Dairy cooperatives bestow upon farmers the virtues of controlling a large supply quantity of a relatively scarce product, i.e., milk (e.g., higher bargaining power, fixed cost sharing, etc.). However, the cooperative structure also encourages individual farmers to free-ride (i.e., provide low-quality milk in the hope that other farmers provide good-quality milk). The question arises: How can the benefits of a cooperative be retained, while eliminating free-riding, especially when individual inspection costs are high? We examine this question from the perspective of a social planner, who wishes to achieve the simultaneous goals of quantity efficiency, quality efficiency, and minimal testing. The basic challenge in this quest is that of countering an endogenous value function associated with a coalition of farmers. This value function emerges from the joint interaction of three forces: (1) an individual farmer’s payoff-maximizing behavior, (2) the testing policies employed by the cooperative, and (3) the manner in which the value earned by a coalition is shared among the farmers (allocation rule). A novel allocation rule – that exploits individual incentives to guide the collective behavior of the farmers and thereby the value-function endogeneity – is proposed that achieves the planner’s goals in the presence of these forces. A modification to this allocation rule is made to address the goals of practicality, e.g., the presence of low-income farmers and unintended variation in the quality of a farmer’s output. We examine our interventions in the light of sample data from actual dairy cooperatives to demonstrate the viability of our proposals.

Key words: cooperative games; dairy cooperatives; allocation rules; quality improvement; testing

1. Introduction

The human consumption of milk (and related dairy products) has strengthened in the last few decades, with both the demand for milk and its price increasing substantially. In many developing countries, however, the production of milk continues to be a cottage industry – milk is produced in relatively small quantities by small farmers and is sold directly or via collection intermediaries to large urban dairies. A significant fraction of the milk produced in the developing world occurs under a cooperative structure, where the farmers (or milk producers) are also the owners of the cooperative. For example, in India – the world’s largest milk producer – dairy cooperatives serve more than 10 million farmers in over 80,000 villages (The Indian Dairy 2016). Reports of successful dairy cooperatives abound: Two prominent examples are the Amul milk cooperative structure in India, which brings together approximately 3.6 million small farmers from thousands of village-level cooperatives (http://www.amul.com) and New Zealand’s Fonterra, the world’s largest exporter of dairy products (http://www.fonterra.com).
In developing countries in Asia and the Pacific Region, the service dairy cooperative has provided an impetus for improving the economic condition of the rural population. Cooperatives assist farmers in planning, decision-making, and implementing schemes that aim to improve their economic standards. Being an extremely perishable commodity, milk requires special handling and care, that is provided by the collective operations of the dairy cooperative. Apart from the collection, distribution and marketing of milk, other services, such as dairy inputs, veterinary services, provision of animal feed, fodder seed, planting material, fertilizers and credit, and training and education, are also provided through cooperatives.

The most appealing aspect of a service cooperative is that it helps low-income farmers obtain a fair (market) value for their produce, rather than have collection and distribution intermediaries reap the surplus. A large cooperative transfers bargaining power to farmers, as they command control over a large production quantity (Levins 2002, Schwettmann 2014, Yoo 2015). There are also the obvious advantages of fixed cost sharing, e.g., testing facilities, storage units, transportation vehicles, etc.

Despite these benefits, smallholder agricultural cooperatives face several operational challenges. Given the small production amounts from individual members, it becomes necessary for the output from a large number of farmers to be mixed prior to marketing the produce. This generates incentives for producers to free-ride on the quality of others. Thus, as the size of a smallholder cooperative increases, an individual farmer’s incentive to produce low quality output also increases. That is, holding the level of testing to be constant, the free-riding effect can be expected to exacerbate with size. For instance, in a recent empirical investigation of a smallholder dairy cooperative, Breza et al. (2016) document the potential (unintended) perverse effects associated with infrastructure investment (e.g., a large chiller) that allows storing large quantities of milk for transport to the dairy firm. While the cooperative they study experienced an increase in quantity with access to a chiller, the average production quality was found to decrease. This decrease in quality is attributed to the reduction in the likelihood of being punished for low quality.

Free-riding can, of course, be kept in check by testing more, e.g., at one extreme, free-riding can be completed eliminated by testing every farmer’s output (for quality). However, excessive individual testing of agricultural produce is clearly expensive for smallholder cooperatives and is also socially inefficient. To achieve the virtues of a large-sized cooperative, one could pay in terms of low quality, or alternatively pay in terms of costly testing. Thus, forming a stable smallholder
cooperative of a large size that produces high-quality produce and yet tests minimally is the gold standard to strive for.

An effective way to address free-riding is through the use of a suitable allocation rule, namely, the manner in which value gets divided within a coalition. In a typical cooperative game, a known value-function is used to determine the value associated with any coalition (formed by a particular subset of agents). On the other hand, the value of a coalition in a dairy cooperative depends not only on the set of agents (farmers) that form the coalition, but also on the individual quality decisions made by each farmer within the coalition. Clearly, to make their quality decisions, individual farmers must know the share of the value they expect to receive as a result of their decisions (i.e., they must know the allocation rule). Thus, the allocation rule together with individual quality decisions determine the coalition value. However, choosing a good allocation rule requires knowledge of the value-function. This allocation rule, value-function circularity, poses a significant obstacle in designing an effective allocation rule. Interestingly, this very feature allows us to develop a novel allocation rule that exploits the endogenous relationship between the allocation rule and the value-function.

Determining a good allocation rule also faces several operational challenges that are unique to the dairy cooperative setting. First, dairy cooperatives usually operate with a large number of low-income farmers that must be paid frequently. As we see later, a delayed payment scheme can help solve some operational issues (specifically, the issue of quality uncertainty), but its use is limited by the presence of low-income farmers that cannot survive very long without being paid. Second, a good allocation scheme should allow farmers to challenge their payment if they believe that they did not get a fair price for their milk. Third, the cooperative itself must remain operationally solvent, that is, it should not run for too long (or not at all) in a deficit.

We now discuss the literature that is most relevant to our work, and also distinguish our relative contribution.

2. Literature Review

In a dairy cooperative setting, there are three key forces of interest: (1) individual goals, (2) testing policies and quality improvement, and (3) allocation rules. The third force, of course, is missing in a supply chain where the milk-collection intermediary is a separate profit-maximizing entity. For a dairy cooperative, the manner in which the surplus is allocated influences both the coalition outcome (e.g., which coalitions will be formed) and the quality decisions from the farmers. Our
brief review of the literature will revolve around these three forces. In addition, we comment on how these forces could interact in a dairy cooperative.

Fundamentally speaking, the problem of managing the output quality of a dairy cooperative arises because the individual outputs of farmers are perfectly miscible. Once the milk from different farmers is mixed, it is impossible to trace back the individual quality supplied by a farmer, unless each farmer’s output is tested prior to mixing. This is different from a typical assembled product, where the failure of the final assembled product can often be traced back to a particular component (Bernstein and DeCroix 2006 and Zhang 2006). In a broader context, the lack of traceability in a milk cooperative is similar to “interdependent security” issues in the literature – where the overall level of security depends jointly on the security investments by individual stakeholders. When the social-security level is low, it is hard to identify which individual has used low investment effort; see, e.g., Kunreuther and Heal (2003) and Baiman et al. (2004). In the absence of traceability, individual agents are encouraged to cheat and must be provided with incentives (and/or penalties) to curb this opportunism.

Mu et al. (2014, 2016) address quality improvement in the milk supply chain under a non-cooperative setting, where farmers sell their milk individually to privately-owned intermediaries. Specifically, Mu et al. (2016) exploit competition between the intermediaries to obtain a scheme, which is based on the testing-standard differential employed by these intermediaries, that improves the quality of milk. Our paper focuses on a fundamentally different and more-popular milk-supply setting that occurs under a cooperative structure, and studies cooperative-related issues such as (a) formation of farmer coalitions, (b) allocation of the total profit earned by a coalition to its members and (c) countering cheating incentives using an allocation rule. Further, the cooperative game in our study is played under a stochastic setting – it is played before the quality produced by each farmer is realized. Thus, quality uncertainty poses a challenge for any allocation rule designed to work in practice. Neither quality uncertainty nor the associated challenge in designing an allocation rule was considered in Mu et al. (2014, 2016).

The larger procurement literature examines a variety of ideas to improve supplier quality under a non-cooperative setting, such as warranty contracts (e.g., Balachandran and Radhakrishnan 2005), deferred payments (e.g., Babich and Tang 2012), shared recall costs (e.g., Chao et al. 2009), and certification programs (e.g., Hwang et al. 2006). These studies typically consider suppliers with private quality information. Once again, unlike in a non-cooperative setting, the added
complexity in a cooperative environment is the need to develop allocation rules that influence the agents to not only make the desired quality decisions but also to form desirable coalitions, e.g., the grand coalition.

