CABE 2017
Math Activities that Maximize Mathematical Reasoning with Minimal Language Demands
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Math Activities that Maximize Mathematical Reasoning with Minimal Language Demands

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Challenges for EL Students to Overcome

—Adapted from California Math Framework, Universal Access, 2015 (30–32)

1. Limited prior or background knowledge and experience with formal schooling
   - May lack basic computation skills and/or have gaps in conceptual knowledge. EL students may be unfamiliar with learning habits expected in the classroom and for homework.

2. Cultural differences
   - May have different learning styles. May have different meanings for certain symbols such as commas and decimal points. Background knowledge may affect interpreting contexts and meanings for word problems, and expressions of commonly used applications for currency and measurements.

3. Linguistics
   - Challenges with understanding differences between academic language and everyday language, as well as math-specific terms and symbols. Included in this is the challenge of grasping the complex structure of the passive voice.

4. Polysemous words
   - Words with identical spellings and pronunciations, but different meanings that are based on context. For example, table, operation, mean, even, etc.

5. Word problems – syntactic features and text analysis
   - Word order is critical, and small words or expressions have precise meaning, such as an and more than. Word problems can have a complex syntax. Additionally, solving a word problem means understanding a context the problem is given in, translating words into English, identifying the question and relevant information, translating the English into math symbols and expressions, setting up and solving the math, and translating the solution into English.

6. Semantic features
   As shown in the following table (adapted from NYU Steinhardt, 2009), many ELs may find semantic features challenging.

<table>
<thead>
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<th>Feature</th>
<th>Examples</th>
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<tbody>
<tr>
<td>Synonyms</td>
<td>add, plus, combine, sum</td>
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<td>Homophones</td>
<td>sum/some, whole/hole</td>
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<td>Difficult expressions</td>
<td>If . . . then; given that . . .</td>
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<tr>
<td>Prepositions</td>
<td>divided into versus divided by; above, over, from, near, to, until, toward, beside</td>
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<td>Comparative</td>
<td>If Amy is taller than Peter, and Peter is taller than Scott, then Amy must be taller than Scott.</td>
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<td>constructions</td>
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<td>Five books were purchased by John.</td>
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<tr>
<td>Conditional clauses</td>
<td>Assuming _____ is true, then . . .</td>
</tr>
<tr>
<td>Language function</td>
<td>Words and phrases used to give instructions, to explain, to make requests, to disagree, and so on</td>
</tr>
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<td>words</td>
<td></td>
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</tbody>
</table>
Addressing the Needs of EL Students in Mathematics

Recommendations for Connecting Mathematical Content to Language

1. Focus on students’ mathematical reasoning, not accuracy in using language.
2. Focus on mathematical discourse practices, not language as words, or grammar.
3. Recognize the complexity of language in math classrooms.
4. Treat everyday language as a resource, not as an obstacle.
5. Uncover the mathematics in what students say and do.

—Moschkovich, 2012 (5–8)

Support English learners as they learn both mathematics and academic language

- Explicitly teach and incorporate into regular practice academic vocabulary for math. Be aware of words that have multiple meanings such as *root*, *plane*, or *table*.
- Provide communication guides, sometimes called sentence frames, to help students express themselves not just in complete sentences, but articulately within the MP standards.
- Use graphic organizers and visuals to help students understand mathematical processes and vocabulary.
- Elementary school English learners’ progress in mathematics may be supported through the intentional lesson planning for content, mathematical practice, and language objectives. Language objectives “…articulate for learners the academic language functions and skills that they need to master to fully participate in the lesson and meet the grade-level content standards.”

—Echevarria, Short, & Vogt, 2008 (55)

The following are examples of possible language objectives for a student in grade 2:

- *Read* addition and subtraction expressions fluently.
- *Explain* the strategies and/or computational estimates used to solve addition and subtraction problems within 100.
- *Describe* the relationship between multiplication and division.
Spend Some Time with 1 to 9

Building Number Sense and Fluency Through Problem Solving for K–8

by Dean Ballard
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Create Equations with the Digits 1–9

Create as many equations as you can with the following conditions:

• Use the digits 1–9 to create many different equations.
• Use some or all of the digits in each equation.
• Do not use any digit more than once within any single equation.
• Do not use the digit zero.
• You may use any math operation, including exponents.

