"The Having of Wonderful Ideas" and other essays on teaching and learning

Eleanor Duckworth
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him or her to grasp the sense of the other representation—no matter how it is explained.

Similarly, two ways of figuring out possible arrangements of three objects are outlined in chapter 10. Both ways are perfectly adequate to the problem, but people who solve it one way often have trouble seeing how the other way can make any sense.

The most striking example in my experience came from a group discussion about arithmetic division (see Duckworth, 1987). "What does it really mean, 24 divided by 8?" one person asked. A second person responded that if you have 24 things, and you distribute them evenly into 8 piles, you then count how many end up in each pile. Yet another person, astonished, said, no, that's not what it means! It means you have 24 things, and you put them in piles of 8, and you see how many piles you get.

Now clearly it means both of those. But imagine a teacher who believes—as some in that group believed—that it could mean only one of them. Imagine that teacher explaining a division problem to a child who was thinking about it the other way. The teacher, with her own way in mind as the only one possible, would not even notice that the child was thinking differently—adequately and differently. With all the good-will in the world, the discussion would nonetheless come to an impasse. Probably both of them—the teacher and the child—would emerge from that discussion believing that the child was hopeless in arithmetic. Imagine how often such an impasse arises in discussing more complicated matters, and how often learners' ways of understanding are left unrecognized.

Textbooks and standardized tests, as well as many teacher education and curriculum programs, feed into this belief that there is one best way of understanding, and that there is one best, clearest way of explaining this way of understanding. Then we need only decide on the best way to come to understand a given topic, and we can tell teachers to present it to students that way.

This idea that there is one best way of understanding is linked to a pervasive pernicious belief—that the students who do not understand it in our way are not smart enough to understand it at all, that their future in the academy is limited, that they need a different kind of education.

And there is another, related pernicious belief: the idea that intelligence is given at birth in a fixed amount. If people have not been given much of it, we can Generally try to do our best with them, but it won't surprise us if they do not manage to understand.

These two ideas support each other quite beautifully: "There is one best way to understand." "Many people are not smart enough to understand." Once we believe these two ideas, a third harmful idea is implied:

"If someone does not understand our way, it is not that there is any problem with our insisting on our way; it is that there is a problem with that learner."

So I find this matter to be critically important. The essays in this book start instead from the premise mentioned above—that there is a vast array of very different adequate ways that people come to their understanding. Curriculum, assessment, teacher education programs—and all of our teaching—must seek out, acknowledge, and take advantage of the diversity of ways that people might take toward understanding. We cannot plan "the logical sequence" through a set of ideas, especially if we want schools to make sense for students whose backgrounds differ from our own. As Lisa Schneier (personal communication, 1997) has said, we must find ways to present subject material that will enable learners to get at their own thoughts about it. Then we must take those thoughts seriously, and set about helping students to pursue them in greater breadth and depth. In this way we can capture the intelligence of all our students, so we do not lose the one-half, three-quarters, nine-tenths that we lose now.

I need to add to this my conviction that students who succeed in our current one-right-way system are similarly missing out on vast possibilities for learning (see Pettigrew, 2006). The world is far more complex and fascinating than the small piecemeal views that the one-right-way allows.

Part of the work described in these chapters involves giving people an appreciation of their own ways of understanding. Part of it involves giving people experience in figuring out and appreciating other people's ways of coming to understand—ways that are different from their own. Much of the learning described here seems light-hearted, playful. It is. But it is also monumentally serious.

There were two major formative influences as I began my work in education: the work of Jean Piaget and Inhelder, and my experience with the Elementary Science Study curriculum development program. These two influences were very powerful.

Piaget's was one of the great minds of the 20th century, and his work, and Inhelder's, are still my most significant source of intellectual stimulation. With Piaget and Inhelder I learned how futile it is to try to change a child's mind by telling her to think something different. It raised the fascinating question of how on earth one can help someone learn, if telling them what you know does not help.

I started to be able to pursue this question when I joined the staff of the Elementary Science Study. My science background was exceedingly limited. The only formal science I had studied since high school was a first-year
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college biology course. But as a staff member of the Elementary Science Study, where the approach to science education was to engage teachers and children directly with fascinating corners of the natural world, my science colleagues and I went about it in ways that never caused me to doubt that what I was engaged in was science. I spent hours exploring materials and phenomena, with my colleagues on hand from time to time to ask how I was getting along and what I was making of things, and to suggest, "Why don't you try it this way?" or "Here, have a look at this." I assembled acoms; I used syringes to move air from one container to another; I explored the effects of different objects as pendulums bobs; I spent a full day with a flashlight battery and bulb by my side, in order to keep trying out ways to arrange a single wire so the bulb would light (I spent about half an hour at the beginning of the morning, went back in it off-and-on all day, and finally succeeded in lighting the bulb in the middle of the afternoon; this was the beginning of many months of explorations with batteries and bulbs); I tried to make liquid mixtures of identical densities—one oil-based and one water-based, so they did not mix with each other. I tried using any long object—a broom, a spoon, a toothbrush—as a balance; I watched frog eggs develop. I worked hard during these explorations, trying to figure out why something had happened, and to think of what I could do next, to shed more light on it. I was captivated by this world of fascinating phenomena, by its accessibility and its complexity. I was intrigued; also, with what it was like to be learning these things in this way, and with how my colleagues thought about opening the world to children.

These two foundations launched me into the world of education, as an elementary school teacher (too briefly to do it successfully, I am afraid), and in curriculum development, program evaluation, teacher education, and research in the development of ideas. The papers in this volume have been written over almost 4 decades of that work.

The earliest one (chapter 4) was written in 1968, the latest (chapter 31) in 2003. The world of schools has changed many times, in many ways, in the intervening decades. Through these changes, my ideas—of what is important in teaching, of what kinds of research are needed, of what teachers’ roles in the organization of schooling can be—have not changed very much. As I work with children, education students, and teachers, the same principles continue to lead to intense involvement of learners with subject matter. My central question has continued to be: How do people learn and what can anyone do to help? Helping people learn is my definition of teaching. Telling, explaining, play a very small part in helping people learn. The emphasis in these pages, rather, is on looking at what happens when teachers engage learners directly with the subject matter—as my science colleagues did with me—and then they, the teachers, do the listening while learners explain their own thoughts (see Duckworth, 2001b).

Although the ideas have not essentially changed since the first edition of this book, through the years there has been an evolution in terminology. When I began to see the work I do as a form of research, as well as a way of teaching, I wanted to give it a name. I think in this volume I finally have a name that I am happy with: “critical exploration in the classroom.”

In these essays, two other names are used to refer to the work. And yet another term—teaching/learning research—was used in a different book (Duckworth, 2001). The first name I tried appears in chapter 12—teaching-research. Teaching-research for me meant research, the act of which was indistinguishable from the act of teaching. It meant teaching (which involves listening to learners explain) as the best way for us to learn about the development of thoughts. I think this name is apt, but it is undistinguished. There is the large field of teacher research, and that term covers a vast range of work—almost any research that is done by a teacher. It would take a careful reader to note the difference in spelling, let alone meaning.

The next name I tried was “extended clinical interviewing.” Pasget called his method for studying children’s thinking, “clinical interviewing.” I wanted to acknowledge my debt to him and to Inhelder (to whom I had apprenticed in learning to do the research). I added the word “extended,” as I explain in chapter 10, because in this work, interviews could be extended in two ways: They could be continued through time, over many different sessions; and they could be carried out with more than one person at once. The drawbacks with this name are that it is difficult mouthful; that it sounds technical—labouratory-based; and that there is in fact far more to this work than asking questions.

