ABSTRACTS

A2.2 Philosophical Logic

Truth and Reference in first-order arithmetic

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According to deflationism, the truth predicate would be entirely dispensable save for the fact that it enables us to express certain generalizations or 'infinite conjunctions' ('all theorems of arithmetic are true'). Several deflationists (e.g. Horwich, Field) claim that the truth predicate can serve this function in virtue of its disquotational nature--i.e. every sentence A is equivalent to "'A' is true" (T-schema).

Accordingly, deflationist truth theories must contain all instances of the T-schema to guarantee the expressive function of truth. However, as is well known, the T-schema is inconsistent with many classical systems, such as Peano arithmetic, due to paradoxes like the liar (given by a sentence that says of itself that it's untrue).

While some authors depart from classical logic (Field), others restrict the T-schema as little as possible, to unproblematic instances (Horwich). Discriminating between safe and unsafe instances isn't straightforward. The so far proposed criteria are either too complex (groundedness, stability) or rather ad hoc (positive instances, typing). Adopting complex criteria means that in many cases there's no way to know whether an instance of the T-schema holds or not; they don't provide a truth predicate we could use to make generalizations, as deflationists want.

Usually, paradoxical expressions are said to display certain characteristic reference patterns (self-reference, non-wellfoundedness). Nonetheless, reference patterns for first-order languages have only been investigated from a semantic standpoint, resulting in too complex criteria. I first provide intuitive proof-theoretic notions of reference for such languages, and show them to be consistent and simple enough (Sigma 1) to serve as a restrictive criterion. Secondly, I put forward a definition of unproblematic sentence based on its reference pattern. Finally, I give a consistent theory of truth that obtains by restricting the T-schema to unproblematic expressions, and prove some metatheoretical results.

Towards a Non-Fregean Axiomatic Theory of Truth

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See page 3
Homotopy Model Theory
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See page 4

Epistemic Truth-Values
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See page 5

An assumption-based logic for the analysis of inconsistent premises
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See page 7
Non-Fregean logics, introduced by R. Suszko, can be seen as a realization of Gottlob Frege’s semantic program with the exception of a postulate, known in the literature as the Fregean axiom, that treats the truth value of a sentence as its denotation, and is a fundamental assumption underlying classical logic.

Non-Fregean logic was explicitly proposed by Suszko as an alternative to the established standard, as it rejects the Fregean axiom and introduces a universe of the semantic correlates of statements, known as the universe of situations. In order to express claims concerning the universe of situations, a new connective $\equiv$, called the identity connective, is added to the language. The identity connective expresses the identity of two statements, which is true whenever the semantic correlates of the statements are the same. Suszko presents the central ideas of the non-Fregean framework and the underlying philosophical motivations extensively in his article [SUS75].

One of the most applicable non-Fregean logics is SCI$_Q$, which is the extension of the minimal non-Fregean propositional logic SCI with quantifiers ranging over propositional variables. The logic SCI$_Q$ offers a wide repertoire of ways to express interesting properties of the universe of situations. In particular, as shown in the paper [GPH14], Peano arithmetic can be coded in SCI$_Q$.

In our talk we will present basics of the logic SCI$_Q$ and give an overview of our first results on the formalization of certain typed and type-free theories of truth with Peano arithmetic as the base theory.

References


Homotopy model theory

“Homotopy Type Theory” connects logic with homotopy theory through type theory. I would like to show that logic can be connected with homotopy theory through model theory.

Given a first-order language $L$ with equality, supposed to contain a unary quantifier $Q$, let $F_n$ be the set of formulas of $L$ with exactly $v_0,\ldots, v_n$ as free variables. The two following applications $d_i: F_n \to F_{n-1}$ (for $n \geq 1$) and $s_i: F_n \to F_{n+1}$ can then be defined:

\[ d_i(\phi(v_0,\ldots, v_n)) = Qx \phi(v_0,\ldots, v_i-1, x, v_{i+1},\ldots, v_{n-1}) \]

\[ s_i(\phi(v_0,\ldots, v_n)) = ((v_j = v_{j+1}) \to \phi(v_0,\ldots, v_j-1, v_{j+1},\ldots, v_{n+1})) \]

Up to logical equivalence between formulas (and provided that the quantifier $Q$ satisfies two very mild conditions), these maps satisfy a set of equalities called “simplicial identities.” In other words, $F^2_n = \langle F_n, (d^n_i)_{0 \leq i \leq n}, (s^n_j)_{0 \leq j \leq n} \rangle_{n \in \mathbb{N}}$ is a simplicial set. So $Q$ can be compared to a “face operator,” while $(s_j)$ is the corresponding sequence of “degeneracy operators.” The boundary of a given formula $\phi$ can then be defined as follows:

\[ \partial \phi := \bigwedge_{i=0}^{n-1} \neg \forall x \phi(v_0,\ldots, v_{i-1}, x, v_{i+1},\ldots, v_{n-1}). \]

One can check that $\partial(\partial \phi) \equiv \bot$ for any formula $\phi$, which prompts a comparison of $\partial$ with a boundary operator pointing to homotopy theory.

