ABSTRACTS

C1.1 Philosophy of the Formal Sciences

Penelope Maddy between realism and naturalism

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In her Realism in Mathematics Maddy defended a realist position in philosophy of mathematics. After criticism from Balaguer, Carson, Lavine, and Riskin she gave up realism and turned to Naturalism in Mathematics. In the paper I will reformulate Maddy's realistic position by stressing the instrumental aspect of mathematics. Mathematical reality is discovered by means of instruments of symbolic representation. These instruments are human creations and as such they change in time. By bringing in the historical dimension we obtain a strong tool for defence of the realist position in the philosophy of mathematics against the above mentioned criticism. We can distinguish different kinds of instruments of symbolic representation, such as the different positional systems in arithmetic, the symbolic notation in linear or polynomial algebra, the functional symbolism in the differential and integral calculus, and the logical symbolism in predicate calculus. By drawing on an analogy between these representational instruments in mathematics and measurement instruments in physics we can refine Maddy's position by grounding intuition in instrumental practice. By introducing several instrumental practices it becomes possible to develop the foundation of a later instrumental practice (say that of the calculus) by means of an earlier one (say arithmetic). In this way we can interpret the sets as objects that are situated not in the space of our immediate perception, as Maddy did (and Carson objected). We situate in that space the objects of elementary arithmetic. Then by ascending the historical sequence of representational instruments we pass through the universes of algebra, calculus and logic to the universe of set theory. These universes are nested, the earlier ones are embedded in the later ones. And by means of this embedding sets obtain realistic status.

The Subject of Mathematics

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There are two possible answers to a question concerning the subject of mathematics:
1. Mathematical reality exists objectively (Penrose, who stands behind this resolution, discusses a Platonic world of mathematical ideas); and

2. Mathematical reality is constructed by mathematicians.

Both of these positions face some essential and intractable difficulties.

Regarding the first position, one has to confront the problem of the truthfulness of mathematical statements. Although Tarski indicated the way to define the notion of truth for formal languages, he did not do so for the criterion of truth. It is even possible to prove that, for richer languages, such a criterion cannot exist. Thus, we have no access to a “Platonian world of ideas.”

In the second position it is mathematical language that creates mathematical reality, so utterances must be “well done” to comprise successful performatives (see works of John L. Austin). The primary condition is consistency. Kurt Gödel demonstrated the complexity of this problem in detail.

The challenge to both of these positions is the preposterous effectiveness of mathematics. It is hard to explain why the world of ideas or constructions of mathematicians matches up so splendidly with empirical reality. An attempt to understand this phenomenon sends us back to the emergence of mathematics, which first involved trading and commercial exchanges, measurements of land, navigation, etc. It also reflects on our ability to perceive a uniform structure within a diversity of phonemes. The development of mathematics, then, does not depend on creating any construction of or penetration into a Platonian world of ideas, but rather on seeking an Aristotelian form in the objects and patterns we pursue in reality in innumerable ways.

This suggests a third position in the debate on the nature of mathematics, alongside those of constructivism and Platonism. We may call this third position, “Aristotelism.”

**Why believe there are infinite sets?**

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The axiom of infinity – the statement that there is an infinite set - is not justified. This should disturb those who believe both that there are infinite sets and that set theory should be foundational of mathematics.

Why should we want a justification for it? In the wake of Hilbert's program, one might call “sets” any objects that satisfy a list of set-theoretic axioms (von Neumann 1925). Fictionalists may deny there really are any sets to make the axiom true. Brouwer (1913) suggested the infinity and sethood of the natural numbers are given directly in intuition. Whatever their
general merits, these positions are dubious when it comes to the axiom of infinity, for reasons anticipated by Dedekind (1888), Russell (1919), and Skolem (1922). So the axiom stands in need of justification.

How could one try to justify the axiom of infinity? P. Maddy distinguishes extrinsic from intrinsic justifications. As for extrinsic justifications, an enhanced indispensability argument (Baker 2005) cannot be run for the axiom itself, nor, I argue, for set theory more generally. Moreover, the foundational role of set theory (Boolos 1998) or its centrality to mathematical practice (Quine 1951) are assumptions that stand in need of justification themselves. And believing the universe of sets is maximal - anything short of contradiction (Zermelo 1908) – itself requires justification. Intrinsic justifications are more interesting. One cannot follow Frege (1893) in supposing that, because there are consistent concepts of infinity, infinite sets do exist, because of Russell’s paradox. Nor can one follow Boolos (1971) in deducing one axiom of infinity (there is a Dedekind-algebra) from an iterative conception of sets that itself contains an axiom of infinity, albeit a different one (there is a limit-level). Deducing existence from concept fails.

So explicit arguments are called for to justify the axiom. Dedekind's foundational study (1888) provides the only explicit arguments I am aware of. First, Dedekind argued that the possible objects of thought form a Dedekind-algebra. Misplaced charges of psychologism aside (Potter 2004), the assumption that the operation “thinking about” is closed over the set of possible objects of thought needs to be justified.

Second, Dedekind wished to represent arithmetic in set theory. Finite arithmetic (Cohen 1966) is sufficient to capture natural and rational arithmetic. Real numbers cannot be so represented (Russell 1908), but why believe real numbers have sets as counterparts? One might say: if finite numbers exist, then by pairing and extensionality their singletons exist, hence by union an ω-large set exists. All this proves is that union is not closed over finite sets. Assuming there were an ω-large set, this would form a Dedekind-algebra only if the axiom of denumerable choice were true (Cohen 1966), and this axiom is considerably more controversial than the axiom of infinity (Skolem 1922).

Perhaps there are other arguments yet. But Russell's early discussions (1908,1919) strongly suggest that the axiom of infinity both needs a justification, and lacks one. We should pay heed to Russell's sentiment.

### Is There an Objective Account of Mathematical Depth?

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Maddy has argued that there are objective facts of mathematical depth that constrain appropriate axiom choice in mathematics, and which undergird a version of mathematical
realism that is compatible with her particular version of philosophical naturalism (Second Philosophy). Perhaps surprisingly given its importance in her account of mathematics, she leaves the concept of mathematical depth largely unanalyzed. What little she does say about the concept by way of linking mathematical depth to fruitfulness indeed seems to pose a threat to the idea that it is an objective matter. After noting the problem here, the paper takes up how mathematical depth might be further elaborated so as to maintain the claim of objectivity. The suggestion considered in some detail is that mathematical depth is associated with general explanatory virtues. It is observed, though, that even if explanation is identified with mathematical depth, serious challenges arise for Maddy's objectivist. These involve spelling out an account of mathematical explanation that will yield a privileged collection of axioms, and defending the account as objective on appropriate naturalistic grounds. I argue that while there are prospects for understanding mathematical explanation in objective terms, the varied nature of mathematical explanations calls into question whether linking mathematical explanation and mathematical depth supports the idea of a privileged set of axioms. Finally, I point to a tension between the basis for the objectivity of theories of mathematical explanation suggested in the paper and Maddy's version of naturalism.