule $x_i(p)$ that specifies her demand for the asset as a function of the price $p$, which in turn is determined to clear the market. The market-clearing price satisfies

$$y + \sum_{i=1}^{n} x_i(p) = 0. \tag{3}$$

Once the market-clearing price $p$ is determined, trader $i$ is allocated $x_i(p)$ units of the asset and pays $px_i(p)$, obtaining a net payoff of $\pi_i = \theta_i x_i(p) - \frac{1}{2} \lambda_i(x_i(p))^2 - px_i(p)$. The consumer’s profit is given by $\pi_c(y) = \alpha y - \frac{\beta y^2}{2} - py$.

With the above framework in hand, we can now define our solution concept:

**Definition 1.** A competitive rational expectations equilibrium consists of demand schedules $x_i$ that are linear in price and agents’ private signals and a price function $p$ such that (i) each trader $i$ maximizes her expected payoff conditional on her information set $p, s_i$ (ii) the market clears.

A few remarks are in order. First, traders rely on the information content of the signal in the rational expectations tradition, such as Grossman and Stiglitz (1980) and Kyle (1989). Second, we assume there are no noise traders, which will allow us to perform a welfare analysis in the following sections. Last but not least, we assume traders are price-takers to ensure that any inefficiencies we identify are not due to market power.\(^2\)

As a preliminary result, we show that an equilibrium in linear strategies always exists and is unique. The characterization in the next result also serves as the basis for the rest of our analysis.

**Proposition 1.** There exists an equilibrium in linear strategies $x_i = a_i s_i + b_i - c_i p$, where the coefficients corresponding to trader $i$’s strategy depend on the price via

$$\lambda_i a_i = \frac{\text{var}(p) - \mathbb{E}[ps_i]\mathbb{E}[p\theta_i]}{(1 + \sigma_i^2) \text{var}(p) - \mathbb{E}^2[ps_i]}$$

$$\lambda_i b_i = \frac{\mathbb{E}[ps_i] - (1 + \sigma_i^2)\mathbb{E}[p\theta_i]}{(1 + \sigma_i^2) \text{var}(p) - \mathbb{E}^2[ps_i]} \mathbb{E}[p]$$

$$\lambda_i c_i = 1 + \frac{\mathbb{E}[ps_i] - (1 + \sigma_i^2)\mathbb{E}[p\theta_i]}{(1 + \sigma_i^2) \text{var}(p) - \mathbb{E}^2[ps_i]} \mathbb{E}[p]$$

whereas the price depends on the equilibrium strategies via

$$p = \frac{\alpha + \beta \sum_{k=1}^{n} (a_k s_k + b_k)}{1 + \beta \sum_{k=1}^{n} c_k}.$$

\(^2\)See Vives (2011), Rostek and Weretka (2012, 2015), and Bergemann, Heumann, and Morris (2015) for some recent studies on the interaction of market power and demand function competition.
Furthermore, coefficients \((a_1, \ldots, a_n)\) are independent of parameters \(\alpha\) and \(\beta\).

The first thing to notice from Proposition 1 is that the price is a weighted average of the private signals. This means that the variable \(\sum_{k=1}^{n} a_k s_k\) is a sufficient statistic for the price. It is easy to see, that when agents are homogeneous the price is simply a constant times the (unweighted) average of the private signals. Because of our Normal information structure, it implies that when traders are homogeneous, the price is a sufficient statistic for the pooled information of the market. The second thing to notice are the two terms in the expression for \(c_i\): the first term represents the responsiveness of trader \(i\) to price - an increase in price would make the asset less desirable. Whereas the second term is due to the information content of the price. Notice that when either \(\sigma_i = 0\) or \(\rho = 0\), this second term equals to zero, as the price does not contain any information that is relevant for trader \(i\).

3 Informational Efficiency

We start our analysis by studying how trader heterogeneity shapes the information content of the price and hence the market’s informational efficiency.

3.1 Privately Revealing Equilibrium

Following Allen (1981), among others, we define informational efficiency relative to a benchmark according to which

Definition 2. The equilibrium is fully privately revealing to trader \(i\) if \(E[\theta_i|s_i, p] = E[\theta_i|s_1, \ldots, s_n]\).

In other words, the equilibrium is fully privately revealing to trader \(i\) if she can construct the same expectation of her value of the good with the information available to her — the price and her private signal — as if she knew all other traders’ private signals (Allen, 1981). Put differently, under full private revelation, the price is a sufficient statistic for all other traders’ private signals. We have the following proposition:

Proposition 2. Suppose \(\rho > 0\) and \(\sigma_i^2 > 0\) for all \(i\). The equilibrium is fully privately revealing to all traders if and only if either

(i) there are only two traders in the market; or

(ii) all trading costs coincide.

In the previous section, we saw that the equilibrium price is a weighted average of agents’ private signals. When there are only two agents in the market, each agent can infer from the price the other agent’s private signal, since the price is simply a linear combination of the two private signals. When there are more than two agents, it is not possible to infer each agent’s private signal separately. Suppose, first, that agents have equal trading costs and their private signals have equal precisions. In this case, the price is simply an average of all agents’ private signals. Since, ex-ante, each agent’s private signal is equally informative, that is exactly the way agent \(i\) would have used her information,
given she knew all the information available in the market (the pooled information). Now, suppose that agents differ in their private signals precision, but have equal trading costs. In this case, the price is a weighted average of private signals, with the weights depending on the precision of the private signals. However, these weights are exactly the weights that trader $i$ would put given she knew the pooled information, since in this case not all signals are equally informative. When agents differ in their trading costs, the weights depend on the trading costs as well as the precision, and these weights differ from the way agent $i$ would have used her information if she knew the pooled information, since agent $j$ trading costs does not affect the informativeness of agent $j$ private signal.

Jordan (1983) shows that the market equilibrium price is generically non-revealing in settings where the dimension of the signal space (in our case $n$, the number of agents receiving private signals) is larger than the number of assets (in our case, one). Jordan (1983) considers a general joint distribution function of signals and future asset returns, with only some regularity assumptions. This results in a lack of a sufficient statistic for all the information available in the market. Grossman (1978) shows that by assuming a normal joint distribution function of signals and future asset returns, a sufficient statistic exists. Indeed, because of our normality assumption, there exists a one-dimensional sufficient statistic for all traders simultaneously, namely, $\sum_{k=1}^{n} s_k/(1-\rho+\sigma_k^2)$. This would fully reveal the information. The lack of full revelation in our setting is because of traders’ asymmetric endogenous responses. We also note that this distinct from the lack of revaluation in Rostek and Weretka (2012), where even though assume normality, does not exhibit a single sufficient statistic for all agents simultaneously.

### 3.2 Information Revelation

We showed that heterogeneity in agents’ trading costs is a source of informational inefficiency. In this subsection we further analyze how this type of heterogeneity affects the informational content of price for each trader $i$. Define the information revelation gap as

$$\phi_i = \frac{\text{var}(\theta_i|s_i, p) - \text{var}(\theta_i|s_1, \ldots, s_n)}{\text{var}(\theta_i|s_i) - \text{var}(\theta_i|s_1, \ldots, s_n)}.$$  

This index, which is always a number between 0 and 1, measures the extent to which price reduces trader $i$’s uncertainty relative to the benchmark that all information would have been revealed (which is equivalent, when assuming normality, to the case when traders are given a fully revealing price). Obviously, when the price is fully revealing, the information revelation gap is equal to zero, and it is equal to one, when it reveals no information at all. Put differently, the information revelation gap decreases as price becomes more informative.

**Proposition 3.** The information revelation gap corresponding to trader $i$ is given by

$$\phi_i = \frac{\Sigma_i^2}{\Sigma_i^2 + (1/\Lambda_i)^2} + o(1),$$  

where

$$\Lambda_i = \left(\frac{\sum_{k \neq i} w_k \lambda_k^{-1}}{\Sigma_{i \neq k} w_k}\right)^{-1},$$  

$$\Sigma_i^2 = \frac{\sum_{k \neq i} w_k (1/\lambda_k - 1/\Lambda_i)^2}{\sum_{k \neq i} w_k}.$$  


are, respectively, the weighted harmonic mean and weighted variance of the reciprocal of trading costs of traders $k \neq i$ with weights $w_k = 1/\text{var}(s_k)$.

First, notice that in line with Proposition 2, revelation is complete (i.e., $\phi_i = 0$) if and only if either there is no heterogeneity or there are only two traders in the market. This is independent of the actual level of trading costs. Second, and more importantly, Proposition 3 illustrates that what matters for trader $i$ is heterogeneity in the rest of the market: as heterogeneity $\Sigma_i$ in other traders’ trading costs increases — while the harmonic mean of trading costs $\Lambda_i$ is kept constant — the price becomes less informative to trader $i$. This means that trader $i$ can be very different than the rest of the market in terms of trading costs, but still be able to infer more information from the price compare to rest of the traders. Finally, the weights reveal that what matters more for $\phi_i$ is less heterogeneity in the trading costs among traders $k \neq i$ with the more precise signals, as heterogeneity among traders that do not have any informative signals should not matter for revelation.

4 Efficiency

Our results in the previous section illustrated that, as long as $n > 2$, the equilibrium is not fully privately revealing to all traders unless their trading costs all coincide. This finding, however, is silent about whether the allocation is constrained-inefficient. In this section, we explore the relationship between allocative and informational inefficiency and illustrate that the presence of heterogeneity also implies that the equilibrium is constrained inefficient.