Let’s turn now to the models of some theory $T$ laid down in $L$, with $Q = \exists$. For such a model $M$, $M_n = F^\exists_n M = \langle D_n(M), (\exists^n_i)_{0 \leq i \leq n}, (s^n_j)_{0 \leq j \leq n} \rangle_{n \in \mathbb{N}}$ where $D_n(M)$ (for $n \geq 0$) is the set of all definable subsets of $|M|^{n+1}$, where $\exists^n_i M : D_n(M) \to D_{n-1}(M)$, $A = \{ \bar{a} \in |M|^{n+1} : M \models \phi_A(v_0,\ldots, v_n)[\bar{a}] \}$ is the face operators, and where $s^n_j M : D_n(M) \to D_{n+1}(M)$, $A \mapsto \{(\bar{x}, y) : \bar{x} \in A$ and $y = v_j\}$ are the degeneracy operators. The resulting $M_n$ is a simplicial complex for any $L$-structure $M$.

**Theorem 1.** A substructure $M$ of a $L$-structure $N$ is an elementary substructure of $N$ iff the corresponding restriction $r_* : N_* \to M_*$ is a simplicial map.

**Corollary 1.** The mapping $(-)_*$ is a contravariant functor from the category of $L$-structures and elementary embeddings, to the category of simplicial sets and simplicial maps.

**Theorem 2.** Let $M$ be an elementary substructure of $N$. Then $M_*$ is a retract of $N_*$ iff the domain $|M|$ of $M$ is definable in $N$.

Other results can be reached which extend these first ones, in particular about spaces of types.
Epistemic truth-values*

Fabien Schang

Abstract.
A number of objections have been addressed by Niiniluoto (2014) against an epistemic definition of truth $T_a p$, according to which the truth of a proposition $p$ is relative to an epistemic agent $a$. These objections are the following:

(1) Relative truth fails to satisfy Von Wright’s truth-logic;
(2) It entails the omniscience of truth: $T_a p$, $p \vdash q \vdash T_a q$;
(3) There are no external constraints for truth and falsity;
(4) Tarski T-equivalence cannot be sustained, both because the equivalence scheme $T_a p \equiv p$ does not make sense and because $B_a p \vdash p$ and $p \vdash B_a p$ are not accepted in doxastic logic;
(5) The definition of relative truth leads either to self-refutation or infinite regress.

The aim of the present paper is to reply to (1)-(5) through an alternative characterization of truth-values.

Logic and epistemology are both related to truth, although these areas of philosophy deal with this central concept from different perspectives. Although the common distinction between formal and material truth is meant to avoid any ambiguity between the two disciplines, I want to discuss the reasons why a pragmatist approach to truth questions this usual borderline. For this purpose, I advocate several topics from epistemic logic to the theory of opposition.

My thesis is that the ensuing coherence theory of truth should justify an alternative semantics, thereby revisiting the usual concept of truth-value in logic through a faillibilist defence of truth in epistemology.

The result is a structural theory of meaning and a Boolean algebra of bitstrings, whereby the so-called truth-value of a proposition is replaced by the logical value of a statement. The transition from a faillibilist theory of truth to a non-Fregean theory of logic will serve as a guideline for the whole talk.

References

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In our talk we present a paraconsistent system based on classical logic. An inspiration comes from the traditional idea in Rescher & Manor (1970) that a necessary condition for being a consequence is to be derivable from a consistent subset of the premises. However, in many applications this criterion is too restrictive because of the resulting syntax-dependency (where different but equivalent formulations of the premise set lead to different conclusions). To overcome this problem our systems are equipped with inference rules that allow for the analysis of the premises. Moreover, unlike traditional systems, consistency assumptions are integrated in a dynamic proof theory. The idea is to protocol significant assumptions about the consistency of formulas that are used in crucial inference steps (such as resolution and aggregation). When these assumptions are violated, the inference gets retracted. This way of integrating consistency assumptions is, for instance, crucial in a predicative setting where no effective test for consistency is available. We will show that depending on what assumptions are protocolled one can obtain either a credulous or skeptical notion of consequence.

We will provide an argumentation-based semantics that is adequate relative to the dynamic proof theory and present meta-theoretic properties of the system. Finally, we compare our logic with similar systems known from the literature such as Quasi-Classical Logic (Besnard & Hunter 2000), the argued consequence (Benferhat et al. 1997), AN(A) (Meheus 2000), CL− (Batens & Provijn 2001), inconsistency-adaptive logics (Batens 2007), and the argumentation systems based on classical logic by Besnard & Hunter (2009).

At the end of the talk we will indicate how by enhancing the system with a non-classical conditional we obtain an interesting variant of default logic.