Abstracts

A1. Mathematical Logic

Constructing normal numbers
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Flip a coin a large number of times and roughly half of the flips will come up heads and half will come up tails. Normality makes similar assertions about the digits in the expansions of a real number. For $b$ an integer greater than or equal to 2, a real number $x$ is simply normal to base $b$ if every digit $d$ in $\{0, 1, \ldots, b-1\}$ occurs in the base $b$ expansion of $x$ with asymptotic frequency $1/b$ (in the above example with coin tosses consider $b$ equal to 2); a real number $x$ is normal to base $b$ if it is simply normal to all powers of $b$; and a real number $x$ is absolutely normal if it is simply normal to all integer bases greater than or equal to 2.

More than one hundred years ago E. Borel showed that almost all (for Lebesgue measure) real numbers are absolutely normal, and he asked for one example. He would have liked some fundamental mathematical constant such as $\pi$ or $e$, but this remains as the most famous open problem on normality. As for other examples, there have been several constructions of normal numbers since Borel's time, with varying levels of effectivity (computability). I will summarize the latest results, including our constructions of numbers normal to selected bases, a fast algorithm to compute an absolutely normal number which runs in nearly quadratic time, and an algorithm to compute an absolutely normal Liouville number. This is joint work with Theodore Slaman and Pablo Heiber.

Entanglement and Formalism Freeness: Templates from Logic and Set Theory
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In his 1946 Princeton Bicentennial Lecture Gödel suggested the problem of finding a notion of definability for set theory that is “formalism free” in a sense similar to the notion of computable function --- a notion that is very robust with respect to its various associated formalisms. One way to interpret this suggestion is to consider standard notions of definability in set theory, which are usually built over first order logic, and change the underlying logic. In joint work with Menachem Magidor and Jouko Väänänen we show that constructibility is not very sensitive to the underlying logic, and the same goes for hereditary ordinal definability (or HOD). This setup can be reformulated as a template for studying the phenomenon of "formalism freeness." In the talk we also consider other templates, which we suggest are ways to consider the dual notion of formalism freeness, namely the phenomenon of "entanglement."