There are several studies on the applications of cooperative game theory in the Operations Management area. The benefits of forming coalitions are numerous – sharing development and operating costs (Granot and Sošić 2005), improving bargaining power (Nagarajan and Bassok 2008), coordinating production schedules at a third party (Aydinliyim and Vairaktarakis 2010, Cai and Vairaktarakis 2014), demand pooling (Nagarajan and Sošić 2007), and inventory pooling (Kemahlioglu-Ziya and Bartholdi 2010). A common feature of the above studies is that the value function associated with any subset of agents is exogenously known. In a dairy cooperative setting, the presence of an exogenously-known value function is not tenable because farmers can freely choose their output quality in response to the allocation rule and in accordance with their individual goals. Thus, the value function in a dairy cooperative needs to depend endogenously on the individual quality decisions of each farmer within a coalition.

It should be clear from the above discussion that the three forces – individual goals, testing policies, and allocation rules – interact with one another to determine what coalition(s) will be formed, as well as the output quality of a coalition. Specifically, the payoff-maximizing behavior of individual agents is influenced by both the testing policy and the allocation rule; therefore, whether or not a particular coalition will be formed and its output quality depends on the interaction of the three forces identified above. Such a subtle interaction is uncommon in a cooperative setting, but the practical operations of a real-world dairy cooperative indeed make this interaction a real issue.

3. Study of Current Practice

In this section, we examine some well-documented operating schemes used in existing dairy cooperatives to see why these schemes could fail to deliver the ideal outcomes of large size, high quality, and minimal testing.

We draw our evidence from KMF (Karnataka Milk Federation), a large dairy cooperative in Southern India that was examined in Breza et al. (2016). In this cooperative, the prices for milk set in a village increase in the quality of the mixed output from the village. Within a village, a farmer is paid directly for quantity. However, unless properly designed, village-level quality incentives have limited power. As we show later, free-ridding incentives are minimized only for very small
sized cooperatives (say, less than about 5 farmers), where the combined milk is of high quality. However, even small villages typically have about 100-200 milk farmers. In addition, farmers may receive year-end bonuses if the average village quality is high. In the KMF cooperative, year-end bonuses are paid at the individual level, but they are based on the quantity of milk supplied by a farmer. Such a bonus scheme is unlikely to foster high quality. In the Amul cooperative, one of the world’s largest, farmers are paid based on a relatively quick, but incomplete test of quality; specifically, only the fat content is tested (Ajwani 2015). Because the true quality of milk is dependent on several other factors (e.g., protein, solids-not-fat, etc.) that are harder to measure, the incomplete test does not correctly reward a farmer for true quality. As a result, there is both the potential for free-riding and adulteration\(^1\) (Sayyed 2014).

We also find evidence of weak quality incentives at dairy cooperatives in other parts of the world. For example, in the cooperatives of Southern Brazil, coarse quality incentives (applied to improve the average quality of bulk, mixed milk) did not result in the desired increase in average milk quality (Botaro et al. 2013). On the other hand, EKO Žemaitija in Lithuania (Skulskis and Girgždienė 2015) compensates farmers solely based on the quantity of milk supplied, while Royal FrieslandCampina N.V. in the Netherlands (Royal FrieslandCampina N.V. 2016) and Agri-Mark in the United States (DiMento 2012) pay farmers primarily based on quantity with a minor quality component.

In light of weak quality incentives, it is not unexpected to find that milk produced under a cooperative structure indeed suffers from quality problems (Khanna and Pandey 2013, Sayyed 2014, Porecha 2015). Furthermore, quality problems can often be traced back to the milk farmers, see, e.g., The Hindu (2013) [India], Omore et al. (2005) [Kenya], Khan (2008) [Pakistan], Souza et al. (2011) [Brazil], Kasemsumran et al. (2007) [Thailand].

The quality problems discussed above are linked to incomplete (not all dimensions of quality are tested) or inadequate (the output of a group, rather than an individual is tested) testing. If it were viable to test fully (comprehensively and at the individual level), perverse incentives to cheat on quality are likely to be completely eliminated. However, from the cooperative’s perspective, full (comprehensive) testing of each farmer’s milk is costly. It is also wasteful or inefficient from a social perspective.

\(^1\) Our study focuses on the non-poisonous contamination of milk, where quality can be controlled in a continuous manner by deciding the amount of an adulterant; common examples include water, vegetable oils, and starch. The case of poisonous adulterants – the addition of even a small amount of which can render the entire milk poisonous – requires an entirely different kind of analysis and is beyond the scope of this study.
3.1. Analysis of Current Practice

To precisely analyze the quality effects of the cooperative arrangements used in practice, we present some basic notation and a setting that will also be used to develop our proposal in the next section.

3.1.1. Basic Notation and Setting

Let \( \mathcal{N} \) be the set of all the farmers. We begin by recalling some basic terminology from the theory of cooperative games. A subset \( S, S \subseteq \mathcal{N} \), is referred to as a coalition and the set \( \mathcal{N} \) itself as the grand coalition. The characteristic function \( v(S) \) of a coalition \( S \) is defined as the total profit received by that coalition and depends on the endogenous quality decisions of its individual farmers. An allocation rule specifies the allocation of the total profit generated by a coalition to each individual farmer in that coalition. Our study considers only efficient allocations, in which the profit from a coalition is fully allocated to its members.

We now discuss our setting. On the supply side, a fixed-quantity contract, where a farmer agrees to supply a fixed quantity daily to the firm, is common in milk procurement in developing countries (Birthal et al. 2005, Kumar et al. 2011, Kolekar et al. 2012, Birthal 2014). Accordingly, we assume that a farmer supplies a fixed quantity of milk\(^2\). Milk farmers form coalitions and then choose their individual target qualities. Each coalition mixes the milk from all the farmers in that coalition and then sells it to a dairy firm. Figure 1 illustrates this process.

![Figure 1](image)

**Figure 1** Milk supply chain under a cooperative structure.

In practice, different tests are available to detect the quality of milk. Some simple tests (e.g., the Gerber test that measures only the fat content, but cannot distinguish between natural

\(^2\)While a fixed quantity is assumed, a farmer can adjust the quality of his milk by changing the level of dilution (i.e., the amount of pure milk); quantity heterogeneity across farmers is allowed.
and artificial fat) are relatively cheap. This provides an incentive for farmers to dilute the milk and add artificial fattening agents to improve the (fake) percentage of fat. On the other hand, performing comprehensive testing (e.g., the UN-recommended Resazurin test) is expensive (Food and Agriculture Organization of the UN 2009). The “test” in this paper refers to a comprehensive one, which accurately detects the true quality of the milk.

The dairy firm offers a common unit-price formula to all coalitions and pays each coalition accordingly after testing the mixed milk from that coalition. Larger coalitions naturally enjoy higher bargaining power, which normally leads to a better price for the coalition (Nagarajan and Bassok 2008, Moran 2009, Kumar et al. 2015, Mshangama and Ali 2016). A quantity premium – a unit price that increases in the quantity of milk supply – is widely used in dairy practice; see, e.g., Stonyfield Inc. (2017), Micobel cvba (2016), Pirisi et al. (2007), Food and Agriculture Organization of the UN (2001). We model this effect by assuming a unit price that increases in both the quality and the quantity of milk. Let the per-unit price offered by the dairy firm be

\[ p(q, Q) = p_0 + p_1 q + p_2 Q, \quad p_1 > 0, \ p_2 > 0, \]

when \( Q \) units milk with quality \( q \) is supplied by a coalition. Here, we assume that the unit price of milk received by any coalition of farmers (from the dairy firm) is a linear function of the quality; see Food and Agriculture Organization of the UN (2013a) and Foreman and Leeuw (2013) for real-world examples of quality-based linear pricing of milk. We further assume that the production cost is a linear function of the quality. Let the per-unit production cost incurred by a farmer be

\[ c(q) = c_0 + c_1 q, \]

when a target quality \( q \) is applied. The most common form of quality degradation is by diluting milk with water (and other ingredients, which help the diluted milk look like pure milk). As more water is added, the quality of milk (i.e., the percentage of nutrition in the milk) roughly decreases in a linear manner and so does its unit production cost (after the dilution process). The robustness of our main findings under nonlinear production costs is verified in Section F of the online appendix. We consider the non-trivial case where supplying high-quality milk is more profitable than supplying low-quality milk, if the true quality is detected. Mathematically, this assumption states that

\[ p(q, Q) - c(q) \]

increases in quality \( q \) for any quantity \( Q \).

