Introduction

Multiplicative Thinking

Sandi Berg
bsdemonstrators@gmail.com
learning.arpdc.ab.ca

• What is it?
• Why is it important?
• What are the Big Ideas?
Introduction

Multiplicative Thinking is:
• A capacity to work flexibly with the concepts, strategies, and representations of multiplication and division as they occur in a wide range of contexts. (Mathematical reasoning)
• Going beyond memorization of basic arithmetic skills, and
• The means to communicate multiplicative understanding effectively in a variety of ways (for example, words, diagrams, symbolic expressions, and written algorithms).

EMPLO, 2015

Introduction

Multiplicative Thinking

Building Toolkits

Strategies

Known Facts

Research says:
• Both mathematical reasoning and arithmetic skills are predictors of mathematical achievement, however, students' ability to reason mathematically was the stronger predictor of success.

Nunes, Bryant, Bamos, & Sylva, 2011
Introduction

Multiplicative Thinking

Basic Facts of Multiplication

Where and how does the curriculum reference knowing your multiplication facts in current curriculum?

Grade 3
- Understand and recall multiplication facts and related division facts to 5x5.

Grade 4
- Understand and apply strategies for multiplication and related division facts to 9x9.
- Recall multiplication and related division facts to 7x7.

Grade 5
- Understand, recall and apply multiplication and related division facts to 9x9.

AB ED's Definitions

Alberta Education, 2016

<table>
<thead>
<tr>
<th>TERM</th>
<th>DEFINITION</th>
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</thead>
<tbody>
<tr>
<td>Number Facts</td>
<td>Single digit addition, subtraction, multiplication and division.</td>
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<tr>
<td>Mastery</td>
<td>Mastery of number facts occurs when students understand and recall facts.</td>
</tr>
<tr>
<td>Recall</td>
<td>Recall of number facts is when students commit them to memory and retrieve</td>
</tr>
<tr>
<td>Strategy</td>
<td>The application of a procedure with reasoning.</td>
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Memorizing words doesn’t make you a reader. Memorizing math facts does not make you a multiplicative thinker.

We want students to know their basic facts but...
Students investigate a variety of strategies for number operations, one of which must be the standard/traditional algorithm. Students are to use a strategy that they understand. A student's strategy may evolve over time as the student continues to develop mathematical understanding.

Alberta Education. (2016) *Clarification of Expectations Regarding Strategies and Maintenance*
Math learning and performance are optimized when the two sides of the brain are communicating – “brain crossing”  

“Park & Brannon, 2013

Multiplication is not just repeated addition

Solve the following using “Repeated Addition”

• 3 x 2
• 2 x 0.5
• -2 x -3
• 0.4 x 3
• 3.5 x 2.6*

*This is above the expectations of K-6 curriculum

Activity: Paper Folding

Fold your paper so you have 3 equal pieces. Unfold. Share your equal pieces with someone.

Did you fold it using an “Accordion fold” or an “Overlap” style?

Did you start from a “Portrait” position or a “Landscape” position?
Activity: Paper Folding

Before you do the next fold, SEE it in your head!
You’re going to fold in the other direction to end up with 4 equal pieces.
BEFORE YOUR START, how many sections will you see in your paper now? Tell your partner.
Once you’re confident, go ahead and fold.

Which images show the result of folding in two directions?

Multiplication moves in two directions. Paper folding can be used to develop multiplicative reasoning.
Introduction

Multiplicative Thinking

Activity: Paper Folding

Do you see any multiplication facts embedded in this picture?

3

4

If this is 1, we modelled $3 \times 4 = 12$ and $4 \times 3 = 12$.

Do you see 3 fours? Do you see 4 threes?

Why do we double count the first box?

If the whole paper is 1,

we just showed that $\frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$ and $\frac{1}{4} \times \frac{1}{3} = \frac{1}{12}$. 
Important Note

The goal is to repeat 4 x 3 and 3 x 4 in so many contexts that students will be able to just answer 12.

