Answering Big Questions with Mathematical Modeling

Elevate Academic Summit
August 2018

Please sit near someone (you will work in groups later) and log in to a computer.
What Mathematical Modeling is NOT:

- Using pictures or manipulatives to model an equation.
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- **Writing an equation** to model either real or contrived data.
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- *Writing an equation* to model either real or contrived data.
- Adding *real-world context* to an otherwise context-less equation.
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- Writing an equation to model either real or contrived data.
- Adding real-world context to an otherwise context-less equation.

While all of these are important skills, none of these are what we mean when we say mathematical modeling. Or rather, each of these make up a small small part of the modeling that those mathematicians use.
If That’s Not Mathematical Modeling, What Is?

- The Mathematics of Ponzi Schemes by Marc Artzrouni
  - Harry Markopolos, *The Guy Who (Tried) to Blow the Whistle on Bernie Madoff*.
- Target Knows You’re Pregnant Before Your Dad Does!!

Spies vs. Analysts

- When creating a mathematical model from scratch, there are essentially two parts:
  - The job of the spy (to figure out what information is needed and to acquire it)
  - The job of the analyst (to take the information and work with it)
- Both parts are equally important.
  - Think about WWII and September 11, 2001.
  - Weather & Earthquakes

Robert Kaplinsky:
Spies: https://robertkaplinsky.com/mathematical-modeling-need-better-spies-analysts/
Plane: https://robertkaplinsky.com/math-modeling-can-get-kicked-off-plane/
Mathematical Modeling in the Early Grades

● Find the sum of 6 and 3
  ○ Not Modeling, strict computation
● Johnny has 6 pretzels and Rosa has 3 pretzels. How many pretzels do they have altogether?
  ○ Not Modeling, computation in context
● Your family is going on a picnic. How many pretzels should you take?
  ○ Modeling

Taken from Guidelines for Assessment and Instruction in Mathematical Modeling
http://www.siam.org/LinkClick.aspx?fileticket=0vLnFjpqYrA%3d&portalid=0
Math Modeling in Middle School

● Find the Lateral Surface Area of a square pyramid that is 28 feet tall and has a base that is 44 feet long on each side.
  ○ Not Modeling, strict computation

● The Great Pyramid of Giza is 280 cubits tall and has a square base that is 440 cubits on one side. What is the total area of the exposed faces of the pyramid?
  ○ Harder, but still not modeling. What is a cubit, anyway? Computation in context

● You have been assigned to create a large, scale model of the pyramid at the Louvre to be displayed at the new art museum. You may use any material you want, but you have a budget of $5,000 for the outer surface. What material should you use, how much will you need and how much will it cost?
  ○ Modeling

Taken from Guidelines for Assessment and Instruction in Mathematical Modeling
http://www.siam.org/LinkClick.aspx?fileticket=0vLnFjpqYrA%3d&portalid=0
Math Modeling in High School

- Graph the line that has a y-intercept of 100 and a slope of 2, then write the equation.
  - Not modeling, strict computation
- Emily works at a retail store that pays her $100 per week plus $2 for each item she sells. Write and graph a linear equation representing the relationship between Emily’s weekly income and the number of items she sells during one week.
  - Not modeling, and ridiculous! Computation in context

Taken from Guidelines for Assessment and Instruction in Mathematical Modeling
http://www.siam.org/LinkClick.aspx?fileticket=0vLnFjqYrA%3d&portalid=0
Math Modeling in High School

- The holidays are approaching and your best friend Karen would like to make some money to purchase gifts. She found one job that will pay $2/hr above the minimum wage. Another job offers to pay half the minimum wage plus commission in the amount of $2 per item she sells. Which job should she take and why? To help Karen understand your analysis, include a useful representation.
  - Modeling

Taken from Guidelines for Assessment and Instruction in Mathematical Modeling
http://www.siam.org/LinkClick.aspx?fileticket=0vLnFjpqYrA%3d&portalid=0
Let’s Try to Answer a Big Question...


Should an airline overbook? By how much?

10 minutes

www.robertkaplinsky.com/lessons - modeling problems
https://m3challenge.siam.org/ - Moody Mega Math Challenge
Are You a Spy or an Analyst at This Point?