The name “critical exploration” comes from Inhelder. Its origins and my choice to use it are taken up in chapter 11.

The three phrases—teaching-research, extended clinical interviewing, and critical exploration in the classroom—are interchangeable. They are three different names for the same work.

I close this Introduction with a word about chapter 9. Just as school practice has, in the last 20 years, been going counter to the ideas expressed here, so has public practice in the realms of peace and social justice. And just as I think there has never been a greater need to consider ideas about learning and teaching such as the ones in this book, I also think that there has never been a greater need to educate children about peace and social justice.
Introduction
college biology course. But as a staff member of the Elementary Science Study, where the approach to science education was to engage teachers and children directly with fascinating corners of the material world, my science education took the form of becoming intrigued by phenomena, and struggling to figure them out. I was in the company of people who presented me with such phenomena, who did not insist on explaining to me what they knew, and who talked about what I was doing in ways that never caused me to doubt that what I was engaged in was science. I spent hour after hour exploring materials and phenomena, with my colleagues on hand from time to time to ask how I was getting along and what I was making of things, and to suggest, “Why don’t you try it this way?” or “Here, have a look at this.” I generated acorns; I used syringes to move air from one container to another; I explored the effects of different objects as pendulum bobs; I spent a full day with a flashlight battery and bulb by my side, in order to keep trying out ways to arrange a single wire so the bulb would light (I spent about half an hour at the beginning of the morning, went back to it off-and-on all day, and finally succeeded in lighting the bulb in the middle of the afternoon; this was the beginning of many months of explorations with batteries and bulbs). I tried to make liquid mixtures of identical densities—one oil-based and one water-based, so they did not mix with each other. I tried using any long object—a broom, a spoon, a toothbrush—as a balance; I watched frog eggs develop. I worked hard during these explorations, trying to figure out why something had happened, and to think of what I could do next, to shed more light on it. I was captivated by this world of fascinating phenomena, by its accessibility and its complexity. I was intrigued, also, with what it was like to be learning these things in this way, and with how my colleagues thought about opening the world to children.

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The Having of Wonderful Ideas

Kevin, Stephanie, and the Mathematician

With a friend, I reviewed some classic Piagetian interviews with a few children. One involved the ordering of lengths. I had cut 10 cellophane drinking straws into different lengths and asked the children to put them in order, from smallest to biggest. The first two 7-year-olds did it with no difficulty and little interest. Then came Kevin. Before I said a word about the straws, he picked them up and said to me, “I know what I’m going to do,” and proceeded, on his own, to order them by length. He didn’t mean, “I know what you’re going to ask me to do.” He meant, “I have a wonderful idea about what to do with these straws. You’ll be surprised by my wonderful idea.”

It wasn’t easy for him. He needed a good deal of trial and error as he set about developing his system. But he was so pleased with himself when he accomplished his self-set task that when I decided to offer them to him to keep (10 whole drinking straws), he glowed with joy, showed them to one or two select friends, and stored them away with other treasures in a shoebox.

The having of wonderful ideas is what I consider the essence of intellectual development. And I consider it the essence of pedagogy to give Kevin the occasion to have his wonderful ideas and to let him feel good about himself for having them. To develop this point of view and to indicate where Piaget fits in for me, I need to start with some autobiography, and I apologize for that, but it was a struggle of some years’ duration for me to see how Piaget was relevant to schools at all.

I had never heard of Piaget when I first sat in a class of his. It was as a philosopher that Piaget won me, and I went on to spend two years in Geneva as a graduate student and research assistant. Then, some years later,
I began to pay attention to schools, when, as a Ph.D. dropout, I accepted a job developing elementary science curriculum, and found myself in the midst of an exciting circle of educators.

The colleagues I admired most got along very well without any special knowledge of psychology. They trusted their own insights about when and how children were learning, and they were right to: Their insights were excellent. Moreover, they were especially distrustful of Piaget. He had not yet appeared on the cover of Saturday Review or the New York Times Magazine, and they had their own picture of him: a severe, humorless intellectual confronting a small child with questions that were surely incomprehensible, while the child tried to tell from the look in his eyes what the answer was supposed to be. No wonder the child couldn't think straight. (More than one of these colleagues first started to pay attention to Piaget when they saw a photo of him. He may be Swiss, but he doesn't look like Calvin! Maybe he can talk to children after all.)

I myself didn't know what to think. My colleagues did not seem to be any worse for not taking Piaget seriously. Nor, I had to admit, did I seem to be any the better. Schools were such complicated places compared with psychology labs that I couldn't find a way to be of any special help. Not only did Piaget seem irrelevant, I was no longer sure that he was right. For a couple of years, I scarcely mentioned him and simply went about the business of trying to be helpful, with no single instance, as I recall, of drawing directly on any of his specific findings.

The lowest point came when one of my colleagues gleefully showed me an essay written in a first grade by 8-year-old Stephanie. The children had been investigating capillary tubes, and were looking at the differences in the height of the water as a function of the diameter of the tube. Stephanie's essay read as follows: I know why it looks like there's more in the skinny tube. Because it's higher. But the other is fatter, so there's the same.

My colleague triumphantly took this statement as proof that 8-year-olds can reason about the compensation of two dimensions. I didn't know what to say. Of course, it should have been simple. Some 6-year-olds can reason about compensation. The ages that Piaget mentions are only norms, not universals. Children develop at a variety of speeds, some more slowly and some more quickly. But I was so unsure of myself at that point that this incident shocked me badly, and all of that only sounded like a lame excuse.

I do have something else to say about that incident later. For now, I shall simply try to describe my struggle. Even if I did believe that Piaget was right, how could he be helpful? If the main thing that we take from Piaget is that before certain ages children are unable to understand certain things—conservation, transitivity, spatial coordinates—what do we do about it? Do we try to teach the children these things? Probably not, because on the one hand Piaget leads us to believe that we probably won't be very successful at it; and on the other hand, if there is one thing we have learned from Piaget it is that children can probably be left to their own devices in coming to understand these notions. We don't have to try to furnish them. It took a few months before that was clear to me, but I did conclude that this was not a very good way to make use of Piaget.

An alternative might be to keep in mind the limits on children's abilities to classify, conserve, order, and so forth, when deciding what to teach them at certain ages. However, I found this an inadequate criterion. There was so much else to keep in mind. The most obvious reason, of course, was that any class of children has a great diversity of levels. Tailoring to an average level of development is sure to miss a large proportion of the children. In addition, a Piaget psychologist has no monopoly here. When trying to approximate the abilities of a group of children of a given age, able teachers like my colleagues could make as good approximations as I.

What I found most appealing was that the people with whom I was working judged the merits of any suggestion by how well it worked in classrooms. That is, instead of deciding on a priori grounds what children ought to know, or what they ought to be able to do at a certain age, they found activities, lessons, points of departure, that would engage children in real classrooms, with real teachers. In their view, it was easy to devise all-embracing schemes of how science (as it was in this instance) could be organized for children, but to make things work pedagogically in classrooms was the difficult part. They started with the difficult part. A theory of intellectual development might have been the basis of a theoretical framework of a curriculum. But in making things work in a classroom, it was but a small part compared with finding ways to interest children, to take into account different children's interests and abilities, to help teachers with no special training in the subject, and so forth. So, the burden of this curriculum effort was classroom trials. The criterion was whether they worked, and their working depended only in part on their being at the right intellectual level for the children. They might be perfectly all right, from the point of view of intellectual demands, and yet fall short in other ways. Most often, it was a complex combination.