We consider the constrained efficient benchmark as in Angeletos and Pavan (2007), according to which the social planner maximizes total expected surplus

$$E[W] = E[\pi_0] + \sum_{i=1}^{n} E[\pi_i]$$

subject to the same informational constraint faced by the traders in the decentralized equilibrium, where $\pi_i$ denotes the profits of trader $i$ and $\pi_0$ is the profit of the uninformed trader. This formulation ensures that the planner internalizes collective welfare while respecting the decentralized information structure of the economy. As in the equilibrium, we restrict the planner to affine strategies of the form $x_i = a_i s_i + b_i - c_i p$, while imposing the market-clearing condition $y + \sum_{i=1}^{n} x_i = 0$. We have the following result:

**Proposition 4.** Suppose $\sigma_i^2 > 0$ for all $i$. The equilibrium is constrained efficient if either

(i) there are only two traders in the market;

(ii) traders have private valuations ($\rho = 0$);

(iii) all traders have identical trading costs.

If the above conditions are violated, then the equilibrium is constrained inefficient for almost all $\beta$. 

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This proposition implies that when there are more than two agents in the market, heterogeneity not only leads to informational inefficiency, but also to allocative constrained inefficiency. This is an implication of the price being an endogenous public signal, resulting in an externality. Part (ii) of the proposition implies that the inefficiency is due to the presence of an informational externality, which is only present when agents are heterogeneous (and $n > 2$). In a heterogeneous market agents do not internalize the fact that by changing their trading actions they can reveal more information to others. When $\rho = 0$, agents’ values are independent and thus, agents do not infer any information that is relevant for them from other agents’ private signals. This, in turn, implies that agents have no informational impact on one another and there are no externalities, and so the equilibrium actions coincide with the constrained efficient actions.

In our next result, we determine the inefficiency identified in Proposition 4 manifests itself in traders’ actions.

**Proposition 5.** Let $a^e_i$ and $a^f_i$ denote the weights that $i$ assigns to her private signal in equilibrium and constrained efficient allocations, respectively. There exist $\bar{\rho} > 0$ and functions $\beta(\rho) < \beta(\rho)$ such that

(a) if $\rho < \bar{\rho}$ and $\beta < \beta$, then $a^e_i < a^f_i$ if and only if

$$\frac{1}{\lambda_i} < \frac{1}{\sum_{k \neq i} 1 - w_k \left( \frac{\sum_{j \neq k} w_j / \lambda_j}{\sum_{j \neq k} w_j / \lambda_j^2} \right)^2}$$

(b) if $\rho < \bar{\rho}$ and $\beta > \beta$, then $a^e_i < a^f_i$ if and only if

$$\frac{1}{\lambda_i} > \frac{1}{\sum_{k \neq i} 1 - w_k \left( \frac{\sum_{j \neq k} w_j / \lambda_j}{\sum_{j \neq k} w_j / \lambda_j^2} \right)^2}$$

where $w_k = 1/(1 + \sigma_k^2)$.

The above result therefore characterizes the set of traders that over- and under-react to their private signals. Crucially, not everyone reacts the same way. While some traders over-react, others under-react relative to the constrained efficient benchmark. The planner would want to change the concentration of the traders’ actions. But, whether the planner would want higher dispersion or less depends on $\beta$. For small $\beta$, the planner would want to reduce the dispersion in actions, whereas for $\beta$ large, the planner would want to increase the dispersion above and beyond the equilibrium. Put differently, when $\beta$ is small, the planner would want traders with relatively high trading costs to increase the weights on their private signals, whereas for traders with relatively low trading costs he would want them to decrease their weights. The opposite is true when $\beta$ is large.

These results therefore point towards the existence of an informational externality as well as a pecuniary externality, with each force dominating the other depending on the value of $\beta$. Even
though these externalities are similar to those identified by Vives (2017), they manifest themselves very differently here: everything relies crucially on the presence of asymmetries among traders. The parameter $\beta$ effectively controls the strength of the pecuniary externality relative to that of the informational externality. Note from Proposition 1, that the weight $a_i$ that trader $i$ assigns to her private signal is independent of $\beta$. As a result, changing $\beta$ has no impact on the informativeness of the price. This can also be seen from Proposition 3, which illustrates that the information revelation gap only depends on the distribution of trading costs, but not on $\beta$. As a result, the extent of informational externality is also independent of $\beta$. The same, however, cannot be said about the strength of the pecuniary externality. More specifically, equation (10) illustrates that $\beta$ impacts the sensitivity of the “price level” to traders’ actions, with a smaller $\beta$ making the price less sensitive to trades’ actions and hence reducing the strength of the pecuniary externality.

It is easiest to understand the case of $\beta \to 0$. In this limit, the price level does not respond to equilibrium actions, even though the informational role of the price is intact. As such, the externality identified in Proposition 4 is purely informational. Indeed, in Proposition 5, when $\beta$ is small, the planner would want to reduce the dispersion in actions. When the informational externality is dominant, the planner would like traders with high trading costs, which in general trade less, to trade more, and put more weight on their private signals. Their high trading costs effectively prevents them from sharing their private information via the price to the good of the other traders.

To further understand the externalities identified in this section, we state the following lemma.

**Lemma 1.** The derivative of expected total surplus with respect to the weight assigned by trader $i$ to her signal is given by

$$\frac{d\mathbb{E}[W]}{da_i} \bigg|_{eq} = \mathbb{E} \left[ \frac{dp}{da_i} \sum_{k=1}^{n} \frac{\partial x_k}{\partial p} \left( \mathbb{E}[\theta_k | s_1, \ldots, s_n] - \mathbb{E}[\theta_k | s_k, p] \right) \right].$$

First, we see that it is crucial for traders to have price-contingent actions in order for the inefficiency to occur (otherwise, the term $\frac{\partial x_k}{\partial p} = 0$ for all $k$). Second, we see from the term $\mathbb{E}[\theta_k | s_1, \ldots, s_n] - \mathbb{E}[\theta_k | s_k, p]$ that inefficiency arises only if there exists informational inefficiency. Thus, the pecuniary externality will only rise with the existence of the informational externality. Finally, we have the term $\frac{dp}{da_i}$, which is exactly the effect trader $i$’s actions have on the price, which is, as we seen, a dual effect - informational and pecuniary.

We would also like to highlight that the inefficiencies we have identified above are distinct from the Hirshleifer effect.

### 5 Application: Centralized vs. Segmented Markets

We conclude the paper with an application, illustrating how the interaction of market heterogeneity and the externalities identified above can have first-order policy implications. More specifically, we focus on whether a centralized market, where all traders can trade with all other traders is socially better than a segmented market, where traders can only trade with other traders in their segment.
To this end, consider a segmented market to be a partition of the market \( \{S_k\}_k \), such that \( \bigcup_k S_k = \{1, \ldots, n\} \) and \( S_k \cap S_j = \emptyset, \forall j \neq k \). Each \( S_k \) is a segment of the market. Agents in the same segment may have different trading costs and signal precisions, and these parameters may also differ across the segments of the market. We then compare the segmented market with a market architecture in which all traders are brought together in a centralized market. The following proposition gives sufficient conditions for which the expected social welfare in the centralized market is higher than in the segmented market.

**Proposition 6.** Expected welfare in the centralized market structure is higher than in the segmented architecture if either of the following conditions are satisfied:

(i) All traders have complete information about their valuations \( (\sigma_i = 0) \);

(ii) traders’ valuations are independent \( (\rho = 0) \);

(iii) All trading costs are identical.

Notice, that the conditions above are also sufficient for market efficiency (both informational efficiency and allocative constrained efficiency). In parts (i) and (ii) of proposition (6), prices only play a role as an index of scarcity. In part (iii) of the proposition, the price is fully privately revealing in equilibrium. Thus, in all cases there is no informational externality (and no pecuniary externality). By centralizing the market, we allow for more trade opportunities, and with no externalities, lead to higher realizations of gains from trade.

However, our next result illustrates that this may not hold in general if the equilibrium is only partially revealing.

**Proposition 7.** There exist constants \( \rho > 0 \) and \( \beta > 0 \) such that if \( \rho < \rho \) and \( \beta < \beta \), then expected welfare in the centralized market structure is higher than the segmented market structure only if

\[
\sum_{i=1}^{n} \frac{1}{\lambda_i} \left( \frac{\sigma_i^2}{1 + \sigma_i^2} \right)^2 \left( (m_i^{\text{cen}} - m_i^{\text{seg}}) - (\phi_i^{\text{cen}} m_i^{\text{cen}} - \phi_i^{\text{seg}} m_i^{\text{seg}}) \right) \geq 0,
\]

where \( \phi_i^{\text{cen}} \) and \( \phi_i^{\text{seg}} \) are trader \( i \)'s information revelation gaps in the centralized and segmented markets, respectively, and \( m_i^{\text{cen}} = \sum_{j \neq i} 1/\text{var}(s_j) \) and \( m_i^{\text{seg}} = \sum_{j \in S(i) \setminus \{i\}} 1/\text{var}(s_j) \).