We now explain the notation that will be used throughout our analysis. Let \( |N| = n \) and let \( Q_i \) be the quantity of milk supplied by farmer \( i, \ i \in N \). A farmer first makes a decision on whether or not to form a coalition, and with whom. Then, the farmer chooses a target quality. Note that even after the target quality is chosen, the realized quality of the farmer’s milk may vary due to (unintended) quality uncertainty. Let \( \tilde{q}_i \) denote the realized quality of milk when a target
quality \(q_i\) is applied (by farmer \(i\)): we assume\(^3\) that \(\tilde{q}_i = q_i + \epsilon\) with probability \(\frac{1}{2}\) and \(\tilde{q}_i = q_i - \epsilon\) with probability \(\frac{1}{2}\). During the milk collection process, before any potential formal testing is conducted, the milk from each individual farmer undergoes a simple visual inspection or smell test upon arrival at the cooperative. Milk that fails to pass this simple inspection is immediately rejected by the cooperative. Let \(q_L\) denote the minimum quality required by this initial examination. Consequently, under quality uncertainty, the lower bound\(^4\) on the target quality one can apply to avoid rejection with certainty is \(q_L + \epsilon\) (in which case, the realized quality is in \([q_L, q_L + 2\epsilon]\)). Let \(q_H\) denote the achievable upper bound on the target quality. We therefore assume that farmer \(i, i \in \mathcal{N}\), chooses a target quality \(q_i \in [q_L + \epsilon, q_H]\). Based on observations from practice, one can comfortably assume that the quality variation \(\epsilon\) due to uncertainty is reasonably small (Tamime 2009). We therefore assume that the realized quality when the high target quality \(q_H\) is applied (in which case, the realized quality is in \([q_H - \epsilon, q_H + \epsilon]\)) is higher than that when the low target quality \(q_L + \epsilon\) is applied (in which case, the realized quality is in \([q_L, q_L + 2\epsilon]\)). Mathematically, this assumption states that \(q_L + 2\epsilon < q_H - \epsilon\), or equivalently, \(q_H - q_L > 3\epsilon\). The validity of our main results when this assumption is not satisfied is discussed in Section G of the online appendix.

Figure 2 shows the sequence of events. The basic notation used in our analysis is summarized below.

**Notation**

\begin{align*}
\text{n} & \quad \text{Number of farmers.} \\
\mathcal{N} & \quad \text{Set of all the farmers.} \\
\mathcal{M} & \quad \text{An arbitrary coalition of the farmers, } \mathcal{M} \subseteq \mathcal{N}. \\
Q_i & \quad \text{Quantity of milk supplied by farmer } i, \ i \in \mathcal{N}. \\
Q_M & \quad \text{Quantity of milk supplied by coalition } \mathcal{M}, \ \mathcal{M} \subseteq \mathcal{N}. \\
Q_{-i} & \quad \text{Quantity of milk supplied by all the farmers in a coalition, other than farmer } i. \\
q_i & \quad \text{Target quality of milk applied by farmer } i, \ i \in \mathcal{N} \ (\text{decision variable}). \\
\tilde{q}_i & \quad \text{Realization of milk quality supplied by farmer } i \text{ when target quality } q_i \text{ has been applied.} \\
\tilde{q}_M & \quad \text{Realization of milk quality supplied by coalition } \mathcal{M}, \ \mathcal{M} \subseteq \mathcal{N}. \\
q_H & \quad \text{Upper bound on the target quality } q. \\
q_L & \quad \text{Lower bound on the quality of milk that is accepted by the dairy firm.} \\
c(q) & \quad \text{Unit production cost incurred by a farmer as a function of target quality } q. \\
p(q, Q) & \quad \text{Unit price offered by the dairy firm as a function of quality } q \text{ and quantity } Q.
\end{align*}

\(^3\)This simple distribution for quality uncertainty is chosen here for simplicity of exposition. A more-general distribution can be used without affecting the nature of our results and insights.

\(^4\)If the farmer reduces his target quality below \(q_L + \epsilon\), then the quality of his milk may fall below the acceptable quality \(q_H\), in which case it will be immediately rejected. Note that the specific value of this lower bound on quality is not the driver of our results.
Having set up the basic notation necessary for the analysis, we next examine two, somewhat extreme, situations that highlight either quality or testing problems in current cooperative settings. First, we consider the case where testing is free. Next, we analyze the situation where testing is not performed at all, presumably because it is too costly.

### 3.1.2. Shapley Values in an Ideal World

Imagine that comprehensive testing is free. Then, it is easy to ensure high-quality milk. For instance, under the well-known Shapley-value allocation (Roth 1988), for any coalition, a farmer receives a payoff based on his average marginal contribution over all possible subsets of that coalition. As shown below in Theorem 1, when testing is free, a Shapley-value-based allocation rule ensures the following properties in our context: (1) the grand coalition will be formed, and (2) the mixed milk will be of high quality.

**Theorem 1.** Under the Shapley-value allocation, the grand coalition is formed. All the farmers apply the high target quality \( q_H \). The expected profit received by farmer \( i \) is \( p(q_H, Q_N)Q_i - c(q_H)Q_i \).

However, the calculation of Shapley values becomes impractical in our context, as we now explain. For the Shapley-value allocation, we need the realized quality of milk from each possible subset of farmers. This would require a substantial amount of testing (e.g., testing each farmer), which is unrealistic in practice, given the large number of farmers and the significant testing cost. Moreover, to apply the Shapley-value allocation for a coalition, one has to calculate the marginal contribution of a farmer to each possible subset in that coalition: If there are \( n \) farmers, then we need the marginal contribution of a particular farmer over \( 2^{n-1} \) possible subsets. This can also be cumbersome in practice.

### 3.1.3. Quality under No Testing

The other extreme (with regard to output quality) occurs when no individual testing is conducted and the unit price a farmer receives corresponds to the quality of the mixed milk supplied by that farmer’s coalition. We refer to this as the
naive allocation rule. As discussed in the beginning of Section 3, this naive rule is widely used in practice. Mathematically, the payment to farmer $i$ in coalition $\mathcal{M}$ is $p(\tilde{q}_M, Q_M)Q_i$, where $Q_i$ is the quantity of milk supplied by farmer $i$ and $p(\tilde{q}_M, Q_M)$ is the unit price received by coalition $\mathcal{M}$ based on the average quality $\tilde{q}_M$.

Farmer $i$ first forms a coalition and then chooses a target quality $q_i$ ($q_L + \epsilon \leq q_i \leq q_H$). Consider a coalition $\mathcal{M}$, $\mathcal{M} \subseteq \mathcal{N}$. We now calculate the expected profit received by farmer $i$, $i \in \mathcal{M}$. Let $Q_{-i}$ (resp., $\tilde{q}_{-i}$) be the total quantity (resp., the realized quality) of milk supplied by all the farmers other than farmer $i$ in coalition $\mathcal{M}$. Recall that $q_i$ (resp., $\tilde{q}_i$) is the target quality (resp., realized quality) supplied by farmer $i$. We have $E[\tilde{q}_i] = q_i$ and $\tilde{q}_M = \frac{\tilde{q}_i Q_i + \tilde{q}_{-i} Q_{-i}}{Q_M}$. Given the realization of milk quality $\tilde{q}_{-i}$ supplied by all the farmers other than farmer $i$ in coalition $\mathcal{M}$, denote $\pi_i(q_i)$ as the expected profit when farmer $i$ applies target quality $q_i$. We have

$$\pi_i(q_i) = E_{\tilde{q}_i}\left[p(\tilde{q}_M, Q_M) - c(q_i)\right]Q_i$$

$$= E_{\tilde{q}_i}\left[p(\tilde{q}_i Q_i + \tilde{q}_{-i} Q_{-i}, Q_M) - c(q_i)\right]Q_i$$

$$= \left\{ p(\tilde{q}_i Q_i + \tilde{q}_{-i} Q_{-i}, Q_M) - c(q_i) \right\} Q_i. \quad (1)$$

The result below shows that quality suffers under the above allocation rule.

**Theorem 2.** Under the naive allocation rule, when the number of farmers in a coalition is not extremely small\(^5\), the unique equilibrium is as follows: The grand coalition is formed. All the farmers apply the low target quality $q_L + \epsilon$. The expected profit received by farmer $i$ is $p(q_L + \epsilon, Q_N)Q_i - c(q_L + \epsilon)Q_i$.