As I was struggling to find some framework within which my knowledge of Piaget would be useful, I found, more or less incidentally, that I was starting to be useful myself. As an observer for some of the pilot teaching of this program, and later as a pilot teacher myself, I found that I had a certain skill in being able to watch and listen to children and that I
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did have some good insights about how they were really seeing the problem. This led to a certain ability to raise questions that made sense to the children or to think of new orientations for the whole activity that might correspond better to their way of seeing things. I don’t want to suggest that I was unique in this. Many of the teachers with whom I was working had similar insights, as did many of the mathematicians and scientists among my colleagues, who, from their points of view, could tell when children were seeing things differently from the ways they did. But the question of whether I was unique is not really pertinent. For me, through my experience with Piaget of working closely with one child at a time and trying to figure out what was really in that child’s mind, I had gained a wonderful background for being sensitive to children in classrooms. I think that a certain amount of this kind of background would be similarly useful for every teacher.

This sensitivity to children in classrooms continued to be central in my own development. As a framework for thinking about learning, my understanding of Piaget has been invaluable. This understanding, however, has also been deepened by working with teachers and children. I may be able to shed some light on that mutual relationship by referring again to 6-year-old Stephanie’s essay on compensation. Few of us, looking at water rise in capillary tubes of different diameters, would bother to wonder whether the quantities are the same. Nobody asked Stephanie to make that comparison and, in fact, it is impossible to tell just by looking. On her own, she felt it was a significant thing to comment upon. I take that as an indication that for her it was a wonderful idea. Not long before, she believed that there was more water in the tube in which the water was higher. She had recently won her own intellectual struggle on that issue, and she wanted to point out her finding to the world for the benefit of those who might be taken in by preliminary appearances.

This incident, once I had figured it out, helped me think about a point that bothered me in one of Piaget’s anecdotes. You may recall Piaget’s account of a mathematician friend who inspired his studies of the conservation of number. This man told Piaget about an incident from his childhood, where he counted a number of pebbles he had set out in a line. Having counted them from left to right and found there were 10, he decided to see how many there would be if he counted them from right to left. Intrigued to find that there were still 10, he put them in a different arrangement and counted them again. He kept rearranging and counting them until he decided that, no matter what the arrangement, he was always going to find that there were 10. Number is independent of the order of counting. My problem was this: In Piaget’s account, if 10 eggs are spread out so they take more space than 10 eggcups, a classic nonconservist will maintain that there are more eggs than eggcups, even if he counts and finds that he comes to 10 in both cases. Counting is not sufficient to convince him that there are enough eggcups for all the eggs. How is it, then, that for the mathematician, counting was sufficient? If he was a nonconservist at the time, counting should not have made any difference. If he was a conservist, he should have known from the start that it would always come out the same.

I think it must be that the whole enterprise was his own wonderful idea. He raised the question for himself and figured out for himself how to try to answer it. In essence, I am saying that he was in a transitional moment, and that Stephanie and Kevin were, too. He was at a point where a certain experience fit into certain thoughts and took him a step forward. A powerful pedagogical point can be made from this. These three instances dramatize it because they deal with children moving ahead with Piaget notions, which are usually difficult to advance on the basis of any one experience. The point has two aspects: First, the right question at the right time can move children to peaks in their thinking that result in significant steps forward and real intellectual excitement; and, second, although it is almost impossible for an adult to know exactly the right time to ask a specific question of a specific child—especially for a teacher who is concerned with 30 or more children—children can raise the right question for themselves if the setting is right. Once the right question is raised, they are moved to test themselves to the fullest to find an answer. The answers did not come easily in any of these three cases, but the children were prepared to work through them. Having confidence in one’s ideas does not mean “I know my ideas are right”; it means “I am willing to try out my ideas.”

As I put together experiences like these and continued to think about them, I started developing some ideas about what education could be and about the relationships between education and intellectual development.

HANK

It is a truism that all children in their first and second years make incredible intellectual advances. Piaget has documented these advances from his own point of view, but every parent and every psychologist knows this to be the case. One recurring question is, why does the intellectual development of vast numbers of children then slow down? What happens to children’s curiosity and resourcefulness later in their childhood? Why do so few continue to have their own wonderful ideas? I think part of the answer is that intellectual breakthroughs come to be less and less
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valued. Either they are dismissed as being trivial—as Kevin’s or Stephanie’s or the mathematician’s might have been by some adults—or else they are discouraged as being unacceptable—the discovering how it feels to wear shoes on the wrong feet, or asking questions that are socially embarrassing, or destroying something to see what it is like inside. The effect is to discourage children from exploring their own ideas and to make them feel that they have no important ideas of their own, only silly or evil ones.

But I think there is at least one other part of the answer, too. Wonderful ideas do not spring out of nothing. They build on a foundation of other ideas. The following incident may help to clarify what I mean.

Hank was an energetic and not very scholarly fifth grader. His class had been learning about electric circuits with flashlight batteries, bulbs, and various wires. After the children had developed considerable familiarity with these materials, the teacher made a number of mystery boxes.* Two wires protruded from each box, but inside, unseen, each box had a different way of making contact between the wires. In one box the wires were attached to a battery; in another they were attached to a bulb; in a third, to a certain length of resistance wire; in a fourth box they were not attached at all; and so forth. By trying to complete the circuit on the outside of a box, the children were able to figure out what made the connection inside the box. Like many other children, Hank attached a battery and a bulb to the wire outside the box. Because the bulb lit, he knew at least that the wires inside the box were connected in some way. But, because it was somewhat dimmer than usual, he also knew that the wires inside were not connected directly to each other by a piece of ordinary copper wire. Along with many of the children, he knew that the degree of dimness of the bulb meant that the wires inside were connected either by another bulb of the same kind or by a certain length of resistance wire.

The teacher expected them to go only this far. However, in order to push the children to think a little further, she asked them if they could tell whether it was a bulb or a piece of wire inside the box. She herself thought there was no way to tell. After some thought, Hank had an idea. He undid the battery and bulb that he had already attached on the outside of the box. In their place, using additional copper wire, he attached six batteries in a series. He had already experimented enough to know that six batteries would burn out a bulb, if it was a bulb inside the box. He also knew that once a bulb is burned out, it no longer completes the circuit. He then attached the original battery and bulb again. This time he found that the bulb on the outside of the box did not light. So he reasoned, rightly, that there had been a bulb inside the box and that now it was burned out. If

*This activity is from the Elementary Science Study’s Batteries and Bulbs, 1969.

there had been a wire inside, it would not have burned through and the bulb on the outside would still light.

Note that to carry out that idea, Hank had to take the risk of destroying a light bulb. In fact, he did destroy one. In accepting this idea, the teacher had to accept not only the fact that Hank had a good idea that even she did not have, but also that it was worthwhile to destroy a small piece of property for the sake of following through an idea. These features almost turn the incident into a parable. Without these kinds of acceptance, Hank would not have been able to pursue his idea. Think of how many times this acceptance is not forthcoming in the life of any one child.

But the main point to be made here is that in order to have his idea, Hank had to know a lot about batteries, bulbs, and wires. His previous work and familiarity with those materials were a necessary aspect of this occasion for him to have a wonderful idea. David Hawkins (2000) has said of curriculum development, “You don’t want to cover a subject; you want to uncover it” (p. 79). That, it seems to me, is what schools should be about. They can help to uncover parts of the world that children would not otherwise know how to tackle. Wonderful ideas are built on other wonderful ideas. In Piaget’s terms, you must reach out to the world with your own intellectual tools and grasp it, assimilate it, yourself. All kinds of things are hidden from us—even though we surround us—unless we know how to reach out for them. Schools and teachers can provide materials and questions in ways that suggest things to be done with them, and children, in the doing, cannot help but become inventive.