Proposition 7 implies that when traders impose an informational externality on other traders, centralizing the market might damage social welfare. Equation (5) captures the difference between the informational gain and loss that are achieved by centralizing the market. More specifically, the first term in the right parenthesis \( m_i^{\text{cen}} - m_i^{\text{seg}} \) measures how much more information is available to market participants post centralization. But then the second term is a penalty term which reduces gains from centralization if the information revelation gap increases. The juxtaposition of this result with Proposition 3 relates welfare gains from centralizing the markets to the heterogeneity in each segment and in the market in general. In particular, when the information revelation gap is small in the segmented markets, implying that the extent of heterogeneity in each of them is low, relative to the information revelation gap in the centralized market, implying that when traders are brought all
together the extent of heterogeneity in the market is high, expected welfare is higher in the segmented market than in the centralized market. Furthermore, when all trading costs are identical, the above result reduces to part \((iii)\) of Proposition 6. Indeed, in this case \(\phi^\text{cen}_i = \phi^\text{seg}_i = 0\) for all \(i\), implying that \(\mathbb{E}[W^\text{cen}] \geq \mathbb{E}[W^\text{seg}]\). On the other hand, when (5) is violated, we get the opposite. Finally, each term is also weighed by trader \(i\)'s trading cost. Traders with high trading costs will trade less and hence, matter less for consideration of expected total welfare.

Centralizing the market not only provides more trading opportunities, but also, in principle, brings traders with more information into the same market. Yet, this new information might not increase price informativeness, but rather may actually hinder the information quality of the signal. More importantly, this reduction in price informativeness also manifests itself as a reduction in welfare, overcoming any possible gains from trade.

This shows that the informational externality and the key role played by heterogeneity can have first-order policy implications for not only price discovery but also welfare.

### 6 Conclusions

Price informativeness is a key feature for analyzing the performance of financial markets and the effects financial markets have on the real economy. By introducing heterogeneity in agents’ preferences we reveal additional insights to the literature: (1) Even when there is no noise in prices, and information is not too rich to be fully incorporated into the price, we show that the price will not be fully revealing, simply by adding heterogeneity in agents preferences; (2) The extent of revelation to a certain trader depends not only on her own characteristics, but rather on the characteristics of the rest of the traders; (3) Rather than agents over- or under-reacting simultaneously to private information, we show responses depend on agents preferences relative to the preferences of other agents; (4) Informational efficiency is a sufficient condition for allocative efficiency, but it is not necessary; and (5) Market centralization may decrease social welfare compared to a segmented market.

These new insights help us better understand the process of information aggregation, and the importance of price informativeness to welfare.
Appendix

A Informationally-Inefficient Efficient Markets

Our results in the main body of the paper establish that when trading costs are heterogenous, (i) the price signal does not fully aggregate the information in the market and (ii) the equilibrium is constrained inefficient, as traders do not internalize the impact of their trading decisions on the information content of the price. In other words, the equilibrium is both informationally and allocatively inefficient. In this appendix, we show that, in general, incomplete aggregation of information does not necessarily imply allocative inefficiency. We illustrate this by contrasting our results to an extension of the model of Rostek and Weretka (2012), where traders have homogenous trading costs but are asymmetric in the correlation between their private valuations. More specifically, we show that even though such heterogeneity leads to a incomplete aggregation of information, the equilibrium is constrained efficient, in the sense that the social planner cannot improve on the allocation.

As in the baseline model in Section 2, consider an economy consisting of $n$ price-taking traders with payoffs given by (1) and private signals $s_i = \theta_i + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma_i^2)$. As in our baseline model, the assumption that traders take the price as given guarantees that any potential inefficiency is not driven by traders’ market power. In a departure from the baseline model, however, suppose that the interdependencies in private valuations can be heterogenous among different pairs of traders. More specifically, suppose $\text{corr}(\theta_i, \theta_j) = \rho_{ij}$, with the assumption that

$$\frac{1}{n-1} \sum_{j \neq i} \rho_{ij} = \bar{\rho}$$

(6)

for some $\bar{\rho} \in (0, 1)$ and all traders $i$. This assumption ensures that all traders face the same average interdependencies in the market. We have the following result:

**Proposition A.1.** Suppose pairwise correlations satisfy (6). Also suppose all trading costs and signal precisions coincide. Then,

(a) The equilibrium is fully privately revealing to all traders if and only if $\rho_{ij} = \bar{\rho}$ for all $i \neq j$.

(b) The equilibrium is constrained efficient, regardless of the pairwise correlations.

The first statement of the above proposition, which generalizes Proposition 3 of Rostek and Weretka (2012), illustrates that heterogeneity in pairwise correlations prevents full private revelation in the sense of Definition 2. This is a consequence of the fact that full private revelation for trader $i$ requires the price to be equal to a specific weighted average of private signals. But the presence of heterogenous correlations means that this weight average may be different for different traders, implying that at least one trader cannot fully extract the sufficient statistic of other traders’ private signals by observing the price.

More importantly for our purposes however, part (b) of Proposition A.1 illustrates that the failure of informational efficiency highlighted in part (a) may not translate into allocative inefficiency: no
matter what the pairwise correlations are, all traders internalize the impact of their actions on others and no policy can improve upon the equilibrium allocation. This result thus underscores that equating informational efficiency with allocative efficiency — without performing a proper welfare analysis — can lead to misleading conclusions.
B Proofs

Proof of Proposition 1

Recall that the (ex ante) expected profit of trader \(i\) is given by \(E[\pi_i] = E[\theta_i x_i] - \frac{1}{2} \lambda_i E[x_i^2] - E[p x_i]\) and suppose trader \(i\) follows a linear strategy given by \(x_i = a_i s_i + b_i - c_i p\), where \(a_i, b_i,\) and \(c_i\) are coefficients that only depend on model parameters. Plugging this expression into \(i\)'s expected profit implies that
\[
E[\pi_i] = a_i - c_i E[\theta_i p] - \frac{1}{2} \lambda_i (1 + \sigma_i^2) a_i^2 - \frac{1}{2} (\lambda_i c_i^2 - 2 c_i) E[p^2] + a_i (\lambda_i c_i - 1) E[p s_i] - \frac{1}{2} \lambda_i b_i^2 + b_i (\lambda_i c_i - 1) E[p].
\]

Trader \(i\)'s objective is to maximize her expected profit while taking the price as given. As a first observation, note that \(i\)'s objective function is jointly concave in \((a_i, b_i, c_i)\). Therefore, the first-order conditions with respect to these parameters are both necessary and sufficient for optimality. Hence, the best-response strategy of trader \(i\) satisfies the following relationships:

\[
\begin{align*}
1 - \lambda_i (1 + \sigma_i^2) a_i + (\lambda_i c_i - 1) E[p s_i] &= 0 \\
-\lambda_i b_i + (\lambda_i c_i - 1) E[p] &= 0 \\
-E[\theta_i p] - (\lambda_i c_i - 1) E[p^2] + \lambda_i a_i E[p s_i] + \lambda_i b_i E[p] &= 0.
\end{align*}
\]

On the other hand, market clearing requires that \(y + \sum_{i=1}^n x_i = 0\), where \(y\) is the quantity demanded by the outside trader. Hence,
\[
\alpha - p + \beta \sum_{i=1}^n (a_i s_i + b_i - c_i p) = 0,
\]
where we are using the fact that the first-order condition of the outside trader is given by \(\alpha - p + \beta y = 0\).

Rearranging the above terms therefore implies that the equilibrium price is given by
\[
p = \frac{\alpha + \beta \sum_{k=1}^n (a_k s_k + b_k)}{1 + \beta \sum_{k=1}^n c_k}.
\]

Equations (7)–(10) provide a system of equations that relate traders’ equilibrium strategies to the model fundamentals. Plugging in the expression for the price (10) in equations (7)–(9) (followed by some tedious calculations) then implies that

\[
\begin{align*}
\lambda_i a_i &= \frac{\sum_{k \neq i} (1 - \rho + \sigma_k^2) a_k^2 + \rho (1 - \rho) (\sum_{k \neq i} a_k)^2 - \rho \sigma_i^2 a_i \sum_{k \neq i} a_k}{(1 + \sigma_i^2) \sum_{k \neq i} a_k^2 (1 - \rho + \sigma_i^2) + \rho (1 - \rho + \sigma_i^2) (\sum_{k \neq i} a_k)^2} \\
\beta \lambda_i b_i &= -\frac{\rho \sigma_i^2 (\sum_{k \neq i} a_k) (\alpha + \beta \sum_{k=1}^n b_k)}{1 + \sigma_i^2) \sum_{k \neq i} a_k^2 (1 - \rho + \sigma_i^2) + \rho (1 - \rho + \sigma_i^2) (\sum_{k \neq i} a_k)^2} \\
\beta (1 - \lambda_i c_i) &= \frac{\rho \sigma_i^2 (\sum_{k \neq i} a_k) (1 + \beta \sum_{k=1}^n c_k)}{1 + \sigma_i^2) \sum_{k \neq i} a_k^2 (1 - \rho + \sigma_i^2) + \rho (1 - \rho + \sigma_i^2) (\sum_{k \neq i} a_k)^2}.
\end{align*}
\]

The proof is complete once we show that the system of equations (11)–(13) has a solution \((a_i, b_i, c_i)_{i=1}^n\).