We now explain the intuition behind this result. Under the naive allocation rule, a farmer’s payment is based on the quality of the mixed milk from all the farmers in a coalition. When a farmer supplies low quality milk, his payment is only affected slightly (since a change in the quality supplied by an individual farmer does not affect the mixed quality much), but the cost incurred by this farmer is substantially reduced (i.e., the farmer takes full advantage of the cost reduction). This free-riding phenomenon provides an incentive for the farmers to supply low quality. Recall that the lower-bound on the target quality for a farmer to avoid being directly rejected (by the initial simple examination) is $q_L + \epsilon$. Accordingly, all farmers apply the low target

\(^5\)The detailed expression of this condition is specified in Section A of the online appendix. This assumption is not restrictive; e.g., in the realistic case when all farmers supply similar quantities of milk, this assumption simply requires that a coalition have at least two farmers to be able to sell directly to the dairy firm. The equilibrium when this assumption is relaxed also conveys a similar message, and is discussed in the online appendix.
quality $q_L + \epsilon$ in equilibrium. Given that all the farmers supply low-quality milk, the profit of a farmer increases in the size of the coalition (i.e., the quantity of milk supplied by the coalition). As a result, the grand coalition $\mathcal{N}$ is formed.

**Remark 1:** To understand current practice, this section studied two somewhat extreme cases: (1) testing is free (Section 3.1.2); thus, the individual quality of each farmer can be detected and high quality is easily achieved in equilibrium, and (2) testing is costly and, therefore, no individual testing is conducted (Section 3.1.3); here, low quality is supplied by the farmers under the natural naive allocation rule. The model can be generalized by incorporating an exogenous testing cost, and it can be shown that when the testing cost is high (resp., low), no (resp., high enough) individual testing is conducted, and each farmer applies the low (resp., high) target quality in equilibrium. Thus, we have either quality or testing inefficiencies in current cooperative settings.

Next, we propose a novel allocation scheme, which eliminates the inefficiencies in both quality and testing stated above.

### 4. A Novel Allocation Rule

As mentioned earlier, our goal is to achieve both quality efficiency and quantity efficiency via minimal testing. From an implementation viewpoint, we also need a scheme that (1) guarantees a lower bound on farmers’ payment, with a reasonably small variation under quality uncertainty and (2) keeps the cooperative solvent, i.e., balanced in terms of receipts and payments. Section 4.1 proposes a basic and novel allocation scheme that achieves the above theoretical goal. A refined scheme which also incorporates several practical considerations, including the two above, is discussed later in Section 4.2.

#### 4.1. A Basic Scheme

Consider the following basic allocation scheme:

In any coalition $\mathcal{M}$, $\mathcal{M} \subseteq \mathcal{N}$, the payoff (i.e., allocation) to farmer $i \ (i \in \mathcal{M})$ is

$$\tilde{r}_i = p(\tilde{q}_M, Q_M)Q_\mathcal{M} - p(q_H, Q_M)Q_{-i}. \quad (2)$$

In words, the payoff to farmer $i$ in a coalition equals the (realization of the) total payoff received by the coalition minus the expected payoff generated by all the farmers other than farmer $i$, assuming that all the other farmers have chosen the high target quality $q_H$. 

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Remark 2: At first glance, it may appear that the allocation in (2) is derived in the spirit of a VCG (Vickrey–Clarke–Groves) payment; see, e.g., Vickrey (1961), Clarke (1971), and Groves (1973). Under (2), the payment to a farmer depends on the value earned by the rest of the coalition under the assumption that the rest of the coalition supplies high-quality milk. We note, however, that (2) is quite different from a VCG payment. First, unlike a VCG payment, observe that both the quality and quantity of a farmer’s milk affect his payment under (2). More importantly, the goal here is not to elicit the true value of privately-held information; rather the payment in (2) influences a farmer to choose a desired level of quality. In contrast, information in a VCG setting is exogenous but privately held and the VCG payment induces the agent to truthfully reveal this information. In our problem, the goal is not to extract exogenous private information, but to induce the agent to choose an action that is optimal from the planner’s perspective.

We now derive the equilibrium outcome under the basic scheme. A farmer first decides with whom to form a coalition and then chooses a target quality \( q_i \), \( q_i \in [q_L + \epsilon, q_H] \). Consider a coalition \( \mathcal{M}, \mathcal{M} \subseteq \mathcal{N} \). Using (2) and \( \tilde{q}_M = \frac{\tilde{q}_i Q_i + \tilde{q}_{-i} Q_{-i}}{Q_M} \), given the realized quality \( \tilde{q}_{-i} \) of the milk from the other farmers in coalition \( \mathcal{M} \), the expected profit of farmer \( i \) from applying target quality \( q_i \) is

\[
\pi_i(q_i) = \mathbb{E}[\tilde{q}_i \tilde{r}_i - c(q_i)Q_i] = p\left(\frac{q_i Q_i + \tilde{q}_{-i} Q_{-i}}{Q_M}\right)Q_M - p(q_H, Q_M)Q_{-i} - c(q_i)Q_i = \left\{p_0 Q_M + p_1 \tilde{q}_{-i} Q_{-i} + p_2 Q_M^2 - p(q_H, Q_M)Q_{-i}\right\} - c_0 Q_i + (p_1 - c_1) Q_i q_i. \tag{3}
\]

We use the well-known notion of core (see, e.g., Driessen 1988) as a measure of the stability of the grand coalition. An allocation rule belongs to the core if the sum of the expected profits (under that allocation) of the farmers in any sub-coalition \( S, S \subset \mathcal{N} \), is at least as much as the amount they could earn on their own. Under a core allocation, no subset of farmers has an incentive to deviate from the grand coalition. The following result provides the equilibrium outcome under the basic scheme. The proof is provided in Section B of the online appendix.

**Theorem 3.** Under the basic scheme, the unique equilibrium is as follows: The grand coalition \( \mathcal{N} \) is formed, all the farmers apply the high target quality \( q_H \), and no individual testing is conducted. The expected profit received by farmer \( i \) (\( i \in \mathcal{N} \)) is \( p(q_H, Q_N)Q_i - c(q_H)Q_i \). Further, the basic scheme belongs to the core, i.e., the grand coalition is stable.

Note that the basic scheme is completely defined by the allocation rule (2) above and, therefore, no testing is needed by the scheme. We now explain the intuition behind the result in Theorem 3.
Under the basic allocation scheme, the payoff to a farmer in a coalition equals the payoff received by that coalition minus a constant. Therefore, if a farmer supplies low-quality milk, then he needs to accept full responsibility for the resulting penalty (i.e., low price). This is different from the naive allocation rule stated in Section 3.1.3 under which, if a farmer supplies low quality, then all the farmers in a coalition share the penalty together (thus inducing free-riding). Consequently, the allocation under the basic scheme directly eliminates the free-riding incentive and perfectly aligns the goal of each individual farmer with that of the coalition. Recall from Section 3.1.1 that, throughout our analysis, we consider the non-trivial case where supplying high-quality milk is more profitable than supplying low-quality milk, if the true quality is detected. Thus, if the coalition were able to select its production quality, its best response would be to supply high-quality milk. Since the goal of each individual farmer is aligned with that of the coalition under the basic allocation scheme, each farmer supplies high-quality milk in equilibrium. The constant (mentioned above) deducted from a farmer’s payoff is chosen carefully such that, in equilibrium, each farmer receives a fair payoff that reflects the true expected quality of his milk.

We now understand the formation of the grand coalition under the basic scheme. Since the price offered by the dairy firm is an increasing function of both quality and quantity, the highest possible benefit the farmers can get is when (i) the high target quality $q_H$ is applied by each farmer and (ii) the grand coalition $\mathcal{N}$ is formed. From Theorem 3 above, the equilibrium outcome under the basic scheme satisfies these two conditions simultaneously. As a result, the maximum payoff is achieved in equilibrium under the basic scheme. If any subset of farmers deviates from the grand coalition, then either quantity or quality (or both) suffers, leading to a lower payoff. Therefore, in equilibrium, no farmer deviates from the grand coalition $\mathcal{N}$.

**Remark 3:** From Theorems 1 and 3, the equilibrium expected payoff of each farmer under the above basic scheme is the same as that under the Shapley-value allocation. Accordingly, the basic scheme enjoys the nice properties of Shapley values, such as (i) *efficiency* – the payoff generated by a coalition is fully allocated to its members, and (ii) *symmetry* – identical players receive identical allocations. From the discussion in Section 3.1.2, implementing Shapley value requires a substantial amount of testing and cumbersome calculations, which is not realistic in our context. However, allocation (2) under the basic scheme requires only the payoff received by a coalition (which is known to the coalition) and, therefore, is much easier to implement.
Under the basic scheme, farmers receive a fair payment for their milk *in expectation* – a farmer receives a high expected payment for applying the high target quality. In a given period, however, the specific *realized* payment in that period could be low due to quality uncertainty. Below, we illustrate this using a simple example.