There are two aspects to providing occasions for wonderful ideas. One is being willing to accept children’s ideas. The other is providing a setting that suggests wonderful ideas to children—different ideas to different children—as they are caught up in intellectual problems that are real to them.

**WHAT SCHOOLS CAN DO**

I had the chance to evaluate an elementary science program in Africa. For the purposes of this discussion it might have been set anywhere. Although the program was by no means a deliberate attempt to apply Piaget’s ideas, it was, to my mind, such an application in the best sense. The assumptions that lay behind the work are consistent with the ideas I have just been developing. The program set out to reveal the world to children. The developers sought to familiarize the children with the material world—that is, with biological phenomena, physical phenomena, and technological phenomena: flashlights, mosquito larvae, coconuts, clay. When I speak of familiarity, I mean feeling at home with these things: knowing what to
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expect of them, what can be done with them, how they react to various circumstances, what you like about them and what you don’t like about them, and how they can be changed, avoided, preserved, destroyed, or enhanced.

Certainly the material world is too diverse and too complex for a child to become familiar with all of it in the course of an elementary school career. The best one can do is to make such knowledge, such familiarity, seem interesting and accessible to the child. That is, one can familiarize children with a few phenomena in such a way as to catch their interest, to let them raise and answer their own questions, to let them realize that their ideas are significant so that they have the interest, the ability, and the self-confidence to go on by themselves.

Such a program is a curriculum, so to speak, but a curriculum with a difference. The difference can best be characterized by saying that the unexpected is valued. Instead of expecting teachers and children to do only what was specified in the booklets, it was the intention of the program that children and teachers would have so many unanticipated ideas of their own about the material that they would never even use the booklets. The purpose of developing booklets at all is that teachers and children start producing and following through their own ideas, if possible getting beyond needing anybody else’s suggestions. Although it is unlikely to be completely realized, this represents the ideal orientation of the program. It is a rather radical view of curriculum development.

It is just as necessary for teachers as for children to feel confidence in their own ideas. It is important for them as people and it is important in order for them to feel free to acknowledge the children’s ideas. If teachers feel that their class must do things just as the book says, and that their excellence as teachers depends on this, they cannot possibly accept the children’s divergence and creations. A teachers’ guide must give enough indications, enough suggestions, so that the teacher has ideas to start with and to pursue. But it must also enable the teacher to feel free to move in her own direction when she has other ideas.

For instance, the teachers’ guides for this program include many examples of things children are likely to do. The risk is that teachers may see these as things that the children in their classes must do. Whether or not the children do them becomes a measure of successful or unsuccessful teaching. Sometimes the writers of the teachers’ guides intentionally omit mention of some of the most exciting activities because they almost always happen even if they are not arranged. If the teacher expects them, she will often force them, and they no longer happen with the excitement of wonderful ideas. Often the writers include extreme examples, so extreme that a teacher cannot really expect them to happen in her class. These examples are meant to convey the message that “even if the children do that it’s OK! Look, in one class they even did this!” This approach often is more fruitful than the use of more common examples where message is likely to be “this is what ought to happen in your class.”

The teachers’ guides dealt with materials that were readily available in or out of schools, and suggested activities that could be done with these materials so that children become interested in them and started asking their own questions. For instance, there are common substances around us that provide the essential basis of chemistry knowledge. They interact in all sorts of interesting ways, accessible to all of us if only we know how to reach out for them. This is a good instance of a part of the world that is waiting to be uncovered. How can it be uncovered for children in a way that gives them an interest in continuing to find out about it, a way that gives them the occasion to take their own initiatives, and to feel at home in this part of the world?

The teachers’ guide suggests starting with salt, ashes, sugar, cassava starch, alum, lemon juice, and water. When mixed together, some of these cause bubbles. Which combinations cause bubbles? How long does the bubbling last? How can it be kept going longer? What other substances cause bubbles? If a combination bubbles, what can be added that will stop the bubbling? Other things change color when they are mixed together, and similar questions can be asked of them.

Written teachers’ guides, however, cannot bear the burden alone, if this kind of reaching is totally new. To get such a program started, a great deal of teacher education is necessary as well. Although I shall not try to go into this in any detail, there seem to be three major aspects to such teacher education. First, teachers themselves must learn in the way that the children in their classes will be learning. Almost any one of the units developed in this program is as effective with adults as it is with children. The teachers themselves learn through some of the units and feel what it is like to learn in this way. Second, the teachers work with one or two children at a time so that they can observe them closely enough to realize what is involved for the children. Last, it seems valuable for teachers to see films or live demonstrations of a class of children learning in this way, so that they can begin to think that it really is possible to run their classes in such a way. A fourth aspect is of a slightly different nature. Except for the rare teacher who will take this leap all on his or her own on the basis of a single course and some written teachers’ guides, most teachers need the support of at least some nearby co-workers who are trying to do the same thing, and with whom they can share notes. An even better help is the presence of an experienced teacher to whom they can go with questions and problems.
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The materials we chose were not, of course, the same as those that children in the program had studied. We chose materials of two sorts on the one hand, imported materials that none of the children had ever seen before—plastic color filters, geometric pattern blocks, folding mirrors, commercial building sets, for example. On the other hand, we chose some materials that were familiar to all the children whether or not they had been in the program—cigarette foil, match boxes, rubber rings from inner tubes, scraps of wire, wood, and metal, empty spoons, and so forth.

From each class we chose a dozen children at random and told them—in their own vernacular—to go into the room and do whatever they wanted with the materials they would find there. We told them that they could move around the room, talk to each other, and work with their friends.

We studied 15 experimental classes and 13 control classes from first to seventh grades. Briefly, and inadequately, summarizing the results of this phase, we found that the children who had been in the program did indeed have more ideas about how to work with the materials. Typically, the children in these classes would take a first look at what was offered, try a few things, and then settle down to work with involvement and concentration. Children sometimes worked alone and sometimes collaborated. They carried materials from table to table, using them in ways we had not anticipated. As time went on, there was no sign that they were running out of ideas. On the contrary, their work became so interesting that we were always disappointed to have to stop them after 40 minutes.

By contrast, the other children had a much smaller range of ideas about what to do with the materials. On the one hand, they tended to copy a few leaders. On the other hand, they tended to leave one piece of work fairly soon and to switch to something else. There were few instances of elaborate work in which a child spent a lot of time and effort to overcome difficulties in what he or she was trying to do. In some of these classes, after 30 to 35 minutes, all the children had run out of ideas and were doing nothing.

We had assessed two things in our evaluation: diversity of ideas in a class, and depth to which the ideas were pursued. The experimental classes were overwhelmingly ahead in each of the two dimensions. This first phase of assessment was actually a substitute for what we really wanted to do. Ideally, we wanted to know whether the experience of these children in the program had the effect of making them more alert, more aware of the possibilities in ordinary things around them, and more questioning and exploring during the time they spent outside school. This would be an intriguing question to try to answer, but we did not have the time to tackle it. The procedure that we did develop, as just described, may have
AN EVALUATION STUDY

What the children are doing in one of these classrooms may be lively and interesting, but it would be helpful to know what difference the approach makes to them in the long run, to compare in some way the children who were in this program with children who were not, and to see whether in some standard situation they now act differently.

One of my thoughts about ways in which these children might be different was based on the fact that many teachers in this program had told us that their children improved at having ideas of what to do, at raising questions, and at answering their own questions; that is, at having their own ideas and being confident about their own ideas. I wanted to see whether this indeed was the case.

My second thought was more ambitious. If these children had really become more intellectually alert, so that their minds were alive and working not only in school but outside school, they might, over a long enough period of time, make significant headway in their intellectual development, as compared with other children.