We first establish that there exists a vector \(a = (a_1, \ldots, a_n)\) that satisfies (11) for all \(i\). To this end, define the mapping \(\Phi : \mathbb{R}_{++}^n \to \mathbb{R}_{++}^n\) as
\[
\Phi_i(a) = \frac{\sum_{k \neq i} (1 - \rho + \sigma_k^2) a_k^2 + \rho (1 - \rho) \left(\sum_{k \neq i} a_k\right)^2}{\lambda_i \left((1 + \sigma_i^2) \sum_{k \neq i} a_k^2 (1 - \rho + \sigma_i^2) + \rho (1 - \rho + \sigma_i^2) (\sum_{k \neq i} a_k)^2\right) + \rho \sigma_i^2 \sum_{k \neq i} a_k}.
\]
Once again, the fact that \( \Phi(a) = a \). Define the set \( A = \prod_{i=1}^{n} [a_i, \bar{a}_i] \), where \( a_i = \lambda_{\max}^{-1}(1 - \rho)/(1 - \rho + \sigma_i^2) \) and \( \bar{a}_i = \lambda_{\min}^{-1}(1 - \rho)/(1 - \rho + \sigma_i^2) \), with \( \lambda_{\max} \) and \( \lambda_{\min} \) denoting the largest and smallest trading costs, respectively. It is easy to verify that \( \Phi_i(a) \geq a_i \) whenever \( \rho \sigma_i^2 \sum_{k \neq i} a_k (\lambda_{\max}(1 - \rho + \sigma_k^2) a_k - (1 - \rho)) \geq 0 \), which holds trivially as long as \( a_k \geq a_i \) for all \( k \neq i \). Similarly, \( \Phi_i(a) \leq \bar{a}_i \) as long as \( \rho \sigma_i^2 \sum_{k \neq i} a_k (\lambda_{\min}(1 - \rho + \sigma_k^2) a_k - (1 - \rho)) \leq 0 \), an inequality that is satisfied when \( a_k \leq \bar{a}_i \) for all \( k \neq i \). These observations therefore imply that \( \Phi \) maps the compact and convex set \( A \) to itself. Thus, by Brouwer’s fixed point theorem, there exists \( a \in A \) such that \( \Phi(a) = a \), hence guaranteeing that there exist coefficients \( a_1, \ldots, a_n \) that satisfy equation (11) for all \( i \) simultaneously.

Next, consider (12). This system of equations has a trivial solution of \( b_i = 0 \) for all \( i \) when \( \alpha = 0 \). We therefore consider the case that \( \alpha \neq 0 \). Dividing both sides of the equation by \( \lambda_i \) and summing over all \( i \) leads to

\[
\beta \sum_{i=1}^{n} b_i = - \left( \alpha + \beta \sum_{i=1}^{n} b_i \right) \sum_{i=1}^{n} \frac{\rho \sigma_i^2}{\lambda_i \delta_i} \sum_{k \neq i} a_k, \tag{14}
\]

where

\[
\delta_k = (1 + \sigma_k^2) \sum_{j \neq k} a_j^2 (1 - \rho + \sigma_j^2) + \rho (1 - \rho + \sigma_k^2) (\sum_{j \neq k} a_j)^2. \tag{15}
\]

Since \( a_i > 0 \) for all \( i \), it must be the case that \( \sum_{i=1}^{n} \frac{\rho \sigma_i^2}{\lambda_i \delta_i} \sum_{k \neq i} a_k \neq -1 \). Therefore, given coefficients \( a_1, \ldots, a_n \), there exists a unique \( \sum_{i=1}^{n} b_i \) that satisfies (14). Plugging back this solution into (12) then implies that there exists a collection of constants \( (b_1, \ldots, b_n) \) that satisfy the equilibrium condition.

Finally, consider (13). This equation implies that

\[
\beta \sum_{i=1}^{n} c_i = \beta \sum_{i=1}^{n} \frac{1}{\lambda_i} - \left( \alpha + \beta \sum_{i=1}^{n} c_i \right) \sum_{i=1}^{n} \frac{\rho \sigma_i^2}{\lambda_i \delta_i} \sum_{k \neq i} a_k.
\]

Once again, the fact that \( \sum_{i=1}^{n} \frac{\rho \sigma_i^2}{\lambda_i \delta_i} \sum_{k \neq i} a_k \neq -1 \) guarantees that the exists a unique \( \sum_{i=1}^{n} c_i \) that satisfies the above equation. Plugging back this solution into (13) then implies that there exists a collection \( (c_1, \ldots, c_n) \) that satisfies the equilibrium conditions. \( \square \)

**Proof of Proposition 2**

**Lemma B.1.** \( a_i (1 - \rho + \sigma_i^2) = a_k (1 - \rho + \sigma_k^2) \) for all pairs \( i \) and \( k \) if and only if all trading costs coincide.

**Proof.** First suppose all trading costs are identical, i.e., \( \lambda_i = \lambda \) for all \( i \). Under such an assumption, it is immediate to verify that \( \lambda a_i = (1 - \rho)/(1 - \rho + \sigma_i^2) \), thus implying that \( a_i (1 - \rho + \sigma_i^2) = a_k (1 - \rho + \sigma_k^2) \) for all pairs of traders \( i \) and \( k \).

To prove the converse implication, suppose \( a_i (1 - \rho + \sigma_i^2) = a_k (1 - \rho + \sigma_k^2) \) for all pairs \( i \neq k \). This means that there exists a constant \( S > 0 \) such that \( (1 - \rho + \sigma_k^2) a_k = S \) for all \( k \). Plugging this expression into equilibrium condition (11) leads to

\[
S \lambda_i \left( (1 + \sigma_i^2) + \rho (1 - \rho + \sigma_i^2) \sum_{k \neq i} (1 - \rho + \sigma_k^2)^{-1} \right) + \rho \sigma_i^2 = (1 + \rho \sigma_i^2) \left( 1 + \rho (1 - \rho) \sum_{k \neq i} (1 - \rho + \sigma_k^2)^{-1} \right).
\]
Solving for the constant $S$ from the above expression implies that $S = (1 - \rho)/\lambda_i$ for all $i$, which can hold only if $\lambda_i = \lambda_k$ for all $i$ and $k$. 

We now turn to the proof of Proposition 2. As a first observation, note that when $n \geq 3$, it is immediate that the equilibrium is fully privately revealing to both traders. Hence, in the rest of the proof we assume that there are at least three traders in the market. Suppose that the equilibrium is fully privately revealing to all traders, where recall from Definition 2 that this is equivalent to assuming that $\mathbb{E}[\theta_i | s_i, i, s_i] = \mathbb{E}[\theta_i | s_1, ..., s_n]$ for all $i$, where

$$
\mathbb{E}[\theta_i | s_1, ..., s_n] = \left( \frac{1 - \rho}{1 - \rho + \sigma_i^2} \right) s_i + \left( \frac{\rho \sigma_i^2}{1 - \rho + \sigma_i^2} \right) \left( \frac{1 + \rho \sum_{j=1}^n (1 - \rho + \sigma_j^2)^{-1}}{1 - \rho + \sigma_i^2} \right) \sum_{k=1}^n \left( \frac{1}{1 - \rho + \sigma_k^2} \right) s_k.
$$

On the other hand, the fact that market-clearing price satisfies (10) means that

$$
\mathbb{E}[\theta_i | s_i, i, p] = \frac{1}{\delta_i} \left( \sum_{k \neq i} (1 - \rho + \sigma_k^2)a_k^2 + \rho(1 - \rho) \left( \sum_{k \neq i} a_k \right)^2 - \rho \sigma_i^2 \sum_{k \neq i} a_k \right) s_i + \frac{1}{\delta_i} \left( \rho \sigma_i^2 \sum_{j \neq i} a_j \right) \sum_{k=1}^n a_k s_k,
$$

where $\delta_i$ is given by (15). Hence, full private revelation requires that the coefficient on signal $s_k$ in the above two expressions coincide for all $k$. Hence, as long as there are at least three traders in the market, full private revelation to all traders $i$ implies that $a_j/a_k = (1 - \rho + \sigma_j^2)/(1 - \rho + \sigma_k^2)$ for all $j, k \neq i$. Consequently, by Lemma B.1, all trading costs have to coincide. 

**Proof of Proposition 3**

As a first observation, note that

$$
\text{var}(\theta_i | s_1, ..., s_n) = \frac{\sigma_i^2}{1 - \rho + \sigma_i^2} \left( 1 - \frac{\rho(1 - \rho + \sigma_i^2)^{-1}}{1 + \rho \sum_{k=1}^n (1 - \rho + \sigma_k^2)^{-1}} \right).
$$

Furthermore, recall from (10) that $\text{var}(\theta_i | s_i, p) = \text{var}(\theta_i | s_i, \sum_{k \neq i} a_k s_k)$. Therefore,

$$
\text{var}(\theta_i | s_i, p) = \frac{\sigma_i^2}{1 - \rho + \sigma_i^2} \left( 1 - \frac{\rho(1 - \rho + \sigma_i^2)^{-1}}{1 + \rho \sum_{k=1}^n (1 - \rho + \sigma_k^2)^{-1}} \right) \frac{\rho \sigma_i^2}{1 + \rho \sum_{k=1}^n (1 - \rho + \sigma_k^2)^{-1}} \sum_{k \neq i} a_k^2(1 - \rho + \sigma_k^2)^{-1} \frac{\rho \sigma_i^2}{1 + \rho \sum_{k=1}^n (1 - \rho + \sigma_k^2)^{-1}} \sum_{k \neq i} a_k^2(1 - \rho + \sigma_k^2)^{-1} + \rho \left( \sum_{k \neq i} a_k \right)^2.
$$