**Example 1:** Consider the following data on the real-world procurement of milk at village-level cooperatives in India (Shah 2012; Food and Agriculture Organization of the UN 2013b): $Q_i = 10$ liters $\forall i \in N$, $p(q_{H_i}, Q_N) = $0.41/liter, $p_1 = $3.09/liter, $p_2 = $0.017/liter, $n = 100$, and $q_H = 6\%$. It can be shown that, (i) the *expected* payment to a farmer under the basic allocation scheme is $4.1$, which is a fair payment corresponding to the high-quality unit price $0.41$/liter and the supply quantity $Q_i = 10$ liters, and (ii) when quality uncertainty is $\epsilon = 10\%q_H$ (resp., $\epsilon = 5\%q_H$), there is a probability of $1.36\%$ (resp., $0\%$) that the *realized* payment to a farmer under the basic scheme is less than or equal to zero. The calculation is provided in Section C of the online appendix.

If the quality variation $\epsilon$ is small, then the adverse impact on the payment is also small and, therefore, not a major concern. Broadly, one would expect this to be the case under prosperous conditions – healthy animals, access to good veterinary services, automated milking units, etc. Such conditions are likely to exist when the farmers themselves are well-to-do. In contrast, economically-strained farmers and conditions would likely indicate an inescapable level of quality uncertainty. In such an environment, farmers are likely to depend heavily on the payment from selling milk for their daily sustenance and, therefore, a low payment (e.g., a negative payment) in a given period could be a concern. Clearly, it is hard to apply a negative payment in practice.

To address this challenge, we now develop an enhanced scheme which

(a) guarantees a lower bound on the payment to each farmer, and

(b) offers the farmers who have received a low payment a chance to challenge that payment.

These two interventions require the cooperative to pay more to the farmers and therefore leave it in a deficit. We then propose a third intervention – imposition of an admission fee – to keep the cooperative solvent. Let us denote this enhanced scheme as the *refined scheme*. As mentioned above, this scheme is suitable for use under the less-than-prosperous economic environment that is typical in developing countries.

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6 For brevity, we only provide the values we use in our assessment.
4.2. Two Practically-Motivated Enhancements: The Refined Scheme

This subsection is organized as follows. We discuss the refined scheme in detail and derive the corresponding equilibrium in Section 4.2.1. We will see that this scheme not only tackles the practical challenges stated above, but also retains the main properties of the basic scheme, namely high-quality milk and the formation of the grand coalition. However, on the flip side, some testing is required under the refined scheme due to the imposition of the lower bound on payment – in Section 4.2.2, we will see that, for realistic data, the amount of testing needed in equilibrium is quite small. Nevertheless, to further reduce testing, we discuss another refinement (namely, delayed payments) in Section 4.2.3 under which testing is close to zero for realistic data.

4.2.1. Equilibrium under the Refined Scheme

We first formally state the refined scheme and then derive the equilibrium outcome.

The Refined Scheme: The main components of the scheme are as follows:

- **Lower Bound on Payment:** Consider a coalition \( \mathcal{M} \subseteq \mathcal{N} \). Let \( L \) denote a guaranteed minimum per-unit payment to the farmers; thus, \( LQ_i \) is the guaranteed minimum payment to farmer \( i \) \((i \in \mathcal{M})\). Recall that \( \tilde{r}_i \) is the allocation to farmer \( i \) under the basic scheme. Let \( \tilde{\theta}_i \) denote the allocation to farmer \( i \) in the presence of this lower-bound guarantee. From (2), we have

\[
\tilde{\theta}_i = \begin{cases} 
\tilde{r}_i, & \text{if } \tilde{r}_i \geq LQ_i, \\
LQ_i, & \text{if } \tilde{r}_i < LQ_i.
\end{cases}
\]  

(4)

- **Challenge Process:** We now allow the farmers who have received the lower-bound payment \( LQ_i \) an opportunity to challenge this payment. If a farmer challenges, then the cooperative tests\(^7\) that farmer’s milk with probability \( \gamma \) (defined below). If farmer \( i \) is not tested, then a payment \( p(q_H - \epsilon, Q_M)Q_i \) corresponding to high quality is paid\(^8\). On the other hand, if farmer \( i \) is tested, then the payment depends on the quality of this farmer’s milk: If the quality is above \( q_H - \epsilon \),

---

\(^7\) When farmers supply milk to a cooperative, a sample of milk is drawn from each individual farmer’s milk before the milk from different farmers is mixed. The individual samples are kept for a certain period of time for the sake of future individual testing (if needed). Therefore, when a farmer challenges and individual testing is triggered, the cooperative can test the aforementioned milk samples to detect individual qualities.

\(^8\) When the high target quality \( q_H \) is applied by a farmer, the realization of the quality could be either \( q_H + \epsilon \) or \( q_H - \epsilon \). Here, the payment is based on quality \( q_H - \epsilon \). Note that the specific value of this payment does not affect the expected payoff in equilibrium: the coalition will choose an appropriate value of the admission fee such that each farmer receives a fair payment that reflects the true quality of the milk he supplies.
then a high-quality payment \( p(q_H - \epsilon, Q_M) \) is paid; if the quality is below \( q_H - \epsilon \), then a low-quality payment \( p(q_L, Q_M) \) is offered and the farmer is charged a penalty \( bQ_i \). We set\(^9\) \( \gamma = \frac{c_1 \max(q_L - q_H - \epsilon, 4\epsilon)}{p_1(q_H - q_L - \epsilon) + b} \); recall from Section 3.1.1 that \( c_1 \) and \( p_1 \) are, respectively, the coefficients in the per-unit production cost incurred by a farmer and the per-unit price offered by the dairy firm.

**Admission Fee:** Motivated by the need to keep the cooperative solvent, farmer \( i \) is required to pay an admission fee \( a_i \) upon joining the coalition; \( i \in M \). The use of admission fees by dairy cooperatives is well-documented in practice; see, e.g., Food and Agriculture Organization of the UN 2014, National Dairy Plan 2016, National Dairy Development Board of India 2016. Let \( \tilde{q}_{HM} \) be the realization of the quality of the mixed milk supplied by coalition \( M \), when all the farmers have applied the high target quality \( q_H \). Let \( f(x) \) be the probability density of the total payoff \( p(\tilde{q}_{HM}, Q_M)Q_M \) received by coalition \( M \). Let \( A = \max \left\{ p(q_H, Q_M)Q_i + LQ_i, p(q_H - \epsilon, Q_M)Q_M \right\} \). Then, we set

\[
a_i = \int_{p(q_H - \epsilon, Q_M)Q_M}^{A} \left\{ p(q_H - \epsilon, Q_M)Q_i + p(q_H, Q_M)Q_{-i} - x \right\} f(x) dx. \tag{5}
\]

We now briefly explain the intuition behind this admission fee. Recall from Section 4.1 that, under the basic scheme, all farmers apply the high target quality \( q_H \) and the expected equilibrium payment received by farmer \( i \) (\( i \in M \)) is \( p(q_H, Q_M)Q_i \), which is a fair payment for an applied target quality of \( q_H \). The refined scheme introduces two new interventions: the lower-bound payment and the challenge process. Both interventions boost the farmers’ payments: Under the lower-bound payment, whenever the initial payment to a farmer is lower than a pre-specified lower bound, the payment to that farmer is increased to meet the lower-bound. Under the challenge process, if a farmer – who has received the lower-bound payment – has supplied high quality, he can increase his payment via a successful challenge. Thus, compared to the (fair) payment under the basic scheme, the payment (which is revenue outflow for the cooperative) is higher after the introduction of the aforementioned two interventions. Therefore, to keep the cooperative solvent, we need to increase the revenue inflow by imposing an admission fee. The precise value of the admission fee is chosen such that, if farmer \( i \) (\( i \in M \)) were to apply the high target quality \( q_H \), then his expected payment under the refined scheme is \( p(q_H; Q_N)Q_i \), which corresponds to high quality. Thus, the admission fee does not impose an extra burden on the farmers in that it is

\(^9\)It can be shown that the desirable outcome (i.e., grand coalition and high quality of milk supply) is achieved under the refined scheme as long as \( \gamma \geq \frac{c_1 \max(q_H - q_L - \epsilon, 4\epsilon)}{p_1(q_H - q_L - \epsilon) + b} \). Thus, to minimize testing, \( \gamma \) is set at its lowest possible value.
returned to them via higher payments. Moreover, in Section 4.2.2, we will see that the ratio of the admission fee to the payment received by a farmer is quite small.