Once students have an automatic response for 4 x 3 and 3 x 4, you can explore other related facts.

Developing Distribution Strategies

Imagine using the grid on your handout:
- Cut out a 3 x 4 grid from cm paper.
- Imagine what it means to double it.
- Explain what you are seeing in your head to your partner.

[Imagine] Lay your array on a new piece of cm paper like this.
Introduction

Multiplicative Thinking

Developing Distribution Strategies

What does the 12 represent?

We're going to double the array to the right. What do you see in your head?

Is this what you saw? What numbers belong where the question marks are?
Introduction
Multiplicative Thinking
Developing Distribution Strategies

We had 3 x 4
We added 3 x 4
(3x4) + (3x4) = 6 x 4

What do the brackets mean?

Is this what you expected?

• Use your 3 x 4 array to outline and cut out the 6 x 4 array.
• Label the back with the distribution.
  (3x4) + (3x4) = 6 x 4

Do you see a different way to double this fact?
Is this what you saw?
What numbers belong where the question marks are?

3 6
4 12 24
7 7

We had 3 x 4
We added 3 x 4
(3x4) + (3x4) = 3 x 8

Why did we end up with 24 again?

• Use your 3 x 4 array to outline and cut out the 3 x 8 array.
• Label the back with the distribution.
  (3x4) + (3x4) = 3 x 8
Introduction

Multiplicative Thinking

Developing Distribution Strategies

• How are these two arrays similar? Different?

\[(3 \times 4) + (3 \times 4) = 6 \times 3\]

\[(3 \times 4) + (3 \times 4) = 3 \times 8\]

• Started with 3 \times 4
• Both have an area of 24
• Both created by a distribution based on doubling
• The perimeters are different
• They are not congruent. (Try fitting one over the other)

• Pick a different array.
• Repeat this process.
Introduction

Multiplicative Thinking

Upcoming Video

Grade 4 student

– Can we expect that the student has mastered any multiplication facts by December?
  Yes. They should have mastered the Grade 3 facts up to 5 x 5. That is ALL we can assume!
  There is NO time expectation for mastery within Grade 4 except for “By the end of the year”

Moving From Additive Thinking

How does the teacher help this student move from additive thinking to multiplicative thinking?

Video
Introduction
Multiplicative Thinking

Important Note

Grade 4 students multiply with more than single digits. They need to know how to multiply by 10 and 100.

Stop Saying
“Multiplying by 10 is just adding a 0!”
Can you just add a 0 for 0.03 x 10?

Stop Saying
“Multiplying by 10 is just adding a 0!”
You’re implying that “Dividing by 10 is just dropping a 0”!
Does this work for 13 ÷ 10
Introduction

Multiplicative Thinking

• Grade 4 student
  – What multiplication facts are they expected to have mastered by December?

Moving From Additive Thinking

How does the teacher help this student move from additive thinking to multiplicative thinking?
Her strategy can be explained with a distribution equation. What is the equation?

Is this what you expected?

\[(7 \times 10) - (7 \times 1) = (7 \times 9)\]
Introduction

Multiplicative Thinking

Student only knows up to 2x6

Student only knows up to 3x6
Introduction

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Imagine 7 dots by 9 dots

Imagine 4 dots by 13 dots

17 x 14
Introduction

Quick Assessment

The Strategy is All

- Counting Strategy
- Counting on/back

Additive Thinking Strategy
- Known Facts
- Making 10
- Using doubles
- (x+3)
- Traditional Algorithm
- Partitioning by Place Value
- Open Number Line

Multiplicative Thinking Strategy
- Known Facts
- Using doubles
- Arrays
- Associative Property
- Commutative Property
- Distributive Property

- Other

Follow up Questions to Ask the Student
- Explain what you did.
- Will this always work?

Notes/Next Steps

Follow up Steps for Student

The Website

http://learning.arpdc.ab.ca
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Session Evaluation