Finally, note that $\text{var}(\theta_i | s_i) = \sigma_i^2/(1 + \sigma_i^2)$. Combining the above expressions implies that trader $i$’s information revelation gap, defined in (4), is given by

$$
\phi_i = \left( \frac{1 + \sigma_i^2}{1 - \rho + \sigma_i^2} \right) \frac{\sum_{k \neq i} a_k^2(1 - \rho + \sigma_k^2)^{-1} - \left( \sum_{k \neq i} a_k \right)^2 \left( \sum_{k \neq i} a_k \right)^2}{\left( 1 + \rho \sum_{k=1}^n (1 - \rho + \sigma_k^2)^{-1} \right) \sum_{k \neq i} a_k^2(1 - \rho + \sigma_k^2)^{-1} + \rho \left( \sum_{k \neq i} a_k \right)^2}.
$$

On the other hand, equation (11) implies that $\lim_{\rho \to 0} a_i = w_i/\lambda_i$ for all traders $i$, where $w_i = 1/(1 + \sigma_i^2)$. Taking the limit as $\rho \to 0$ from both sides of the above equation implies that

$$
\lim_{\rho \to 0} \phi_i = \left( \frac{\sum_{k \neq i} w_i/\lambda_k^2}{\sum_{k \neq i} w_i/\lambda_k^2} \right) / \left( \frac{\sum_{k \neq i} w_i/\lambda_k^2}{\sum_{k \neq i} w_i/\lambda_k^2} \right).
$$

Dividing both the numerator and the denominator by $\sum_{k \neq i} w_i$ then complete the proof. \qed
Proof of Proposition 4

Lemma B.2. Let \( x_i = a_i s_i + b_i - c_i p \) denote traders' equilibrium strategies. Then,

\[
\frac{\beta c_k}{1 + \beta \sum_{j=1}^{n} c_j} = \frac{\beta Q_k - M_k}{\lambda_k (1 + \beta \sum_{j=1}^{n} 1/\lambda_j)},
\]

where \( Q_k \) and \( M_k \) are independent of the value of \( \beta \).

Proof. Recall that equilibrium coefficients \( (a_i, b_i, c_i) \) satisfy equations (11)–(13). Summing both sides of (13) over all traders \( i \) and solving for \( 1 + \beta \sum_{i=1}^{n} c_i \) implies that

\[
1 + \beta \sum_{i=1}^{n} c_i = \frac{1 + \beta \sum_{j=1}^{n} 1/\lambda_j}{1 + \beta \sum_{j=1}^{n} \sum_{r \neq j} a_r \sigma^2_i / (\lambda_j \delta_j)},
\]

where \( \delta_k \) is given by (15). Plugging the above back into the expression for \( c_k \) in (13) then establishes (17), where \( Q_k \) and \( M_k \) are given by

\[
Q_k = 1 + \beta \sum_{j=1}^{n} \sum_{r \neq j} a_r \sigma^2_i / (\lambda_j \delta_j),
\]

and

\[
M_k = \rho \sum_{j=1}^{n} \sum_{r \neq j} a_r.
\]

Finally, to establish that \( Q_k \) and \( M_k \) are independent of \( \beta \), recall that the coefficients \( (a_1, \ldots, a_n) \) are solutions to the system of equations given by (11), which does not depend on \( \beta \). Hence, \( Q_k \) and \( M_k \) are independent of \( \beta \).

With the above lemma in hand, we now return to the proof of Proposition 4. We prove this result by determining the conditions under which the equilibrium strategies identified in Proposition 1 satisfy the optimality conditions of the planner’s problem.

Recall that the total ex ante surplus in the economy is given by \( E[W] = E[\pi_0] + \sum_{i=1}^{n} E[\pi_i] \), where \( \pi_0 \) is the surplus of the outside trader and \( \pi_i \) is the profit of trader \( i \). Therefore, the market-clearing condition \( y + \sum_{i=1}^{n} x_i = 0 \) implies that

\[
E[W] = \sum_{i=1}^{n} E[\theta_i x_i] - \frac{1}{2} \sum_{i=1}^{n} \lambda_i E[x_i^2] + \alpha E[y] - \frac{\beta}{2} E[y^2].
\]

When agents follow linear strategies in the form of \( x_i = a_i s_i + b_i - c_i p \), the expected total surplus is given by

\[
E[W] = \sum_{i=1}^{n} E[(\theta_i - \alpha)(a_i s_i + b_i - c_i p)] - \frac{1}{2} \sum_{i=1}^{n} \lambda_i E[(a_i s_i + b_i - c_i p)^2] - \frac{\beta}{2} E \left[ \sum_{i=1}^{n} (a_i s_i + b_i - c_i p)^2 \right],
\]

where once again we are using the market-clearing condition. Thus the social planner chooses the constants \( a_i, b_i, \) and \( c_i \) to maximize the total expected surplus in (20). We now determine the conditions under which the equilibrium strategies identified in Proposition 1 satisfy the first-order conditions corresponding to the planner’s problem.

First, consider the planner’s first-order condition with respect to coefficients \( (b_1, \ldots, b_n) \). Differentiating (20) with respect to \( b_i \) and using the fact that the market-clearing price satisfies (10) implies that

\[
\frac{dE[W]}{db_i} = -\lambda_i b_i + \lambda_i c_i E[p] - \frac{\alpha + \beta \sum_{k=1}^{n} b_k}{1 + \beta \sum_{k=1}^{n} c_k} + \frac{\beta}{1 + \beta \sum_{k=1}^{n} c_k} \sum_{k=1}^{n} c_k (\lambda_k b_k + (1 - \lambda_k c_k) E[p]). \tag{21}
\]
On the other hand, recall from equation (8) that equilibrium coefficients satisfy $(\lambda_i c_i - 1)\mathbb{E}[p] = \lambda_i b_i$. Consequently, the first-order condition of the planner’s problem with respect to $b_i$ evaluated at the equilibrium strategies is given by

$$
\left. \frac{d\mathbb{E}[W]}{db_i} \right|_{eq} = \mathbb{E}[p] - \frac{\alpha + \beta \sum_{k=1}^{n} b_k}{1 + \beta \sum_{k=1}^{n} c_k}.
$$

But note that (10) implies that the right-hand side of the above expression is equal to zero, thus implying that equilibrium strategies always satisfy the planner’s first-order conditions with respect to $b_i$ for all parameter values.

Next, consider the social planner’s first-order condition with respect to coefficients $(c_1, \ldots, c_n)$. Differentiating (20) with respect to $c_i$ leads to

$$
\frac{d\mathbb{E}[W]}{dc_i} = \lambda_i a_i \mathbb{E}[p s_i] + \lambda_i b_i \mathbb{E}[p] - \lambda_i c_i \mathbb{E}[p^2] - \mathbb{E}[\theta_i p] + \frac{\alpha + \beta \sum_{k=1}^{n} b_k c_k}{1 + \beta \sum_{k=1}^{n} c_k} \mathbb{E}[p] + \frac{\beta \sum_{k=1}^{n} a_k \mathbb{E}[s_k p]}{1 + \beta \sum_{k=1}^{n} c_k},
$$

where once again we are using the fact that the market-clearing price satisfies (10). On the other hand, recall from equation (8) that equilibrium coefficients satisfy (9). Therefore, the first-order condition of the planner’s problem with respect to $c_i$ evaluated at equilibrium strategies is equal to

$$
\left. \frac{d\mathbb{E}[W]}{dc_i} \right|_{eq} = -\mathbb{E}[p^2] + \frac{\alpha + \beta \sum_{k=1}^{n} b_k c_k}{1 + \beta \sum_{k=1}^{n} c_k} \mathbb{E}[p] + \frac{\beta \sum_{k=1}^{n} a_k \mathbb{E}[s_k p]}{1 + \beta \sum_{k=1}^{n} c_k}.
$$

Equation (10) then implies that the right-hand side of the above equation is equal to zero. In other words, no matter the parameter values, the equilibrium strategies always satisfy the planner’s first-order conditions with respect to $(c_1, \ldots, c_n)$.

Finally, we consider the planner’s first-order condition with respect to $(a_1, \ldots, a_n)$. Differentiating (20) with respect to $a_i$, and using the fact that the market-clearing price satisfies (10) implies that

$$
\frac{d\mathbb{E}[W]}{da_i} = 1 - \lambda_i (1 + \sigma_i^2) a_i + (\lambda_i c_i - 1) \mathbb{E}[p s_i] + \frac{\beta}{1 + \beta \sum_{j=1}^{n} c_j} \sum_{k=1}^{n} c_k \left( \lambda_k a_k \mathbb{E}[s_i s_k] - \mathbb{E}[\theta_k s_i] + (1 - \lambda_k c_k) \mathbb{E}[s_i p] \right).
$$