- **Support for Testing:** Being an essential commodity that is consumed by a majority of the population, the quality of milk is an important concern for governments worldwide as well as for non-governmental organizations that are involved in ensuring the safety of food. As a result, it is common to see governmental as well as non-governmental support aimed at improving the quality of milk; see, e.g., Haartsen et al. (2010) and Ministry of Agriculture, Government of India (2013). It is therefore reasonable to assume third-party support for the testing of milk, provided of course that the amount of testing is minuscule. Later, in Section 4.2.3, we will show that the probability of a farmer being tested is close to zero under real-world parameter settings.

We now analyze the equilibrium outcome under the refined scheme. First, a farmer decides with whom to form a coalition. Then, the farmer determines (1) which target quality $q_i$ to apply, and (2) whether or not to challenge if he receives the lower-bound payment. Using backward induction, we first study the quality and challenge decisions of the farmers in any given coalition $M, M \subseteq N$. Then, we examine the formation of coalitions.

Consider a coalition $M, M \subseteq N$. For any farmer $i, i \in M$, we first calculate the expected profit when this farmer does not challenge and then when he does. The equilibrium quality and challenge decisions are derived by comparing these cases.

Given the realized average quality $\tilde{q}_{-i}$ from the other farmers in coalition $M$, let us calculate the expected profit when farmer $i$ applies target quality $q_i$ and does not challenge if the lower-bound payment $LQ_i$ is received. Let $\beta_i$ be the probability that the payment $\tilde{\theta}_i$ to farmer $i$ in (4) reaches its lower bound. From (2) and (4), we have $\beta_i = \text{Prob}[\tilde{\theta}_i = LQ_i] = \text{Prob}[p(\tilde{q}_M, Q_M)Q_M - p(q_H, Q_M)Q_{-i} \leq LQ_i]$. This probability depends on the realization of the average quality $\tilde{q}_M$ supplied by coalition $M$, which is, in turn, a function of the target qualities $\{q_j, j \in M\}$ applied by each member in coalition $M$. In a large coalition, the quality decision of an individual farmer does not much affect the average quality of the mixed milk (supplied by all the farmers in the coalition). Therefore, purely for simplicity of our exposition here, we assume that $\beta_i$ does not depend on the individual target quality $q_i$ applied by farmer $i$. It can be shown that the same equilibrium outcome – high quality and formation of the grand coalition – is achieved in the general setting without this assumption. We briefly summarize the reasoning behind the retention of this equilibrium outcome even without our simplifying assumption: Under the general setting,
the probability $\beta_i$ decreases in the quality $q_i$. Therefore, as compared to the simpler case when $\beta_i$ does not depend on $q_i$, if farmer $i$ applies a low target quality $q_i$, then there is a higher chance that he will receive the lower-bound payment $LQ_i$ under the general setting. As a result, as compared to the simpler case, farmers have a lesser incentive to supply low quality under the general setting. Thus, since high quality is achieved under the simpler case (Theorem 4 below), the same equilibrium outcome remains valid occurs under the general setting as well.

Denote $\pi_i(q_i)$ as the expected profit when farmer $i$ applies target quality $q_i$, given the realized average quality $\bar{q}_{-i}$ from the other farmers in coalition $\mathcal{M}$. Using (2) and (4), the expected profit when farmer $i$ applies target quality $q_i$ and does not challenge is

$$\pi_i(q_i) = \beta_i LQ_i + (1 - \beta_i) E_{\tilde{q}_i} \bar{r}_i - c(q_i) Q_i - a_i$$

$$= \beta_i LQ_i + (1 - \beta_i) E_{\tilde{q}_i} \left( p(\tilde{q}_i|Q_i, Q_M) Q_M - p(q_H, Q_M) Q_{-i} \right) - c(q_i) Q_i - a_i$$

$$= \beta_i LQ_i + (1 - \beta_i) \left\{ p(q_i, Q_M) Q_i + p(\tilde{q}_{-i}, Q_M) Q_{-i} - p(q_H, Q_M) Q_{-i} \right\} - c(q_i) Q_i - a_i. \quad (6)$$

Note that the admission fee $a_i$ defined in (5) is a constant and does not affect the quality decision of farmer $i$.

Given the realized average quality $\bar{q}_{-i}$ from the other farmers in coalition $\mathcal{M}$, we now derive the expected profit when farmer $i$ applies target quality $q_i$ and challenges if the lower-bound payment $LQ_i$ is received. When the farmer challenges, if he is not tested, then a high-quality payment $p(q_H - \epsilon, Q_M)Q_i$ is offered. On the other hand, if farmer $i$ is tested, then the payment depends on the realized quality $\tilde{q}_i$. Let $\rho(q_i) = \text{Prob}[\tilde{q}_i \geq q_H - \epsilon]$ be the probability that the realized quality $\tilde{q}_i$ (which is a function of the target quality $q_i$) is at least $q_H - \epsilon$, when farmer $i$ applies target quality $q_i$. The expected payment when farmer $i$ is tested is $\rho(q_i)p(q_H - \epsilon, Q_M)Q_i + (1 - \rho(q_i))\{p(q_L, Q_M) - b\}Q_i$. Recall that the testing probability is $\gamma$. Thus, the expected payment when farmer $i$ challenges is

$$(1 - \gamma)p(q_H - \epsilon, Q_M)Q_i + \gamma \left\{ (1 - \rho(q_i))\{p(q_L, Q_M) - b\}Q_i + \rho(q_i)p(q_H - \epsilon, Q_M)Q_i \right\}.$$
refined scheme. The proof is provided in Section D of the online appendix.

Thus, to minimize overall testing, the lower-bound payment $L$ naturally imposes a lower bound on $L$ to guarantee a minimum income for small farmers who depend on it for their daily sustenance—this economic environment that the cooperative faces. The idea behind the lower-bound payment is to set its lowest (resp., highest) possible value. These values will, of course, depend on the specific equilibrium is as follows: The grand coalition $N$ is formed and all the farmers apply the high target quality $q_H$. The expected profit received by farmer $i$ ($i \in N$) is $p(q_H, Q_N)\tilde{q}_i - c(q_H)q_i$. Farmer $i$ challenges on receiving the lower-bound payment $\tilde{q}_i = L\tilde{q}_i$. The overall testing probability for farmer $i$ is

$$
\delta_i = \beta_i \gamma = \frac{c_1 \max\{q_H - q_L - \epsilon, 4\epsilon\}}{P_i(q_H - q_L - \epsilon) + b} \Pr[p(\tilde{q}_N, Q_N)Q_N - p(q_H, Q_N)Q_{-i} \leq L\tilde{q}_i].
$$

Moreover, the refined scheme belongs to the core, i.e., the grand coalition is stable.

Remark 4: The overall equilibrium testing probability $\delta_i$ for farmer $i$ increases in the per-unit lower-bound payment $L$ and decreases in the per-unit penalty $b$ for an unsuccessful challenge. Thus, to minimize overall testing, the lower-bound payment $L$ (resp., penalty $b$) should be kept at its lowest (resp., highest) possible value. These values will, of course, depend on the specific economic environment that the cooperative faces. The idea behind the lower-bound payment is to guarantee a minimum income for small farmers who depend on it for their daily sustenance—this naturally imposes a lower bound on $L$. Similarly, if the farmers that supply to the cooperative are largely poor, then the upper bound on the penalty will naturally be tighter relative to when they are well-to-do.

