Granular Mining of Logical Rules from Relational Structures

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Granular computing (GrC) is an emerging technology with problem-solving concepts deeply rooted in human thinking, and rough set theory is an effective GrC tool that has been successfully applied to knowledge discovery from data tables. In recent years, the application of rough set analysis has been extended to relational structures such as ontology graphs or social networks. Unlike classical rough set theory, in which the attribute values of objects fully determine the indiscernibility relation, the rough set analysis of relational structures must account for the relationships between objects. In the previous study, the indiscernibility relation with respect to relational structures is defined by using the notions of positional equivalences in social network analysis. The indiscernibility relation can partition a relational structure into elementary information granules (IG) which are used to define the lower and upper approximations of an arbitrary subset as in classical rough set theory. However, to induce rules from such approximations, we need a knowledge representation formalism. In this paper, we use description logics (DL) to represent knowledge discovered from relational structures and present a constructive procedure to find a characterizing DL concept for each IG. Because the lower and upper approximations of a target set are unions of IG's, we can induce rules with the disjunction of such characterizing concepts as their antecedents. This leads to a complete process of granular data mining from relational structures.

Finitely Unstable Theories and Computational Complexity

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Stability theory separates first order (FO) theories with infinite models into stable ones that have a "small" number of non-isomorphic models and unstable ones that have a large - in fact exponential - number of non-isomorphic models with respect to the infinite cardinality of the corresponding set of constants. In this presentation we pursue an analogue of this distinction in the case of FO theories over a finite, even bounded, set of parameters. Such a bound is induced by a bound M on the definition length of the corresponding binary encoding of the theory. As a vehicle for this analysis we use the propositional satisfiability problem SAT restricted to bounded definition length. In this restricted setting the bounded restriction of SAT is rendered definable in FO and it can be shown to be finitely unstable, in the sense that the bounded FO theory SAT_M only has models with cardinality that is exponential in M. The property of finite instability is further translated into a lower bound on the deterministic computational time complexity of the corresponding bounded decision problem by encoding in FO an arbitrary deterministic Turing machine (DTM) that decides SAT_M. It can be shown that there is an isomorphism between an FO-definable set of non-isomorphic propositional models of SAT_M and the FO-definable set of equivalence classes of...
DTM computations that decide all the corresponding instances of SAT_M. This isomorphism yields a lower bound on the size of any model of SAT_M as a FO theory, and also one on the size of any model of the corresponding FO-definable set of DTM computations. This lower bound on model size also carries over to a lower bound on the deterministic time complexity of SAT when the bound M is allowed to grow without limit.

Bi-Logic Via Infinite Singletons

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Bi-logic [1] describes two sides of the human thinking, the rational reasoning (asymmetric mode) and the symmetric mode, also termed indivisible mode, where any relation is symmetric and any set is infinite.

In a quantum model [2], we have characterized the class of finite sets for which the membership relation can be expressed as a finite propositional disjunction of equalities, in the object language. Finiteness can be recognized, at the object level, only when this happens. In particular, one can conceive an infinite singleton, dropping, in the object language, the closed term which denotes its unique element. Infinite singletons satisfy the symmetry property as well, since the class of sets where any relation is symmetric is exactly the class of singletons.

The logic of infinite singletons represents a symmetric kernel, in sequent calculus [3]. It has the features of the symmetric mode: absence of mutual contradiction and condensation, absence of negation, absence of time, and displacement. The direction of logical consequence becomes irrelevant. In this setting, one can develop the definition of a generalized quantifier, disappearing once consequence is recovered, that represents correlations. Considering the structural rules of sequent calculus, this suggests a possible approach to the problem of the representation of contextual reasoning and other kinds of human reasoning in artificial intelligence [4].


Level system of formulas for decreasing the number of proof steps of formulas simulating some Artificial Intelligence problems

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The solving of many Artificial Intelligence problems may be reduced to the proof of a series of formulas (for $k = 1, \ldots, K$) in the form

$$S(a_1, \ldots, a_t) \Rightarrow \exists (x_1, \ldots, x_m). A_k(x_1, \ldots, x_m),$$

(1)

where $(a_1, \ldots, a_t)$ are constants, $(x_1, \ldots, x_m)$ are variables, $S(a_1, \ldots, a_t)$ is a set of atomic formulas or their negations, $A_k(x_1, \ldots, x_m)$ are elementary conjunctions. The notation $\exists (x_1, \ldots, x_m). P$ is used for the formula $\exists x_1 \ldots x_m (x_1 \neq x_2 \& \cdots \& x_i \neq x_j \& \cdots \& x_{m-1} \neq x_m \& P)$. The verification problem of such a formula is NP-complete and the complexity of an algorithm based on the derivation in sequential calculus or on the use of resolution method is $O(s^a)$ where $s$ and $a$ are the numbers of atomic formulas in $S(a_1, \ldots, a_t)$ and $A_k(x_1, \ldots, x_m)$ respectively.

Below it is suggested an algorithm of extracting common (up to the names of variables) sub-formulas $P_i^l(y_1, \ldots, y_n)$ of $A_i(x_1, \ldots, x_m)$, $A_k(x_1, \ldots, x_m)$. These sub-formulas are changed in $A_i(x_1, \ldots, x_m)$ by new predicates $p_i^l(y_1^l)$ with new variables $y_1^l$ for the lists of initial variables. The obtained formulas are denoted by $A_i^l(x_1, \ldots, x_m, y_1^l, \ldots, y_n^l)$. The use of such common sub-formulas allows to construct a level system of formulas in the form (1) with the change of $A_i(x_1, \ldots, x_m)$ by $A_i^l(x_1, \ldots, x_m)$. The use of such a level system decreases the exponent in the complexity bound of the mentioned algorithms up to the $O(s^{\alpha})$, where $\alpha$ is the maximal number of atomic formulas in $P_i^l(y_1, \ldots, y_n)$ and $A_i^l(x_1, \ldots, x_m, y_1^l, \ldots, y_n^l)$. 