Recall that we have already established that $d\mathbb{E}[W] / db_i = d\mathbb{E}[W] / dc_i = 0$ at the equilibrium strategies. Therefore, the equilibrium is constrained efficient if only if the above expression is equal to zero when evaluated at the equilibrium strategies. Furthermore, recall that equilibrium strategies satisfy equations (7)–(9). Hence, by (7), it is immediate that

$$
\left. \frac{d\mathbb{E}[W]}{da_i} \right|_{eq} = \frac{\beta}{1 + \beta \sum_{j=1}^{n} c_j} \sum_{k=1}^{n} c_k \left( \lambda_k a_k \mathbb{E}[s_i s_k] - \mathbb{E}[\theta_k s_i] + (1 - \lambda_k c_k) \mathbb{E}[s_i p] \right),
$$

which by using (7) one more time further simplifies to

$$
\left. \frac{d\mathbb{E}[W]}{da_i} \right|_{eq} = \frac{\beta}{1 + \beta \sum_{j=1}^{n} c_j} \sum_{k \neq i}^{n} c_k \left( \rho (\lambda_k a_k - 1) + (1 - \lambda_k c_k) \mathbb{E}[s_i p] \right).
$$
Replacing for coefficients \(a_i\) and \(c_i\) from equations (11) and (13) and using the fact that the market-clearing price satisfies (10) implies that

\[
\frac{dE[W]}{da_i}\bigg|_{\text{eq}} = \frac{\beta \rho}{1 + \beta} \sum_{j=1}^{n} c_j \sum_{k \neq i} \frac{\sigma_k^2}{\delta_k} a_j \left(a_i(1 - \rho + \sigma_i^2) - a_j(1 - \rho + \sigma_j^2)\right),
\]

where \(\delta_k\) is defined in (15). Thus, by Lemma B.2,

\[
\frac{dE[W]}{da_i}\bigg|_{\text{eq}} = \frac{\rho}{1 + \beta} \sum_{k \neq i} \frac{\sigma_k^2}{\delta_k\lambda_k} (\beta Q_k - M_k) \sum_{j \neq k} a_j \left(a_i(1 - \rho + \sigma_i^2) - a_j(1 - \rho + \sigma_j^2)\right). \tag{24}
\]

We now use (24) to prove Proposition 4. As a first observation, note that when \(\rho = 0\), the right-hand side of the above equation is equal to zero, thus implying that the equilibrium is constrained efficient for all profiles of trading costs. Next, consider the case that \(n = 2\). With only two traders, it is immediate that the right-hand side of (24) is also equal to zero for all parameter values, thus once again implying constrained efficiency. To establish that the equilibrium is constrained efficient when all trading costs coincide, recall from Lemma B.1 that \(\lambda_i = \lambda\) guarantees that \(a_i(1 - \rho + \sigma_i^2) = a_j(1 - \rho + \sigma_j^2)\) for all \(i\) and \(j\). Therefore, when all trading costs are identical, the right-hand side of (24) is equal to zero, thus guaranteeing constrained efficiency.

Finally, we show that as long as \(n \geq 3\), trading costs are heterogenous, and \(\rho > 0\), the equilibrium is constrained inefficient for almost all values of \(\beta\). We establish this by contradiction. Suppose there exist \(\beta \neq \tilde{\beta}\) for which the equilibrium is constrained efficient. Hence, the right-hand side of (24) is equal to zero for both \(\beta\) and \(\tilde{\beta}\) and all traders \(i\). Since \(\rho \neq 0\), this implies that

\[
\sum_{k \neq i} \frac{\sigma_k^2}{\delta_k\lambda_k} (\beta Q_k - M_k) \sum_{j \neq k} a_j \left(a_i(1 - \rho + \sigma_i^2) - a_j(1 - \rho + \sigma_j^2)\right) = 0
\]

\[
\sum_{k \neq i} \frac{\sigma_k^2}{\delta_k\lambda_k} (\tilde{\beta} Q_k - M_k) \sum_{j \neq k} a_j \left(a_i(1 - \rho + \sigma_i^2) - a_j(1 - \rho + \sigma_j^2)\right) = 0,
\]

where recall that the coefficients \((a_1, \ldots, a_n)\) are the solution to the fixed point equation (11) and hence are independent of the value of \(\beta\). Subtracting the above two equations from one another and using the fact that \(\beta \neq \tilde{\beta}\) leads to

\[
\sum_{k \neq i} \frac{\sigma_k^2 M_k}{\delta_k\lambda_k} \sum_{j \neq k} a_j \left(a_i(1 - \rho + \sigma_i^2) - a_j(1 - \rho + \sigma_j^2)\right) = 0 \tag{25}
\]

for all traders \(i\). Since not all trading costs are identical, Lemma B.1 in the proof of Proposition 2 guarantees that there exists a \(i\) such that \(a_i(1 - \rho + \sigma_i^2) \leq a_j(1 - \rho + \sigma_j^2)\) for all \(j\), with at least one inequality holding strictly. But since \(M_k > 0\), this means that the left-hand side of (25) has to be strictly negative, leading to a contradiction. \(\square\)

**Proof of Proposition 5**

**Proof of part (a)** Recall from equation (24) that

\[
\lim_{\beta \to 0} \frac{dE[W]}{da_i}\bigg|_{\text{eq}} = \rho^2 \sum_{k \neq i} \frac{\sigma_k^4}{\delta_k\lambda_k^2} \left(\sum_{j \neq k} a_j \left(a_j(1 - \rho + \sigma_j^2) - a_i(1 - \rho + \sigma_i^2)\right)\right),
\]

19
where $\delta_k$ is given by (15). The above expression therefore implies that

$$\lim_{\rho \to 0} \lim_{\beta \to 0} \left| \frac{1}{\rho^2} \frac{d\mathbb{E}[W]}{da_i} \right|_\text{eq} > 0$$

if and only if

$$\lim_{\rho \to 0} a_i (1 - \rho + \sigma_i^2) < \lim_{\rho \to 0} \frac{\sum_{k \neq i} \frac{\sigma_k^4}{\lambda_k} \sum_{j \neq k} a_i \left( \sum_{j \neq k} a_j^2 (1 - \rho + \sigma_j^2) \right)}{\sum_{k \neq i} \frac{\sigma_k^4}{\lambda_k} \left( \sum_{j \neq k} a_j^2 \right)}.$$ 

On the other hand, equation (7) implies that $\lim_{\rho \to 0} \lambda_i a_i = 1/(1 + \sigma_i^2)$. Consequently, replacing for $a_i$ in the above equation implies that inequality (26) holds if and only if

$$\frac{1}{\lambda_i} > \frac{\sum_{k \neq i} \frac{\sigma_k^2}{\lambda_k (1 + \sigma_k^2)} \left( \sum_{j \neq k} a_j \right) \left( \sum_{j \neq k} a_j^2 (1 - \rho + \sigma_j^2) \right)}{\sum_{k \neq i} \frac{\sigma_k^2}{\lambda_k (1 + \sigma_k^2)} \left( \sum_{j \neq k} a_j^2 \right)}.$$ 

\[\square\]

**Proof of part (b)** Next, consider the case that $\beta \to \infty$. In this case, we have

$$\lim_{\rho \to 0} \lim_{\beta \to \infty} \left| \frac{1}{\rho} \frac{d\mathbb{E}[W]}{da_i} \right|_\text{eq} = \frac{1}{\sum_{r=1}^n 1/\lambda_r} \sum_{k \neq i} \frac{\sigma_k^2}{\lambda_k (1 + \sigma_k^2)} \left( \frac{1}{\lambda_i} \sum_{j \neq k} \frac{1}{\lambda_j (1 + \sigma_j^2)} - 1 \right).$$

Therefore,

$$\lim_{\rho \to 0} \lim_{\beta \to \infty} \left| \frac{1}{\rho} \frac{d\mathbb{E}[W]}{da_i} \right|_\text{eq} > 0$$

if and only if

$$\frac{1}{\lambda_i} > \frac{\sum_{k \neq i} \frac{\sigma_k^2}{\lambda_k (1 + \sigma_k^2)} \left( \sum_{j \neq k} a_j \right) \left( \sum_{j \neq k} a_j^2 (1 - \rho + \sigma_j^2) \right)}{\sum_{k \neq i} \frac{\sigma_k^2}{\lambda_k (1 + \sigma_k^2)} \left( \sum_{j \neq k} a_j^2 \right)}.$$ 

\[\square\]
Proof of Proposition 6

Before presenting the proof, we state and prove two simple lemmas.

**Lemma B.3.** Suppose $\zeta_1, \ldots, \zeta_m \geq 0$ and $\sum_{k=1}^{m} \zeta_k = 1$. Then,

$$\sum_{k=1}^{m} \frac{y_k}{\zeta_k + z_k} \geq \frac{\left(\sum_{k=1}^{m} \sqrt{y_k}\right)^2}{1 + \sum_{k=1}^{m} z_k} \tag{27}$$

for any collection of non-negative numbers $y_1, \ldots, y_m$ and $z_1, \ldots, z_m$.