In sum, these two aspects would put to the test my notions that the development of intelligence is a matter of having wonderful ideas and feeling confident enough to try them out, and that schools can have an effect on the continuing development of wonderful ideas. The study has been written up elsewhere (Duckworth, 1978) but let me give a summary of it here.

The evaluation had two phases. The procedure developed for the first phase was inspired in part by a physics examination given to students at Cornell University by Philip Morrison. His examination was held in the laboratory. The students were given sets of materials, the same set of materials for each student, but they were given no specific problem. Their problem was to find a problem and then to work on it. For Morrison, the crucial thing is finding the question, just as it was for Kevin, Stephanie, and the mathematician. In this examination, clear differences in the degree of both knowledge and inventiveness were revealed in the problems the students set themselves, and the work they did was only as good as their problems.

In our evaluation study, we had to modify this procedure somewhat to make it appropriate for children as young as six years of age. Our general question was what children with a year or more of experience in this program would do with materials when they were left to their own resources without any teacher at all. We wanted to know whether children who had been in the program had more ideas about what to do with materials than did other children.

The materials we chose were not, of course, the same as those that children in the program had studied. We chose materials of two sorts: on the one hand, imported materials that none of the children had ever seen before—plastic color filters, geometric pattern blocks, folding mirrors, commercial building sets, for example. On the other hand, we chose some materials that were familiar to all the children whether or not they had been in the program—cigarette foil, match boxes, rubber rings from inner tubes, scraps of wire, wood, and metal, empty spools, and so forth.

From each class we chose a dozen children at random and told them—in their own vernacular—to go into the room and do whatever they wanted with the materials they would find there. We told them that they could move around the room, talk to each other, and work with their friends.

We studied 15 experimental classes and 15 control classes from first to seventh grades. Briefly, and inadequately, summarizing the results of this phase, we found that the children who had been in the program did indeed have more ideas about how to work with the materials. Typically, the children in these classes would take a first look at what was offered, try a few things, and then settle down to work with involvement and concentration. Children sometimes worked alone and sometimes collaborated. They carried materials from table to table, using them in ways we had not anticipated. As time went on, there was no sign that they were running out of ideas. On the contrary, their work became so interesting that we were always disappointed to have to stop them after 40 minutes.

By contrast, the other children had a much smaller range of ideas about what to do with the materials. On the one hand, they tended to copy a few leaders. On the other hand, they tended to leave out one piece of work fairly soon and to switch to something else. There were few instances of elaborate work in which a child spent a lot of time and effort to overcome difficulties in what he or she was trying to do. In some of these classes, after 30 to 35 minutes, all the children had run out of ideas and were doing nothing.

We had assessed two things in our evaluation: diversity of ideas in a class, and depth to which the ideas were pursued. The experimental classes were overwhelmingly ahead in each of the two dimensions. This first phase of assessment was actually a substitute for what we really wanted to do. Ideally, we wanted to know whether the experience of these children in the program had the effect of making them more alert, more aware of the possibilities in ordinary things around them, and more questioning and exploring during the time they spent outside school. This would be an intriguing question to try to answer, but we did not have the time to tackle it. The procedure that we did develop, as just described, may have
been too close to the school setting to give rise to any valid conclusions about what children are like in the world outside school. However, if you can accept with me, tentatively, the thought that our results might indicate a greater intellectual alertness in general—a tendency to have wonderful ideas—then the next phase takes on a considerable interest.

I am hypothesizing that this alertness is the motor of intellectual development (in Piaget's terms, operational thinking). No doubt there is a continuum: No normal child is completely unalert. But some are far more alert than others. I am also hypothesizing that a child's alertness is not fixed. I believe that, by opening up to children the many fascinating aspects of the ordinary world and by enabling them to feel that their ideas are worthwhile having and following through, their tendency to have wonderful ideas can be affected in significant ways. This program seemed to be doing both those things, and by the time I evaluated it, some children had been in the classes for up to three years. It seemed to me that we might—just might—find that the two or three years of increased alertness that this program fostered had made some difference to the intellectual development of the children.

In the second phase, then, we examined the same children individually, using Piaget problems administered by a trained assistant who spoke the language of the children. A statistical analysis revealed that on five of the six problems we studied, the children in the experimental classes did significantly better than the children in the comparison classes.

I find this a pretty stunning result on the whole. But I want to insist on one particular view of the result. I do not, in any way, want to suggest that the important thing for education to be about is acceleration of Piaget stages (see chapter 5). I want to make a theoretical point. My thesis at the outset of this chapter was that the development of intelligence is a matter of having wonderful ideas. In other words, it is a creative affair. When children are afforded the occasions to be intellectually creative—by being offered matter to be concerned about intellectually and by having their ideas accepted—then not only do they learn about the world, but as a happy side effect their general intellectual ability is stimulated as well.

Another way of putting this is that I think the distinction made between "divergent" and "convergent" thinking (Hudson, 1968) is oversimplified. Even to think a problem through to its most appropriate end point (convergent) one must create various hypotheses to check-out (divergent). When Hank came up with a closed end point to the problem, it was the result of a brilliantly imaginative—that is, divergent—thought. We must conceive of the possibilities before we can check them out.
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**CONCLUSION**

I am suggesting that children do not have a built-in pace of intellectual development. I would temper that suggestion by saying that the built-in aspect of the pace is minimal. The having of wonderful ideas, which I consider the essence of intellectual development, would depend instead to an overwhelming extent on the occasions for having them. I have dwelt at some length on how important it is to allow children to accept their own ideas and to work them through. I would like now to consider the intellectual basis for new ideas.

I react strongly against the thought that we need to provide children with only a set of intellectual processes—a dry, contentless set of topics that they can go about applying. I believe that the tools cannot help developing once children have something real to think about; and if they don't have anything real to think about, they won't be applying tools anyway. That is, there really is no such thing as a contentless intellectual tool. If a person has some knowledge at his disposal, he can try to make sense of new experiences and new information related to it. He fits it into what he has.

By knowledge I do not mean verbal summaries of somebody else's knowledge. I am not urging textbooks and lectures. I mean a person's own repertoire of thoughts, actions, connections, predictions, and feelings. Some of these may have as their source something read or heard. But the individual has done the work of putting them together for himself or herself, and they give rise to new ways to put them together.

The greater the child's repertoire of actions and thoughts—in Piaget's terms, schemes—the more material he or she has for trying to put things together in his or her own mind. The essence of the African program I described is that children increase the repertoire of actions that they carry out on ordinary things, which in turn gives rise to the need to make more intellectual connections.

Let us consider a child who has had the world of common substances opened to him, as described earlier. He now has a vastly increased repertoire of actions to carry out and of connections to make. He has seen that when you boil away sea water, a salt residue remains. Would some residue remain if he boiled away beer? If he dissolved this residue in water again, would he have beer again—flat beer? He has seen that he can get a colored liquid from flower petals if he crushes them. Could he get that liquid to go into water and make colored water? Could he make colored coconut oil this way? All these questions and the actions they lead to are based on the familiarity the child has gained with the possibilities contained in this world of common substances.
Learning with Breadth and Depth

If ideas develop on their own so slowly, what can we do to speed them up? In chapter 3, we pointed out that Piaget referred to this as the American question. For him the question is not how fast you go but how far you go. He delighted in the results of a study of kittens carried out by Howard Gruber. Studying his own children, Piaget had concluded that they were about a year old before they realized that an object had its own continuing existence and location even when out of their reach and out of their sight. Gruber found that kittens go through all the same steps that children do, but instead of taking a year, they take six weeks (Gruber, Grzus, & Banazizi, 1971). Piaget cheerfully pointed out that you can scarcely say that kittens are better off for having cut almost a year off the time. After all, they don't get much further.