Note that some amount of testing is conducted under the refined scheme. In the next subsection, we evaluate the extent of testing in equilibrium using realistic data. We will then introduce another enhancement to the refined scheme with the aim of reducing testing in Section 4.2.3.

\[\alpha_M = p(q_H - \epsilon, Q_M) - c(q_H - q_L - \epsilon) = \left\{p(q_H - \epsilon, Q_M) - c(q_H - \epsilon)\right\} + c(q_L)\] is the summation of (i) the per-unit profit from supplying high-quality milk and (ii) the per-unit cost of producing low-quality milk. For instance, the choice $L = p(q_L, Q_M)$ (i.e., the unit price corresponding to low quality) seems reasonable in practice and satisfies $L \leq \alpha_M$. 

\[\left(1 - \beta_i\right)E_{\tilde{q}_i, \tilde{q}_i} - c(q_i)Q_i - \alpha_i\]

\[
= \beta_i \left\{(1 - \gamma)p(q_H - \epsilon, Q_M)Q_i + \gamma(1 - \rho(q_i)\{p(q_L, Q_M) - b\})Q_i + \gamma \rho(q_i)p(q_H - \epsilon, Q_M)Q_i \right\}
+ \left(1 - \beta_i\right) \left\{p(q_i, Q_M)Q_i + p(\tilde{q}_i, Q_M)Q_{-i} - p(q_H, Q_M)Q_{-i}\right\} - c(q_i)Q_i - \alpha_i.
\]
4.2.2. The Extent of Testing under the Refined Scheme

Recall that $\beta_i$ is the probability that farmer $i$ receives the lower-bound payment and that $\gamma$ is the probability of being tested (by the cooperative) when a farmer challenges. Thus, $\delta_i = \gamma \beta_i$ is the overall probability of farmer $i$ being tested. This probability is expected to be small due to the following reasons: (1) The probability $\beta_i$ measures the chance that the realized quality of the mixed milk supplied by the grand coalition is low when all the farmers have applied the high target quality $q_H$. Clearly, when quality uncertainty is small, $\beta_i$ is expected to be quite small. (2) The testing probability $\gamma$ decreases in the penalty coefficient $b$. Thus, for a sufficiently large penalty coefficient $b$, the probability $\gamma$ (and therefore the probability $\delta_i = \gamma \beta_i$) is small.

The grand coalition collects an admission fee (defined in (5)) from each farmer upon joining the coalition. From an implementation point of view, we would also like this admission fee to be small, relative to the payoff received by a farmer. From Theorem 4, the expected payoff received by farmer $i$ is

$$p(q_H, Q_N) Q_i.$$  

Let $\rho_i = \frac{\alpha}{p(q_H, Q_N) Q_i}$ be the ratio of the admission fee over the payment to farmer $i$. Below, we examine the values of $\delta_i$ and $\rho_i$ on realistic data. For simplicity, we assume that each farmer supplies the same quantity; i.e., $Q_i = Q$, $i \in N$. Let $\delta_i = \delta$ and $\rho_i = \rho$. The result below provides useful properties of the equilibrium values of $\delta$ and $\rho$.

**Theorem 5.** For the equilibrium outcome under the refined scheme:

- The testing probability $\delta$ increases in the number of farmers $n$ and in the guaranteed per-unit lower-bound payment $L$, and decreases in the penalty coefficient $b$.
- The ratio $\rho$ of the admission fee over the payment to a farmer increases in the level of the quality uncertainty $\epsilon$ and in the guaranteed per-unit lower-bound payment $L$, and is independent of the penalty coefficient $b$.

To examine practical values of $\delta$ and $\rho$, we obtained data for our basic parameters from several publicly-available documents on the real-world procurement of milk at village-level cooperatives in India: Shah (2012), Shah (2011), Food and Agriculture Organization of the UN (2013b), Hemme et al. (2003), Kumar et al. (2011), and Micobel cvba (2016). For brevity, we only provide the values we use in our assessment: $Q_i = 10$ liters $\forall i \in N$, $p(q_H, Q_N) = \$0.41$/liter, $p(q_L, Q_N) = \$0.29$/liter, $p_2 = \$0.017$/liter, $c(q_H) = \$0.24$/liter, $c(q_L) = \$0.18$/liter, $n = 100$, $q_H = 6\%$, $q_L = 2\%$, and $t = \$1.8$. Recall that $p(q_H, Q_N)$ (resp., $p(q_L, Q_N)$) is the per-unit price received by a farmer based on high quality $q_H$ (resp., low quality $q_L$), $t$ is the cost of performing a comprehensive test on a milk sample,

\[^{11}\text{In equilibrium, a farmer challenges on receiving the lower-bound payment.}\]
and \( n \) is the number of farmers in a village-level milk cooperative. Using the values of \( p(q_H, Q_N) \), \( p(q_L, Q_N) \), \( c(q_H) \), and \( c(q_L) \), we have \( c_1 = \frac{c(q_H) - c(q_L)}{q_H - q_L} = 1.62 \) and \( p_1 = \frac{p(q_H, Q_N) - p(q_L, Q_N)}{q_H - q_L} = 3.09 \), which are, respectively, the coefficients in the expressions (defined in Section 3.1.1) for the per-unit production cost \( c(q) \) incurred by a farmer and the per-unit price \( p(q, Q) \) offered by the dairy firm.

To examine the values of \( \delta \) and \( \rho \), we set the guaranteed per-unit lower bound payment to \( L = p(q_L, Q_N) \), which corresponds to the payment when low quality \( q_L \) has been supplied. Table 1 summarizes the values of \( \delta \) and \( \rho \) under different levels of the quality uncertainty \( \epsilon \) and the penalty coefficient \( b \). The calculation of \( \delta \) is provided in Section E of the online appendix; the calculation of \( \rho \) is similar and therefore avoided for brevity. We consider two levels of the penalty coefficient \( b \): one is the extreme case, where an infinite penalty is charged; the other is a modest one, where a farmer receives zero profit if not only low quality of his milk is detected but also he chooses to challenge when the correct price (corresponding to low quality) is offered. We note again that a high admission fee \( a \) (and therefore a high value of \( \rho \)) is not an extra burden to the farmers, since, in equilibrium, each farmer receives a fair expected payment based on the true quality of his milk.

<table>
<thead>
<tr>
<th>( \epsilon ) (in % of ( q_H ))</th>
<th>( b = \infty )</th>
<th>( b = p(q_L, Q_N) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \delta = 0% )</td>
<td>( \delta = 1.4% )</td>
<td>( \delta = 1.4% )</td>
</tr>
<tr>
<td>( \rho = 3.7% )</td>
<td>( \rho = 3.7% )</td>
<td>( \rho = 3.7% )</td>
</tr>
<tr>
<td>( \epsilon = 10% q_H )</td>
<td>( \delta = 0% )</td>
<td>( \delta = 3.5% )</td>
</tr>
<tr>
<td>( \rho = 13.4% )</td>
<td>( \rho = 13.4% )</td>
<td>( \rho = 13.4% )</td>
</tr>
</tbody>
</table>

Table 1 The testing probability \( \delta \) and the ratio \( \rho \) of the admission fee to the payment under the refined scheme.

While the values \( \delta \) and \( \rho \) in Table 1 are reasonably low, we can further reduce them by enhancing the refined scheme. We discuss this enhancement next.

### 4.2.3. The Refined Scheme with Delayed Payments: The Final Scheme

Consider the following “delayed payment” enhancement to the refined scheme: For an integer \( k \geq 1 \), a coalition pays each farmer once every \( k \) periods (i.e., \( k \) collection cycles) based on the average quality of the mixed milk from that coalition over \( k \) periods. Such payment schemes have been widely implemented by cooperatives (see, e.g., Sankar and Yoganandham 2016, Brulin and Klingzell-Brulin 2012, Indian Merchants’ Chamber 2016). To define this scheme precisely, let \( M \)

\[ L = p(q_L, Q_N) \]

From Theorem 4, the desirable equilibrium outcome (i.e., grand coalition and high quality of milk) is achieved for any value of \( L \), \( L \leq p(q_H - \epsilon, Q_N) - c_1(q_H - q_L - \epsilon) \). Note that \( p(q_L, Q_N) \leq p(q_H - \epsilon, Q_N) - c_1(q_H - q_L - \epsilon) \).
\((\mathcal{M} \subseteq \mathcal{N})\) denote an arbitrary coalition, let \(Q_{\mathcal{M}}\) denote the total quantity supplied by the coalition in each period, and let \(\tilde{q}_{iM}^j\) be the realized quality of milk supplied by coalition \(\mathcal{M}\) in period \(j, j = 1, 2, \ldots, k\). Thus, \(\tilde{q}_{avg}^M = \frac{1}{k} \sum_{j=1}^{k} \tilde{q}_{iM}^j\) is the average quality supplied by the coalition over the \(k\) periods. Then, farmer \(i\) \((i \in \mathcal{M})\) receives an amount \(\tilde{\theta}_i = \max\{\tilde{r}_i, kLQ_i\}\) from coalition \(\mathcal{M}\) every \(k\) periods, where \(\tilde{r}_i = kp(\tilde{q}_{avg}^M, Q_{\mathcal{M}})Q_{\mathcal{M}} - kp(q_H, Q_{\mathcal{M}})Q_{\mathcal{M}}\). Other than the change that a coalition pays each farmer once every \(k\) periods, all other aspects of the scheme remain the same as those of the refined scheme. In particular, coalition \(\mathcal{M}\) receives an amount \(p(\tilde{q}_{iM}^j, Q_{\mathcal{M}})Q_{\mathcal{M}}\) from the dairy firm in period \(j, j = 1, 2, \ldots, k\). Thus, the total payoff received by coalition \(\mathcal{M}\) over the \(k\) periods is \(\sum_{j=1}^{k} p(\tilde{q}_{iM}^j, Q_{\mathcal{M}})Q_{\mathcal{M}} = kp(\tilde{q}_{avg}^M, Q_{\mathcal{M}})Q_{\mathcal{M}}\).