**Proof.** We establish the lemma by showing that $\min_{\zeta} f(\zeta)$ subject to the constraint that $\sum_{k=1}^{n} \zeta_k = 1$ is equal to the right-hand side of (27), where $f(\zeta) = \sum_{k=1}^{m} y_k/(\zeta_k + z_k)$. First, note that $f(\zeta)$ is convex in $\zeta$, thus implying that the first-order condition is a sufficient for optimality. This implies that $\eta = y_k/(\zeta_k + z_k)^2$, where $\eta$ is the Lagrange multiplier corresponding to the constraint. Plugging this expression into the constraint implies that the optimal value of $\zeta_k$ is given by

$$\zeta_k = \left(1 + \frac{\sum_{j=1}^{m} z_j}{\sum_{j=1}^{m} \sqrt{y_j}}\right) \sqrt{y_k} - z_k.$$

Evaluating $f(\zeta)$ at the above values leads to the right-hand side of (27), thus completing the proof. ∎

**Lemma B.4.** Suppose $\alpha = 0$. The equilibrium welfare in a market consisting of $n$ traders is

$$\mathbb{E}[W] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} \left(1 - \text{var}(\theta_i|s_i, p)\right) - \left(\frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i}\right) \mathbb{E}[p^2].$$

**Proof.** Recall from the proof of Proposition 4 that the expected welfare in the economy is given by (19). Furthermore, note that the first-order condition of trader $i$ is given by $x_i = (\mathbb{E}[\theta_i|s_i, p] - p)/\lambda_i$, whereas that of the outside trader is given by $y = (\alpha - p)/\beta$. Plugging these expressions into (19) therefore implies that

$$\mathbb{E}[W] = \sum_{i=1}^{n} \frac{1}{\lambda_i} \mathbb{E}[\mathbb{E}^2[\theta_i|s_i, p]] - \sum_{i=1}^{n} \frac{1}{\lambda_i} \mathbb{E}[\theta_i p] - \sum_{i=1}^{n} \frac{1}{2\lambda_i} \mathbb{E}[(\mathbb{E}[\theta_i|s_i, p] - p)^2] + \frac{\alpha}{\beta} \mathbb{E}[\alpha - p] - \frac{1}{2\beta} \mathbb{E}[(\alpha - p)^2].$$

Consequently,

$$\mathbb{E}[W] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} \left(\mathbb{E}[\mathbb{E}^2[\theta_i|s_i, p]] - \left(\frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i}\right) \mathbb{E}[p^2] + \frac{\alpha^2}{2\beta}\right)$$

$$= \sum_{i=1}^{n} \frac{1}{2\lambda_i} \left(\text{var}(\theta_i) - \mathbb{E}[\text{var}(\theta_i|s_i, p)]\right) - \left(\frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i}\right) \mathbb{E}[p^2] + \frac{\alpha^2}{2\beta},$$

where the second equality is a consequence of the fact that $\mathbb{E}[\mathbb{E}[\theta_i|s_i, p]] = \mathbb{E}[\theta_i] = 0$ and the law of total variance. Noting that $\text{var}(\theta_i) = 1$ and $\mathbb{E}[\text{var}(\theta_i|s_i, p)] = \text{var}(\theta_i|s_i, p)$, which is a consequence of normality, and setting $\alpha = 0$ completes the proof. ∎

With the above lemmas in hand, we now proceed to proving Proposition 6.
Proof of part (a). Suppose traders face no uncertainties about their private valuations, i.e., \( \sigma_i = 0 \) for all \( i \). This means that \( \text{var}(\theta_i|s_i, p) = 0 \) for all traders regardless of the market structure. Thus, by Lemma B.4, expected welfare in the centralized market is given by

\[
\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \left( \frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2].
\]

On the other hand, equations (11)–(13) imply that when \( \sigma_i = 0, \) equilibrium strategies satisfy \( a_i = c_i = \lambda_i^{-1} \). Thus, by equation (10), the market clearing price in the centralized market is equal to \( p = \beta \sum_{i=1}^{n} s_i \lambda_i^{-1} / (1 + \beta \sum_{i=1}^{n} \lambda_i^{-1}) \). Therefore,

\[
\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta}{2} \left( \frac{(1 - \rho) \sum_{i=1}^{n} 1/\lambda_i^2 + \rho(\sum_{i=1}^{n} 1/\lambda_i)^2}{1 + \beta \sum_{i=1}^{n} 1/\lambda_i} \right).
\]

Following similar steps implies that expected welfare in the segmented architecture is given by

\[
\mathbb{E}[W^{\text{seg}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta}{2} \sum_{i=1}^{n} \frac{(1 - \rho) \sum_{i \in S_k} 1/\lambda_i^2 + \rho(\sum_{i \in S_k} 1/\lambda_i)^2}{\zeta_k + \beta \sum_{i \in S_k} 1/\lambda_i},
\]

where \( S_k \) denotes the set of traders in the \( k \)-th segment and \( \zeta_k \) is the fraction of outside traders that are active in that segment. Applying Lemma B.3 to the second term on the right-hand side above and noting that \( \sum_{S_i \in S} \zeta_k = 1 \) leads to

\[
\mathbb{E}[W^{\text{seg}}] \leq \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta/2}{1 + \beta \sum_{i=1}^{n} 1/\lambda_i} \left( \sum_{S_i \in S} \left( \frac{(1 - \rho) \sum_{i \in S_k} 1/\lambda_i^2 + \rho(\sum_{i \in S_k} 1/\lambda_i)^2}{\sum_{\lambda_i \in S_k} \lambda_i} \right)^2 \right),
\]

which in turn implies that

\[
\mathbb{E}[W^{\text{seg}}] \leq \sum_{i=1}^{n} \frac{1}{2\lambda_i} - \frac{\beta/2}{1 + \beta \sum_{i=1}^{n} 1/\lambda_i} \left( \frac{(1 - \rho) \sum_{i=1}^{n} 1/\lambda_i^2 + \rho \sum_{S_i \in S} \left( \sum_{i \in S_k} 1/\lambda_i \right)^2 + \rho \sum_{S_i \neq S_j} \left( \sum_{i \in S_k} 1/\lambda_i \right) \left( \sum_{i \in S_j} 1/\lambda_i \right)}{\sum_{\lambda_i \in S_k} \lambda_i} \right).
\]

Equation (28) implies that the right-hand side of the above inequality coincides with \( \mathbb{E}[W^{\text{cen}}] \), thus establishing that expected welfare is weakly higher in the centralized market structure. \( \Box \)

Proof of part (b). Suppose \( \rho = 0 \). This means that \( \text{var}(\theta_i|s_i, p) = \text{var}(\theta_i|s_i) = \sigma_i^2 / (1 + \sigma_i^2) \) regardless of the market structure. Consequently, Lemma B.4 implies that expected welfare in the centralized architecture is equal to

\[
\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i(1 + \sigma_i^2)} - \left( \frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2].
\]

Equations (11)–(13) imply that when \( \rho = 0, \) the coefficients corresponding to equilibrium strategies satisfy \( a_i = \lambda_i^{-1} / (1 + \sigma_i^2), \) \( b_i = 0, \) and \( c_i = \lambda_i^{-1} \). Replacing \( p \) from (10) leads to

\[
\mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i(1 + \sigma_i^2)} - \frac{\beta \sum_{i=1}^{n} 1/\lambda_i^2 (1 + \sigma_i^2)}{2(1 + \beta \sum_{i=1}^{n} \lambda_i^{-1})}.
\]

(29)
Following similar steps for the segmented market structure implies that

\[ \mathbb{E}[W_{\text{seg}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i(1 + \sigma_i^2)} \sum_{S_k \in S} \frac{\beta \sum_{i \in S_k} \lambda_i^{-1}}{2(\zeta_k + \beta \sum_{i \in S_k} \lambda_i^{-1})}, \]

and as a result,

\[ \mathbb{E}[W_{\text{seg}}] \leq \sum_{i=1}^{n} \frac{1}{2\lambda_i(1 + \sigma_i^2)} \sum_{S_k \in S} \frac{\beta \sum_{i \in S_k} \lambda_i^{-1}}{2(1 + \beta \sum_{i=1}^{n} \lambda_i^{-1})}. \]

Note that, by (29), the right-hand side of the above inequality is equal to \( \mathbb{E}[W_{\text{cen}}] \), thus implying that expected welfare in the centralized architecture is higher than the segmented architecture.  

**Proof of part (c).** Suppose all traders have identical trading costs, i.e., \( \lambda_i = \lambda \) for all \( i \). Since all trading costs are identical, Proposition 2 implies that the equilibrium in the centralized market structure is fully privately revealing to all traders, i.e., \( \text{var}(\theta_i|s_i, p) = \text{var}(\theta_i|s_1, \ldots, s_n) \) for all \( i \). A similar argument also guarantees that the equilibrium of the segmented market structure is also fully privately revealing to all traders within that segment. Therefore, Lemma B.4 implies that the expected welfare gain from market centralization is given by

\[ \mathbb{E}[W_{\text{cen}}] - \mathbb{E}[W_{\text{seg}}] = \frac{1}{2\lambda} \sum_{i=1}^{n} \left( \text{var}(\theta_i|s_k : k \in S_{(i)}) - \text{var}(\theta_i|s_1, \ldots, s_n) \right) \]

\[ + \frac{1}{2} \sum_{i=1}^{n} \left( \frac{\zeta_{(i)}}{\beta|S_{(i)}|} + \frac{1}{\lambda} \right) (\mathbb{E}[p_i^2] - \mathbb{E}[p^2]), \]

where \( S_{(i)} \) denotes the set of traders that belong to the same segment as trader \( i \) and \( p_{(i)} \) is the market-clearing price in that segment. Since \( S_{(i)} \subseteq \{1, \ldots, n\} \), it is immediate that the first term on the right-hand side above is always non-negative. It is therefore sufficient to establish that the second term is also non-negative. To this end, first consider the centralized market structure. Recall that when all trading costs are identical (11) implies that

\[ a_i = \frac{1 - \rho}{\lambda(1 - \rho + \sigma_i^2)}, \]

whereas (18) leads to

\[ 1 + \beta \sum_{j=1}^{n} c_j = \frac{(1 - \rho)(1 + n\beta/\lambda)(1 + \rho \sum_{k=1}^{n} (1 - \rho + \sigma_k^2)^{-1})}{1 + (n - 1)\rho}. \]