How could it be that going fast does not mean going far? A useful metaphor might be the construction of a tower—all the more appropriate given that Piaget thinks of the development of intelligence as continual construction. Building a tower with one brick on top of another is a pretty speedy business. But the tower will soon reach its limits, compared with one built on a broad base or a deep foundation—which of course takes a longer time to construct.

What is the intellectual equivalent of building in breadth and depth? I think it is a matter of making connections: Breadth could be thought of as the widely different spheres of experience that can be related to one another; depth could be thought of as the many different kinds of connections that can be made among different facets of our experience. I am not sure whether intellectual breadth and depth can be separated from each other, except in talking about them. In this chapter I shall not try to keep them separate, but instead try to show how learning with breadth and depth is a different matter from learning with speed.
PRODUCTIVE WRONG IDEAS

If a child spends time exploring all the possibilities of a given notion, it may mean that she holds onto it longer, and moves onto the next stage less quickly; but by the time she does move on, she will have a far better foundation—the idea will serve her far better, will stand up in the face of surprises. Let me develop a hypothetical example to show what I mean, based on the notion of the conservation of area.

Imagine two identical pieces of paper; you cut one in half and rearrange the pieces so the shape is different from the original one, while preserving the same area, as in the example in Figure 6.1. One might think that it would be to anyone’s advantage to realize early in life that a change in shape does not affect area; that no matter how a shape is transformed, its area is conserved. But I can imagine a child not managing to settle that question as soon as others because she raises for herself the question of the perimeter. In fact the perimeter does change, and thinking about the relationship between those two is complicated work. One child might, then, take longer than another to come to the conclusion that area is conserved, independent of shape, but her understanding will be the better for it. Most children (and adults) who arrive smartly at the notion that area is independent of shape do not think about the perimeter and are likely to become confounded if it is brought up. Having thought about perimeter on her own, she has complicated the job of thinking about area, but once she has straightened it out, her understanding is far deeper than that of someone who has never noticed this difference between area and perimeter.

![Figure 6.1](image)

Exploring ideas can only be to the good, even if it takes time. Wrong ideas, moreover, can only be productive. Any wrong idea that is corrected provides far more depth than if one never had a wrong idea to begin with. You master the idea much more thoroughly if you have considered alternatives, tried to work it out in cases where it didn’t work, and figured out why it was that it didn’t work, all of which takes time. After this hypothetical introduction, here are some examples where making the mistakes and correcting them reveal and give rise to a far better grasp of the phenomenon than there would have been if no mistakes were made at all.

One experiment involves an odd-shaped lake like the one in Figure 6.2, with a road around it, and a bi-colored car on the road, one side black and one side white. Let’s say the white side is next to the water to start with; the question is, after the car drives around a corner, or around several corners, which color will be beside the water? Six-year-olds, after one or two mistaken predictions, usually come to be quite sure that it will always be the white. Eight-year-olds, on the other hand, can be very perplexed, and not quite get it straight; no matter how often they see the white side come out next to the water. They keep predicting that this time the black side will be next to the water.

Now one might be tempted to think that 6-year-olds know more than 8-year-olds. They, after all, do not make mistakes. But I think it is the greater breadth and depth of the 8-year-olds’ insight that leads to their perplexity. Eight-year-olds are often just at the point of organizing space into some interleaved whole: Your left is opposite my right; something that you can see from your point of view may be hidden from my point of view; if a car in front of me is facing right, I see its right side, and if it turns 180°, I’ll see its left side. With all these shifting, relative relationships, what is it about the lake that makes that relationship an absolute? No matter how many curves there are in the road, the same side is always next to the water. If a car turns 180°, I thought I would see its other side; well, how is it that the same side is next to the water? What is it that stays the same.
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and what is it that changes, after all? The 6-year-old, who has no idea of the systematic changes involved in some spatial relationships, has no difficulty seeing the constant in the lake problem; it is because the 6-year-old is trying to make sense of the lake in a far broader context that the right answer is not so immediate. The dawning organization of something new throws into confusion something that had been simple before. But when, a few months later, the 8- or 9-year-old does start to understand that the same side must always stay next to the lake, his or her understanding is far deeper than that of the 6-year-old; it is set in the context of an understanding of spatial relationships as a whole.

Here is another example, where what appears to be less facility really indicates greater understanding. I was working with two children, who happened to be brother and sister, and they were making all possible arrangements of three colors. After each of them had found all six possibilities, I added a fourth color, and they tried again. The sister, who was younger, rapidly produced a dozen, and was still going. The older brother stopped at four, and declared that that's all there were. But look at what he had done. With three colors, he had made the arrangement shown in Figure 6.3. Now, into what he had already, he inserted the fourth color, in each of the possible positions as shown in Figure 6.4. It was from his sense of system—his sense (which can only be called mathematical) that there was a fixed and necessary number of placements—that he stopped there: The new color was in each possible position, within a system that had all of the other colors already in each possible position. It is true that his thinking left out one step, but nonetheless his was a far deeper understanding of permutations than his sister's facile but random generation of yet more arrangements that looked different.

\[
\begin{array}{l}
RBY \\
RYB \\
BRY \\
BYR \\
YRB \\
YBR
\end{array}
\]

Figure 6.3

\[
\begin{array}{l}
GRBY \\
RGYB \\
BRGY \\
BYRG \\
YBR \\
YBR
\end{array}
\]

Figure 6.4

WAYS OF MEASURING—PRODUCTIVE AND UNPRODUCTIVE

Getting closer to everyday concerns in the classroom, think of measurements. It can seem very straightforward—count the number of units that apply to some quantity and there it is, measured: so many foot-long rulers in a table, plus a number of inches; so many minutes in the running of a mile, plus a number of seconds. But take this example, for which I am indebted to Strauss, Stavy, and Orpaz (1981): You've measured the temperature of one glass of water—100°C; you add to it another glass of water, which is also 100°C. What will the temperature be now? Most of our measurement experience would lead us to say 200°C. And that is what a lot of children do in fact say—having easily understood how to add measurements together—but never having wondered where or whether to add measurements together.

Let me, by contrast, give some examples of invention of ways of measuring, which might seem foolish and inefficient, but which are thoroughly understood by their inventors. The first one deserves a better accounting than I can undertake here. In a class studying (once again) pendulums, children had explored coupled pendulums, set up like the example in Figure 6.5. If everything is symmetrical when you start one bob, then after a few swings the other bob starts to move; gradually both A's movement diminishes and bob B's movement increases, until A is stopped and B is swinging wildly. Then the movement passes back to A, and so on. Suppose, however, that everything is not symmetrical—the stick is tilted, or one string is longer than the other; or one bob weighs more than the other. In that case, the bob that starts swinging does pass some of its movement on to the other, but it does not come to a halt itself; the bobs are asymmetric—they belong only to the bob that was at rest when the other started swinging.
and what is it that changes, after all? The 6-year-old, who has no idea of the systematic changes involved in some spatial relationships, has no difficulty seeing the constant in the lake problem; it is because the 6-year-old is trying to make sense of the lake in a far broader context that the right answer is not so immediate. The dawning organization of something new throws into confusion something that had been simple before. But when, a few months later, the 8- or 9-year-old does start to understand that the same side must always stay next to the lake, his or her understanding is far deeper than that of the 6-year-old; it is set in the context of an understanding of spatial relationships as a whole.