This part of the modeling process has three pieces:

- Brainstorming
- Making Assumptions
- Defining Variables

While you start with Brainstorming, this process is not strictly linear. It is nearly impossible to move straight through each piece in order and not revisit any.
Brainstorming

- Often, these questions are written in a way that allows for multiple interpretations or approaches. The goal of brainstorming is to develop a concise restatement of the question.
  - Focus on subjective words that can be interpreted in different ways and words that are not easily quantified (best, efficient, optimal, etc.)
  - Explore the problem by doing a combination of brainstorming and research. Keep the time constraints in mind, though.
  - Write down EVERY idea, no matter how crazy. You can prune unrealistic ideas later.
  - Approach brainstorming as if you had access to any data you need.
  - Use a graphic organizer such as a mind map to keep track of your ideas.
  - Your concise restatement of the problem should explain precisely what your model will measure.

What Were Some of Your Thoughts?
Making Assumptions

- This often comes straight out of the Brainstorming process.
- Do some research. The kind of data that you can find may guide you in what assumptions you will need to make. In the absence of any data, make a reasonable assumption and justify it in your write-up.
- Different assumptions can lead to very different, yet equally valid outcomes, often dependent upon the mathematical ability of the students.
- Not all of your assumptions will be made during the Brainstorming process. Some will be made during the modeling process. You should keep track of them and include them in your write up.

Defining Variables

- The problem statement should determine the output of the model and this or these will be the dependent variables.
- The results of the Brainstorming process can give insight into which variables will be independent variables and which will be fixed parameters.
- Keep careful track of the units. This may reveal relationships between variables.
- You may have to do additional research and make additional assumptions to identify parameters.
- Submodels may be needed to provide input for your model.

Focus Question for Today

Is it cost-effective for Southwest Airlines to overbook on direct flights from Midland, Texas? By how much?

What information do you need?
What information do you have already?
What assumptions have been made?
What sort of assumptions do you need to make to move forward?

20 minutes
Building a Model to Get a Solution

- How you build a solution may depend on what mathematical tools are available to you.
- There is often more than one way to tackle a problem, so just start and see what happens.
- If you don’t immediately know how to solve the problem at hand, ask yourself the questions from the handout to get you started.
- Different solution methods can lead to solutions of different natures. This is perfectly acceptable.

Analysis and Model Assessment

- Be sure to allocate time to analyze your results since it is indeed a critical part of the entire modeling process.
- Always examine the output you get from your model and ask yourself if it makes sense. If your answer doesn’t make sense, verify that you haven’t made a mistake in implementing your model.
- If your solution is consistent with your assumptions, but not consistent with the real-world phenomenon you are trying to describe, you may need to refine your model by adjusting your assumptions.
- **List strengths and weaknesses of your model.**
- Try to determine how sensitive your model is to parameters and assumptions.
- **Include specific improvements you would have incorporated given more time.**

Putting it All Together (for a larger project)

- Take notes throughout the process so that you do not leave out anything important, especially assumptions made along the way.
- Give yourself enough time to focus on the writing process and to proofread the report.
- Keep in mind that this is a technical document, not a story about your modeling experience.
- Follow the guidelines for technical writing.

https://amatyc.site-ym.com/page/SRLPastQuestions
Putting it All Together (for today)

- Oral presentation that includes:
  - Assumptions, model(s) used and the solution
  - A strength and a weakness of your model
  - One thing you would do to improve the model

- Questions to Ask When Building a Model
  - Have I seen this type of problem before?
  - If so, how did I solve it? If not, how is this problem different?
  - Do I have a single unknown, or is this a multivariable problem with many interdependent variables?
  - Is the model linear or nonlinear?
  - Am I solving a system of equations simultaneously, or can I solve sequentially?
  - What software or computational tools are available to me?
  - Would a graph or other visual schematic help provide insight?
  - Could I approximate my complicated model with a simpler one?
  - Can I hold some values constant and allow others to vary to see what is going on?

No later than 11:00
5 minutes per group
Use algebra tiles to solve $3x + 4 = 13$.

$x = $
EXAMPLE 3  Write an equation to model data

BIRD POPULATIONS

The table shows the number of active red-cockaded woodpecker clusters in a part of the De Soto National Forest in Mississippi. Write an equation that models the number of active clusters as a function of the number of years since 1990.

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</thead>
<tbody>
<tr>
<td>Active clusters</td>
<td>22</td>
<td>24</td>
<td>27</td>
<td>27</td>
<td>34</td>
<td>40</td>
<td>42</td>
<td>45</td>
<td>51</td>
</tr>
</tbody>
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From an unknown text book

https://slideplayer.com/slide/7812749/
Both images taken from Robert Kaplinsky “Beware of Fake Math Modeling Problems”

https://robertkaplinsky.com/beware-fake-math-modeling-problems/