Plugging the above two expressions into the expression for the market-clearing price in (10), we obtain

\[ \left( \frac{1}{2\beta} + \sum_{i=1}^{n} \frac{1}{2\lambda_i} \right) \mathbb{E}[p^2] = \frac{\beta(1 + (n - 1)\rho)^2 \sum_{k=1}^{n} (1 - \rho + \sigma_k^2)^{-1}}{2\lambda(1 + n\beta)(1 + \rho \sum_{k=1}^{n} (1 - \rho + \sigma_k^2)^{-1})}. \]
Proof of Proposition 7

First consider the centralized market architecture. By Lemma B.4,

$$\lim_{\beta \to 0} \mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} (1 - \var(\theta_i|s_i, p)),$$

where we are using the fact that, by equation (10), $$\lim_{\beta \to 0} p^2 / \beta = 0$$. Replacing for $$\var(\theta_i|s_i, p)$$ in terms of the information revelation gap defined in (4) leads to

$$\lim_{\beta \to 0} \mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} \left( 1 - \phi_i^{\text{cen}} \var(\theta_i|s_i) - (1 - \phi_i^{\text{cen}}) \var(\theta_i|s_1, \ldots, s_n) \right).$$

On the other hand, recall that $$\var(\theta_i|s_i) = \sigma_i^2 / (1 + \sigma_i^2)$$, whereas (16) implies that $$\var(\theta_i|s_1, \ldots, s_n) = \frac{\sigma_i^2}{1 + \sigma_i^2} - \frac{\rho^2 \sigma_i^4}{(1 + \sigma_i^2)^2} \sum_{j \neq i} (1 + \sigma_j^2)^{-1} + o(\rho^2)$$. Consequently,

$$\lim_{\beta \to 0} \mathbb{E}[W^{\text{cen}}] = \sum_{i=1}^{n} \frac{1}{2\lambda_i} \left( 1 - \phi_i^{\text{cen}} \frac{\sigma_i^2}{1 + \sigma_i^2} - (1 - \phi_i^{\text{cen}}) \left( \frac{\sigma_i^2}{1 + \sigma_i^2} - \frac{\rho^2 \sigma_i^4}{(1 + \sigma_i^2)^2} \sum_{j \neq i} (1 + \sigma_j^2)^{-1} \right) \right) + o(\rho^2).$$

Following similar steps for the segmented market structure implies that

$$\lim_{\beta \to 0} \mathbb{E}[W^{\text{seg}}] = \sum_{S_k \subset S} \sum_{i \in S_k} \frac{1}{2\lambda_i} \left( 1 - \phi_i^{\text{seg}} \frac{\sigma_i^2}{1 + \sigma_i^2} - (1 - \phi_i^{\text{seg}}) \left( \frac{\sigma_i^2}{1 + \sigma_i^2} - \frac{\rho^2 \sigma_i^4}{(1 + \sigma_i^2)^2} \sum_{j \neq i} (1 + \sigma_j^2)^{-1} \right) \right) + o(\rho^2),$$

where $\phi_i^{\text{seg}}$ is trader $i$’s information revelation gap in the segmented market structure. Subtracting the above two equations from one another implies that

$$\lim_{\beta \to 0} \left( \mathbb{E}[W^{\text{cen}}] - \mathbb{E}[W^{\text{seg}}] \right) = \rho^2 \sum_{S_k \subset S} \sum_{i \in S_k} \frac{\sigma_i^4}{2\lambda_i(1 + \sigma_i^2)^2} \left( 1 - \phi_i^{\text{cen}} \right) \sum_{j \neq i} \frac{1}{1 + \sigma_j^2} - \left( 1 - \phi_i^{\text{seg}} \right) \sum_{j \in S_k} \frac{1}{1 + \sigma_j^2} \right) + o(\rho^2),$$

thus completing the proof. 

Proof of Proposition A.1

Proof of part (a) The proof of part (a) is similar to that of Proposition 3 of Rostek and Weretka (2012). First suppose that $\rho_{ij} = \rho$ for all $i \neq j$. Since all trading costs coincide, then Proposition 2 guarantees that the equilibrium is fully privately revealing to all traders simultaneously.

To prove the converse implication, suppose the price is fully privately revealing to all traders. That is, $\mathbb{E}[\theta_i|s_i, p] = \mathbb{E}[\theta_i|s_1, \ldots, s_n]$ for all $i$. In addition, recall that when traders follow linear strategies in the form of $x_i = a_i s_i + b_i - c_i p$, the corresponding coefficients satisfy (7)–(9). Consequently,

$$\lambda a_i = \frac{\var(p) - \mathbb{E}[ps_i]\mathbb{E}[p\theta_i]}{\left(1 + \sigma^2\right) \var(p) - \mathbb{E}^2[ps_i]},$$
where we are using the fact that all traders have identical trading costs and signal precisions. Also recall that the market-clearing price satisfies (10). Replacing for the price in the above expression therefore implies that coefficients \(a_i, \ldots, a_n\) are the solution to the following system of equations:

\[
\lambda a_i = \frac{\sum_{k\neq i} a_k^2(1 + \sigma^2) \rho_{kj} a_k a_j - (\sum_{k\neq i} \rho_{ik} a_k)^2}{(1 + \sigma^2) \sum_{k\neq i} a_k^2(1 + \sigma^2)} - a_i \sigma^2 \sum_{k\neq i} \rho_{ik} a_k^2. 
\]

(30)

It is easy to verify that the solution to the above system of equations is given by

\[
a_i = \frac{1 - \bar{\rho}}{\lambda(1 - \bar{\rho} + \sigma^2)},
\]

where \(\bar{\rho}\) is defined (6). Since \(a_i = a_j\) for all pairs of traders \(i\) and \(j\), equation (10) implies that the price is a sufficient statistic for the unweighted average of traders' signals, namely, \((1/n) \sum_{k=1}^{n} s_k\). Therefore,

\[
\mathbb{E}[\theta_i|s_1, \ldots, s_n] = \mathbb{E}[\theta_i|s_i, p] = \left( \frac{1 - \bar{\rho}}{1 - \bar{\rho} + \sigma^2} \right) s_i + \frac{\bar{\rho} \sigma^2}{(1 - \bar{\rho} + \sigma^2)(1 + \sigma^2 + \bar{\rho}(n - 1))} \sum_{k=1}^{n} s_k,
\]

where we are using the fact that the equilibrium is fully privately revealing to trader \(i\). Consequently,

\[
\mathbb{E}[\theta_i s_j] = \left( \frac{1 - \bar{\rho}}{1 - \bar{\rho} + \sigma^2} \right) \rho_{ij} + \frac{\bar{\rho} \sigma^2}{(1 - \bar{\rho} + \sigma^2)(1 + \sigma^2 + \bar{\rho}(n - 1))} \left( 1 + \sigma^2 + \sum_{k\neq j} \rho_{jk} \right)
\]

for any \(j \neq i\). Replacing the left-hand side of the above equation with \(\rho_{ij}\) and noting that \(\sum_{k\neq j} \rho_{jk} = (n - 1)\bar{\rho}\) implies that the above equality is satisfied for all \(i \neq j\) only if \(\rho_{ij} = \bar{\rho}\) for all pairs of traders \(i \neq j\).

**Proof of part (b)** Recall from the proof of Proposition 1 that equilibrium strategies satisfy equations (7)–(9). Furthermore, recall from the proof of Proposition 4 that the first-order conditions of the planner’s problem with respect to coefficients \(a_i, b_i,\) and \(c_i\) are given by (23), (21), and (22), respectively. As in the proof of Proposition 4, it is immediate to verify that, as long as (8) is satisfied, the right-hand side of (21) is equal to zero, thus implying that equilibrium strategies satisfy the planner’s first-order condition with respect to \(b_i\). Similarly, using (9) to simplify (22) implies that the right-hand side of the latter equation is also equal to zero for all parameter values, which establishes that equilibrium strategies satisfy the planner’s first-order condition with respect to \(c_i\).

Having established \(d\mathbb{E}[W]/db_i = d\mathbb{E}[W]/dc_i = 0\) for all \(i\), it is therefore sufficient to verify that the right-hand side of (23), when evaluated at equilibrium strategies, is equal to zero. The fact that equilibrium strategies satisfy (7) implies that

\[
\frac{d\mathbb{E}[W]}{da_i} \bigg|_{eq} = \frac{\beta}{1 + \beta \sum_{j=1}^{n} c_j \sum_{k\neq i} c_k \left( \rho_{ik}(\lambda a_k - 1) + (1 - \lambda(1 + \sigma^2) a_k) \frac{\mathbb{E}[s_i|p]}{\mathbb{E}[s_k|p]} \right)},
\]

where we are using the fact that all traders have identical trading costs and signal precisions. Plugging for equilibrium actions from (31) and noting that equilibrium strategies are symmetric lead to

\[
\frac{d\mathbb{E}[W]}{da_i} \bigg|_{eq} = \frac{\beta}{1 + n\beta c \bar{\rho} + \sigma^2} \sum_{k\neq i} \left( \bar{\rho} \frac{1 + \sigma^2 + \sum_{j\neq i} \rho_{ij}}{1 + \sigma^2 + \sum_{j\neq i} \rho_{jk}} - \rho_{ik} \right) = \frac{\beta}{1 + n\beta c \bar{\rho} + \sigma^2} \sum_{k\neq i} (\bar{\rho} - \rho_{ik}).
\]

The definition of \(\bar{\rho}\) in (6) now guarantees that the right-hand side of the above equality is equal to zero, thus completing the proof.
References


