Mathematical beauty: A challenge for empirically informed philosophy of mathematics

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This symposium is organized by the corresponding author, who is NOT a speaker at the symposium. Description of the symposium and abstracts are in the Appendix.

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General Description (max. 500 words):
The aim of this symposium is to present some recent philosophical and empirical work on mathematical beauty as the basis for a discussion of some of the methodological challenges that are raised for philosophy by the use of empirical methods.

In the past decades, philosophy of mathematics has increasingly turned away from treating mathematics as an idealized and static subject matter and has moved towards incorporating episodes from the history of mathematics and views expressed by individual mathematicians into its analyses. More recently, also work from cognitive science and results from empirical studies have been taken into consideration. The participants of this symposium have all, in one way or another, contributed to these latter developments.

For the case of mathematical beauty in particular, Inglis and Aberdein (2014) have probed the judgments of working mathematicians about aesthetic properties of proofs. They asked 255 mathematicians to think of a proof they’d recently read, and state how well each of 80 adjectives described it. Their results cause problems for classical accounts of mathematical beauty: for instance, there was no significant correlation between a proof’s perceived level of beauty and its perceived level of simplicity or explanatory power. They argued that this result poses a serious challenge to traditional approaches in philosophy of mathematics, which often focus on the philosopher’s intuitions or on a single case study to draw some general philosophical conclusions; for example, Euclid’s proof of the infinitude of prime numbers or proofs of Pythagoras’ Theorem are often discussed as “beautiful” proofs and their analysis then is supposed to give us universal insights into the notion of mathematical beauty.

This symposium is intended as a forum to discuss how this challenge can be met from the point of view of an empirically informed philosophy of mathematics, i.e., without dismissing the empirical findings as being irrelevant for philosophical analyses. The invited participants have all worked on the problem of characterizing mathematical beauty from a number of different perspectives, which include aesthetics, mathematical cognition, and mathematical education. Thus, this symposium promises to advance both the philosophical discussion of mathematical beauty as well as the methodological reflections that are prompted by the recent opening of philosophy of mathematics to inter-disciplinary approaches.

The format of the symposium will be four talks (20min each with 5min for questions) and a final general discussion with all participants (20min).

1. Abstracts of individual papers (max. 300 words each): 1) Diversity in proof appraisal

We investigated whether mathematicians typically agree about the qualities of mathematical proofs. Between mathematician consensus in proof appraisals is an implicit assumption of many arguments made by philosophers of mathematics, but to our knowledge the issue has not previously been empirically investigated. We asked a group of 112 mathematicians to assess a specific proof on four dimensions, using the framework identified by Inglis and Aberdein (2014). We found widespread disagreement between our participants about the aesthetics, intricacy, precision and utility of the proof, suggesting that a priori assumptions about the consistency of mathematical proof appraisals are unreasonable.

2) Beauty in the eyes of the beholder? Approaching mathematical beauty in an empirically-informed way

It is well known that mathematicians often attribute aesthetic properties such as ‘beautiful’, ‘elegant’, ‘ugly’ etc. to proofs. However, there is no consensus among mathematicians and philosophers on how to interpret these judgments: are aesthetic properties primary, indefinable features of mathematical proofs, or are these aesthetic terms used to refer, in a roundabout way, to non-aesthetic properties? A case in point: Montano (2013) introduces the useful distinction between literal and non-literal interpretations of aesthetic vocabulary in mathematics; he sides with McAllister to defend the literal interpretation, against Rota’s non-literal interpretation of ‘beauty as enlightenment’. But what kind of ‘data’ could count as evidence in this debate? So far the debates have been mostly conducted on a purely conceptual level, but the recent publication of Inglis
and Aberdein (2014) opened up a new, empirical way to approach these issues. Some of the pertinent questions that can be investigated empirically are: do mathematicians converge in their attributions of aesthetic properties to mathematical proofs? Are there significant correlations between attributions of certain aesthetic properties and attributions of non-aesthetic properties to a given proof? Answers to these questions would provide important (even if not decisive) data for the issues surrounding the phenomenon of aesthetic judgments in mathematics. For example, if it turns out that mathematicians by and large agree on their attributions of aesthetic properties to specific proofs, this would lend support to the idea that these are ‘robust’, primary features of mathematical proofs. In contrast, if there is no convergence, then we may conclude that for proofs too, ‘beauty is in the eyes of the beholder’, not in the proofs themselves. In my talk, I will raise some of the questions that could be investigated empirically, discuss existing results, and draw implications of these results for some of the philosophical questions pertaining to beauty in mathematics.

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3) Approaches to mathematical aesthetics
This talk will cast doubt on both of two opposing views about how to investigate mathematical aesthetics. One view recommends philosophical reflection without regard to the findings of empirical studies. The other recommends taking empirical findings about aesthetic (and other) judgments at face value and using them as the basis of one’s philosophical conclusions. Looking at one or two empirical studies, I will try to give my reasons and (time permitting) make some positive suggestions about the right approach.

4) There is no beauty there
This paper explores the possibility that when it comes to beauty in mathematics, we oversubscribe, attributing properties of beauty where there may be none. We take as an example a topologist at a moment of insight. When interviewed about this moment she claimed there was nothing aesthetic about it—she just wanted to produce new mathematics and this was her result. Work on philosophical questions using empirical methods is increasingly common (e.g. Knobe (2008), Inglis and Aberdein (2014)). However this example raises questions of what we can reliably infer about our data. The term “beauty” has a wealth of connotations and its own place in mathematical lore. However this does not mean we cannot make progress on what the term means, regardless of what people say.