As long as the length \(k\) of the payment cycle is not too large, such a delayed-payment scheme is reasonable; for instance, since milk is collected either once or twice daily (i.e., a period corresponds to a day or half a day), paying farmers every 3-5 days is tenable. We refer to the refined scheme with the above \(k\)-period delayed-payment enhancement as our \textit{final scheme}. It is easy to show that, in the unique equilibrium under the final scheme, the grand coalition \(\mathcal{N}\) is formed and all the farmers apply the high target quality \(q_H\). Let \(\tilde{q}_{avg}^{H\mathcal{N}}\) be the average realized quality supplied by the grand coalition \(\mathcal{N}\) over \(k\) periods, when all the farmers have applied the high target quality \(q_H\). Theorem 6 states the equilibrium outcome formally; the proof of this result is similar to that of Theorem 4 and therefore not provided for brevity.

\textbf{Theorem 6.} \textit{Under the final scheme, the unique equilibrium is as follows: The grand coalition \(\mathcal{N}\) is formed and all the farmers apply the high target quality \(q_H\). The overall testing probability for farmer \(i\) \((i \in \mathcal{N})\) is}

\[
\delta_i = \beta_i \gamma = \frac{c_i \max\{q_H - q_L - \epsilon, 4\epsilon\}}{p_i(q_H - q_L - \epsilon) + b} \text{Prob}\left[p(\tilde{q}_{avg}^{H\mathcal{N}}, Q_{\mathcal{N}})Q_{\mathcal{N}} - p(q_H, Q_{\mathcal{N}})Q_{\mathcal{N}} \leq LQ_i\right].
\]

\textit{The expected profit received by farmer \(i\) is} \(p(q_H, Q_{\mathcal{N}})Q_i - c(q_H)Q_i\). \textit{Further, this scheme belongs to the core, i.e., the grand coalition is stable.}

We now examine the overall testing probability \(\delta\) and the ratio \(\rho\) of the admission fee to the expected payoff per \(k\) periods of a farmer under the final scheme. As in Section 4.2.2, we assume that each farmer supplies the same quantity of milk for simplicity of exposition. The result below provides useful properties of the equilibrium values of \(\delta\) and \(\rho\) under the final scheme.

\textbf{Theorem 7.} \textit{In the unique equilibrium outcome under the final scheme,}
• The testing probability $\delta$ increases in the number of farmers $n$ and in the per-unit lower-bound payment $L$, and decreases in the length of a payment cycle $k$ and the penalty coefficient $b$.

• The ratio $\rho$ of the admission fee to a farmer’s expected payment per $k$ periods increases in the level of quality uncertainty $\epsilon$ and in the per-unit lower-bound payment $L$, decreases in the length of a payment cycle $k$, and is independent of the penalty coefficient $b$.

Both $\delta$ and $\rho$ decrease in the length $k$ of the payment cycle. Therefore, when setting an appropriate value for $k$, one needs to consider the tradeoff between (a) the reduced testing probability $\delta$ and the ratio $\rho$ of the admission fee to the expected payment and (b) the potential implementation challenges from a larger payment cycle. To illustrate, using the practical data specified in Section 4.2.2, Tables 2 shows the values of $\delta$ and $\rho$ under the final scheme for $k = 6$. Since cooperatives typically collect milk from farmers twice a day, $k = 6$ corresponds to paying farmers once every 3 days. Thus, considering the benefits to the farmers, this slight delay in payment is both justifiable and reasonable.

<table>
<thead>
<tr>
<th>$\epsilon$</th>
<th>$b = \infty$</th>
<th>$b = p(q_L,Q_N)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5%q_H$</td>
<td>$\delta = 0%$</td>
<td>$\delta = 0.01%$</td>
</tr>
<tr>
<td>$0.02%$</td>
<td>$\rho = 0.02%$</td>
<td>$\rho = 0.02%$</td>
</tr>
<tr>
<td>$10%q_H$</td>
<td>$\delta = 0%$</td>
<td>$\delta = 0.72%$</td>
</tr>
<tr>
<td>$1.84%$</td>
<td>$\rho = 1.84%$</td>
<td>$\rho = 1.84%$</td>
</tr>
</tbody>
</table>

Table 2: The testing probability $\delta$ and ratio $\rho$ of the admission fee over a farmer’s expected payment per $k$ periods under the final scheme for $k = 6$.

Notice that both the values of $\delta$ and $\rho$ are sufficiently small. Thus, the admission fee paid by a farmer is a negligible fraction of the payment received by that farmer. Testing is negligible too; justifying its support by a third party (see Section 4.2.1).

**Remark 5:** Consider a benchmark setting where a cooperative applies the naive allocation rule (Section 3.1.3) and conducts a simple random testing on the farmers within the cooperative, it can be shown that (i) when testing cost is high ($t > \frac{(q_H-q_L-\epsilon)Q_N}{n(p_1-c_1)}$), no individual testing is conducted and all farmers apply the low target quality, and (ii) when testing cost is low ($t \leq \frac{(q_H-q_L-\epsilon)Q_N}{n(p_1-c_1)}$), the cooperative applies a testing probability of $\frac{\alpha}{p_1} - \frac{1}{n}$ and all farmers apply the high target quality. To illustrate, using the practical data specified in Section 4.2.2, the required testing probability to guarantee a high-quality supply in the latter case is 51.4%. Thus, if a simple random testing is applied, the cooperative suffers either in terms of low quality or in terms of extensive testing. In contrast, the quality under our proposed refined scheme is
high and, as shown in Table 2, the required testing is negligible – when \( b = p(q_L, Q_N) \), \( \delta = 0.01\% \) (resp., \( \delta = 0.72\% \)) if \( \epsilon = 5\% \) (resp., \( \epsilon = 10\% \)).

5. Conclusion

Dairy cooperatives are intrinsically appealing, modern organizations, where producers (typically small-income farmers) combine their individual outputs to reap the benefits of scale. But the blessing of scale, unless properly managed, can turn into a curse. This is because an individual farmer can cheat on quality, in the hope that his low quality milk can hide behind the good quality milk supplied by other farmers. Naturally, if a significant fraction of farmers think the same way, the mass free-riding can result in poor quality at the level of the cooperative. Testing is a solution, but it comes at a cost. Besides, testing is not a socially-productive activity. We therefore set ourselves the task of developing allocation rules and testing policies for a dairy cooperative to achieve: (1) quantity efficiency, (2) quality efficiency, and (3) minimal testing.

Our analysis in this study exploited the following features of smallholder dairy cooperatives: (a) a large number of small-scale farmers; (b) output from the farmers is mixed prior to marketing the produce (to exploit economies of scale), thus generating incentives for producers to free-ride on the quality of others; (c) the quality of the mixed output is a weighted average of the qualities of the inputs. This is unlike the case where all the produce becomes of low quality if a portion of it is of low quality; (d) testing for quality is expensive. Given our main goals (quantity efficiency, quality efficiency, and low testing) and essential operating conditions (mixing of output and material testing costs), we proposed a basic allocation scheme under which all three goals were achieved.

We then enhanced the basic scheme to address additional operating conditions often found in practice: (1) farmers with low working-capital buffers, (2) the unintended quality variation in a farmer’s output, and (3) a cooperative that also operates with a low working-capital buffer. We proposed a lower bound on a farmer’s payment in light of the “hand-to-mouth” nature of smallholder dairy operations, especially in developing countries. The second practically-motivated intervention was made to address quality variation, that if left unattended, could lead to excessive testing. Here, we proposed a delayed-payment scheme that exploited the virtues of variation reduction in larger samples. The third practical issue was to balance the payments of the cooperative, to keep it solvent – this was achieved by imposing an admission fee (paid annually, for example) to stay in the cooperative. Another modification to the basic scheme was made to
improve the acceptability, at the ground level, of our enhanced allocation scheme. Here, we provided farmers with a recourse if they felt that they were not adequately compensated for their output. A farmer could challenge the cooperative if the payment to him was perceived as unfair: a successful challenge increased the farmer’s payment, whereas a lost challenge required the farmer to pay a penalty.

References