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R BY
RY B
B R Y
B Y R
Y RB
Y BR

Figure 6.3

GR BY
RG YB
BR GY
BY RG

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Figure 6.5
That is a long introduction. The point is that in this class, a time came
when the children were interested in comparing the weights of the wood-
en bob and the steel bob. Scales were available, and most of the children
got to them. But Elliott, who happened to be the least scholarly child in
the class, had a different idea. He set up a coupled pendulum, hung a steel
bob on one string, and then added wooden bobs to the other, trying the
coupled motion each time he added a bob—until, at four wooden bobs,
the halts were alternating symmetrically from string to string. So he knew
the four on one string must weigh the same as the one on the other. This
astonishingly imaginative grasp of what it means to compare weights of
things should be contrasted with the following tale.

In a different pendulum class, junior high school students had just
previously been taught the equilibrium formula that applies to balances:
Distance times weight on one side must equal distance times weight on
the other. The only weighing mechanism available to them now was a
strip of pegboard, supported in the center (Figure 6.6). When the stu-
dents became interested in weighing the bobs, they hung a wooden bob
on one end, and then a steel bob on the other side, not on the end, but
placed so as to make the pegboard horizontal, announcing, “There, they
weigh the same. We learned that just last week, they weigh the same.” It
seemed clear that that formula had been hastily learned, and remained
quite unexplored.

Figure 6.6

The next example comes from work done with Jeanne Langer and
Magdalene Lampert at the Massachusetts Institute of Technology (Lan-
ger, Duckworth, & Lampert, 1981). We were working with a group
of Cambridge teachers, helping them examine their own ways of know-
ing in order to understand better children’s ways of knowing. During

Figure 6.7

a musical exploration, they were building tunes, and at one point they
wanted to know whether a tune they had built had sections that were
the same length. They didn’t know how to think about that. They tried to
use a stopwatch but couldn’t tell from this whether the first half of the
tune was the same length as the second half. This led us to invent time-
measuring machines. We took a recorded tune, as the standard event,
and they were to construct time machines (without using a stopwatch)
to tell whether some other piece of music, which we were subsequently
going to play, was as long as that first piece, or longer, or shorter. They
all knew what we called tune-specific time-measuring machines; that is,
they did not set out to find some unit that would be repeated a number
of times, but instead tried to make something that measured just the
length of the standard piece: water dripping out of a cup, down to a line
that indicated the end of the piece; or a candle burning down just to the
end of the piece.

One team made a ramp of two pieces of metal, each about 4 feet long.
To their dismay, the ball rolled off the 8 feet of ramp before the music
stopped. They changed the slope; the ball still rolled off. They made a
pathway on the floor at the end out of tongue depressors so that the ball
could keep rolling along the floor, but now the ball stopped too soon. They
changed the slope—very steep, or barely any slope at all—but no matter
what they did with the slope, the ball stopped too soon. They finally con-
cluded that they would have to make the ball do something else after the
roll down the ramp; otherwise they would simply have to abandon the
ramp idea. So they moved the ramp up onto a long table, set it up with
hardly any slope at all, and arranged it so the ball could drop off at the
end. Now what could they have it do when it dropped off? Casing about
for available material, they took a pan from one of the pan balances, and
suspended it at the end of the ramp, so the ball would fall into it (Figure
6.7). As the recorded tune started, the ball started rolling slowly down
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In a different pendulum class, junior high school students had just previously been taught the equilibrium formula that applies to balances: Distance times weight on one side must equal distance times weight on the other. The only weighing mechanism available to them now was a strip of pegboard, supported in the center (Figure 6.6). When the students became interested in weighing the bobs, they hung a wooden bob on one end, and then a steel bob on the other side, not on the end, but placed so as to make the pegboard horizontal, announcing, “There, they weigh the same. We learned that just last week, they weigh the same.” It seemed clear that that formula had been hastily learned, and remained quite unexplored.

Figure 6.6

The next example comes from work done with Jeanne Bamberg and Magdalene Lampert at the Massachusetts Institute of Technology (Bamberger, Duckworth, & Lampert, 1981). We were working with a group of Cambridge teachers, helping them examine their own ways of knowing in order to understand better children’s ways of knowing. During a musical exploration, they were building tunes, and at one point they wanted to know whether a tune they had built had sections that were the same length. They didn’t know how to think about that. They tried to use a stopwatch, but couldn’t tell from this whether the first half of the tune was the same length as the second half. This led us to invent time-measuring machines. We took a recorded tune, as the standard event, and they were to construct time machines (without using a stopwatch) to tell whether some other piece of music, which we were subsequently going to play, was as long as that first piece, or longer, or shorter. They all made what we called tune-specific time-measuring machines; that is, they did not set out to find some unit that would be repeated a number of times, but instead tried to make something that measured just the length of the standard piece: water dripping out of a cup, down to a line that indicated the end of the piece; or a candle burning down just to the end of the piece.

One team made a ramp of two pieces of metal, each about 4 feet long. To their dismay, the ball rolled off the 8 feet of ramp before the music stopped. They changed the slope; the ball still rolled off. They made a pathway on the floor at the end out of tongue depressors so that the ball could keep rolling along the floor, but now the ball stopped too soon. They changed the slope—very steep, or barely any slope at all—but no matter what they did with the slope, the ball stopped too soon. They finally concluded that they would have to make the ball do something else after the roll down the ramp; otherwise they would simply have to abandon the ramp idea. So they moved the ramp up onto a long table, set it up with barely any slope at all, and arranged it so the ball could drop off at the end. Now what could they have it do when it dropped off? Cautiously, for available material, they took a pan from one of the pan balances, and suspended it at the end of the ramp, so the ball would fall into it (Figure 6.7). As the recorded tune started, the ball started rolling slowly down
the ramp, fell into the pan at the end, thus setting it swinging, and at 32 swings of the pan the tune was ended. A single-purpose time machine it was, but a perfectly dependable one—it was a roll down the ramp followed by 32 swings of the pan, every time. The tune that was to be compared with it, moreover, turned out to be a roll down the ramp followed by 37 swings of the pan; so their machine was shown to be adequate to its time-measuring task.

These stories can be thought of as comic relief. In a sense, they are. But the comedy of the coupled pendulum and the ball on the ramp is very different from the comedy of the 20°F water and the misuse of the pegboard balance. The latter two are sad tales of too rapid assumption of understanding. The other two are the rather appealing consequences of avoiding such facile rapidity. How to measure can be taught rapidly, but when it is, the inadequacies are stunning. It is quite different from the breadth and depth of understanding involved in messily constructing your own ways of measuring, knowing what they mean, how they are applicable or not applicable, and how they inform each new situation.

**RAISING QUESTIONS ABOUT SIMPLE ANSWERS**

Readers may think that any adult must of course know what time measurement is about, and that the only challenge in the work of these teachers was the technological one of getting some machine to work dependably. But it is worth reflecting on how you would know, without having some other ready-made timer, whether a candle burns with the same speed during its first quarter-inch and during its last quarter-inch. How do we know that a second hand takes the same time for each of its sweeps? How, back there in history, did anyone conclude that some event always takes the same amount of time, and so could be used to measure the time of other events? Without a standard unit, how did they establish a standard unit? This group of teachers gave those questions a lot of thought. And here is a question that gave them pause for a long time: One of them had heard that between five and seven in the evening, demands on electricity are such that electric clocks always run slower. Is that true? If it were, how would we ever know? If it is not, why isn’t it? Wouldn’t any time piece, in fact, keep going slower and slower as the battery wears out, or as the spring unwinds? As teachers, I think one major role is to undo rapid assumptions of understanding, to slow down closure, in the interests of breadth and depth, which attach our knowledge to the world in which we are called upon to use it. There may, for some given situation, be one right answer, even one that is quite easily reached. But I think a teacher’s job is to raise questions about even such a simple right answer, to push it to its limits, to see where it holds up and where it does not hold up. One right answer unconnected to other answers, unexplored, not pushed to its limits, necessarily means a less adequate grasp of our experience. Every time we push an idea to its limits, we find out how it relates to areas that might have seemed to have nothing to do with it. By virtue of that search, our understanding of the world is deepened and broadened.

I would like to develop this thought in the context of the adult thinking of this same group of teachers. Having started with music and proceeding to measure time, they came to the study of ramps, and the main interest of this study was that they pushed the limits of what seemed to be ordinary, even obvious, thoughts about time, speed, and space.

The time-specific time-measurement machines developed in the direction of a search for units of time measurement—calibrating the candle as it burned, counting the water drips, looking for natural phenomena that keep a steady rhythm. The search applied to ramps, too. Could a ball rolling down a ramp give rise to units of time? This led to another question, as a preliminary: What does the speed of a ball do as it rolls down a ramp? Does it remain constant? Increase? Decrease and then increase? Increase and then remain constant?

One group, watching a ball in order to make an initial guess about the answer, noticed a spot on it. The spot came up faster and faster as the ball rolled, until by the last part of the ramp its occurrences were no longer distinguishable—it looked like a blurred continuous line. This supported the idea that the ball was going faster and faster as it rolled down the ramp, but this group wanted to do a better job of it than that. It occurred to one of them that if the dot left a mark as it rolled they would be able to see better what the speed of the ball was doing. A bit of experimenting and they found a substance they could mark the dot with that would leave a spot each time it hit a long sheet of computer printout paper that was stretched down the ramp. The reader might want to predict what the spots did. We have since discovered that about half the adults we ask predict that the dots will get closer together, a few predict they will get farther apart, and the rest predict they will remain at a constant distance. The roll of computer paper with the spots left by the ball looked like the graph in Figure 6.8 (see next page). The reaction of at least one member of the group was to take a piece of string and measure the distances, saying something to the effect of, "Go, those dots don't get closer together as noticeably as I had thought they would!"
the ramp, fell into the pan at the end, thus setting it swinging, and at 32 swings of the pan the time was ended. A single-purpose time machine it was, but a perfectly dependable one—it was a roll down the ramp followed by 32 swings of the pan, every time. The time that was to be compared with it, moreover, turned out to be a roll down the ramp followed by 32 swings of the pan; so their machine was shown to be adequate to its time-measuring task.

These stories can be thought of as comic relief. In a sense, they are. But the comedy of the coupled pendulum and the ball on the ramp is very different from the comedy of the 200° water and the misuse of the pegboard balance. The latter two are said to take no more time to understand the other two are the rather appealing consequences of avoiding such facile rapidity. How to measure can be taught rapidly, but when it is, the inadequacies are stunning. It is quite different from the breadth and depth of understanding involved in messily constructing your own ways of measuring, knowing what they mean, how they are applicable or not applicable, and how they inform each new situation.

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That turned out to be just the beginning of many perplexities in this consideration of speed-space-time relationships. Another group, also trying to establish what the speed of a ball does as the ball rolls down a ramp, produced the graph shown in Figure 6.8.

I am not going to say here how the second graph came about. My purpose is better served if readers put themselves to the task—because in this case the answer to the ball-ramp problem is really beside the point; what I would rather do is make vivid how much harder it is to think coherently about space-speed-time phenomena than it is to enunciate formulas.

At a subsequent seminar, the teachers who had been absent when the two graphs were produced were given the job of interpreting them—trying to establish how each had been made, and what each of them said about the speed of the balls rolling down the ramps.

Here are a couple of the inferences made by the teachers who had been absent. One person thought the spots on the first graph looked as if the ball had left its own mark as it rolled; but then, she went on to say, it would have to have been rolling at the same speed all the way, so it couldn’t have been rolling down a ramp. The second graph was thought not to have been made by the ball itself. This inference was made not on the basis of the distances between the marks, but because the marks looked as if they were drawn by a hand-held felt marker. One generally accepted thought was that marks were made indicating where the ball was after equal time intervals.

The discussion of these two graphs went on for two hours. The members of the group who had been present to generate them got caught up in considering what interpretations were possible in addition to those they knew to be the case. Does the first graph say anything about speed? Is anything to be learned by superimposing the first graph on the second? What picture would you get if you made both graphs at once, of one ball rolling down a ramp? What does the speed of the ball do in the second graph?
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TIME FOR CONFUSION

One other topic that this group of teachers worked on was the moon. All of us know that the earth turns upon itself, the moon goes around the earth, and while both these things are going on, the earth is also going around the sun. All of us also see the sky get light and dark again every day, see the sun pass overhead, often see the moon, sometimes full and sometimes not. But how many of us can make a connection between these two kinds of experiences? On a given afternoon in Massachusetts, for example, at 5:00, the moon was slightly less than half, and it was visible quite high in the sky. Now, in a model of sun, earth, and moon, could you place them in the relative positions to indicate where they would be in order for the sky to look like that? Almost nobody I’ve run into can do that. Those two kinds of knowledge about the moon are, for the most part, quite separate. Bringing them together, moreover, is a difficult job, which makes this a marvelous subject through which to study one’s ways of making sense of one’s experience, and especially to realize how a simple formal model can have almost no connection with the experience it is meant to describe.

It takes months of watching and finding some order in the motions before one can know, when looking at the moon, in what direction it will move from there, where it will be an hour later, or 24 hours later; how the crescent will be tipped 2 hours from now; whether it has yet reached its highest point of the night; whether, tomorrow it will be visible in the daytime. Does the moon pass every day straight overhead? Does the moon ever pass straight overhead? Does it depend on where you are on the earth? If, right now, from here, it was up at a 70° angle from me, at what angle would it be if I climbed up to the top of that building? If I were sitting down, at what angle would it be? Or if I walked down the block toward it?
One friend claimed he had seen the moon like the drawing in Figure 6.10. How was it possible, he asked, for the round earth to have cast a crescent-shaped shadow on the moon? He could understand seeing the moon itself like a crescent, as in Figure 6.11, but he could not understand what he claimed to have seen. It is a good question for moon-watchers, and I put it to the readers, with what seem to me three possible explanations: Either he did not see the moon shaped that way, or there are circumstances under which a sphere (the earth, in this case) can cast a crescent-shaped shadow; or the crescent that is missing from the side of the moon is not the shadow of the earth.

Another friend confessed he perplexed she had been when she realized that people standing on the moon looked up to see the earth. Surely, from the moon, one should look down at the earth if, from the earth, one looks up at the moon? Figuring out that puzzle for herself was a source of considerable joy.

In our seminar, moon questions took us into sun-earth questions that were no less difficult. How, with models of earth and sun, do you represent the sun coming up over the horizon? What is the horizon, anyway? If the sun is, for you, on the horizon, where is it for everybody else? If the sun is straight overhead at noon (and is it straight overhead at noon?), is it straight underground at midnight? If the sun’s rays go out in all directions, past the earth, can we see them? Does that mean that the part that is in darkness on earth is smaller than the part that is in light?

One of the teachers, Joanne Cleary, drew on the blackboard this picture of the earth in the midst of the sun’s rays (Figure 6.12), and was trying to articulate her thoughts about it. Another member of the group was asking her to be more precise. Did she mean exactly half the earth was in darkness? Did it get suddenly black at the dividing line, or was there some gray stripe? The one who was trying to articulate her thoughts grew angry, and gave up the attempt. She said later that she knew the questions were necessary at some point, but she had not been ready to be more precise. She was struggling to make sense of a morass of observations and models, an idea was just starting to take shape, and, she said, “I needed time for my confusion.”
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