For example:

\[ 8 ÷ 4 = 5 − 3 \rightarrow \text{uses the digits 3, 5, 4, and 8} \]

\[ 5 + 6 \times 4 = 29 \times 1 \rightarrow \text{uses the digits 1, 2, 4, 5, 6, and 9} \]
What Can You Do with 5, 6, 8, and 2?

1. Using any two numbers from above, what is the greatest possible sum you can create?

2. Using any two numbers from above, what is the least possible sum you can create?

3. Using any two numbers from above, what is the greatest possible difference you can create?

4. Using any two numbers from above, what is the least possible difference you can create?

5. For any of your answers for 1–4, explain or show how you know you have the right answer.
What Else Can You Do with 5, 6, 8, and 2?

We can use the four digits 5, 6, 8, and 2 to make many other numbers. For example, we can create 56 + 82, or 26 + 58, or 5 + 6 + 8 + 2. The rules are as follows:

- Use all four digits exactly once each time.
- Use only addition and subtraction, and use at least one operation in each expression.

1. What is the greatest possible sum you can create?

2. What is the least possible sum you can create?

3. What is the greatest possible difference you can create?

4. What is the least possible difference you can create?

5. For any of your answers for 1–4, explain or show how you know you have the right answer.
How Many Ways Can You Make a Sum of 18?

Use different numbers from 1 to 9 to create a sum of 18. For example, $4 + 6 + 8 = 18$ is correct because we used three different numbers 4, 6, and 8 to get a sum of 18. However, $5 + 5 + 8$ is not correct because we used 5 twice.

1. Create all possible expressions with sums of 18 that use three different numbers from 1 to 9.

2. Create all possible expressions with sums of 18 that use four different numbers from 1 to 9.

3. If possible, create an expression with a sum of 18 that uses five different numbers from 1 to 9.

4. If possible, create an expression with a sum of 18 that uses six different numbers from 1 to 9.
Spend a Fraction of Unequal Time with 1 to 9
Make the Inequality Statements True

1. Place any of the digits from the set above into the numerators in each inequality shown to the right to make the statement true.

For example, below we have used 1, 3, and 7 to make a true statement:

\[
\begin{array}{c}
\frac{1}{2} < \frac{3}{4} < \frac{7}{8}
\end{array}
\]

You may use a digit more than once, even in the same statement.

You may only create proper fractions.

2. Show at least two possible solutions for any problem that can have more than one solution.

3. Is there any statement in which you can use the same digit in all three boxes? Why does this work? Why won’t this work in other statements?

4. What ideas or strategies did you use to help you solve some or all of these problems? Why do your ideas/strategies work?

a. \[
\begin{array}{c}
\frac{}{2} < \frac{}{4} < \frac{}{8}
\end{array}
\]

b. \[
\begin{array}{c}
\frac{}{4} < \frac{}{2} < \frac{}{8}
\end{array}
\]

c. \[
\begin{array}{c}
\frac{}{8} < \frac{}{4} < \frac{}{2}
\end{array}
\]

d. \[
\begin{array}{c}
\frac{}{8} < \frac{}{2} < \frac{}{4}
\end{array}
\]

e. \[
\begin{array}{c}
\frac{}{4} < \frac{}{8} < \frac{}{2}
\end{array}
\]

f. \[
\begin{array}{c}
\frac{}{2} < \frac{}{8} < \frac{}{4}
\end{array}
\]
Solutions to
Spend Some Time with 1 to 9 (K–8)
Challenges
Create Equations with the Digits 1–9

There are many different possible equations. Here are several examples of equations, including one that uses all nine digits:

18 − 5 − 7 = 6

7² = 49

2(3 + 4) = 19 − 5

54 = 9 × 6

54 × 1 = 9 × 6

9 − 4 + 3 + 2 + 8 = 7 + (6 × 1) + 5
What Can You Do with 5, 6, 8, and 2?

1. \(8 + 6 = 14\)
2. \(2 + 5 = 7\)
3. \(8 - 2 = 6\)
4. \(6 - 5 = 1\)

5.
   a. 8 and 6 are the greater numbers in the set; therefore, they would have the greatest sum.
   b. 2 and 5 are the lesser numbers in the set; therefore, they would have the least sum.
   c. 8 is the greatest value in the set, and 2 is the least value in the set; therefore, they have the greatest difference.
   d. The least possible difference would be two consecutive numbers; therefore, 5 and 6 have the least possible difference (given that we are working only with different whole numbers).
What Else Can You Do with 5, 6, 8, and 2?

1. \(862 + 5 = 867\)
2. \(2 + 5 + 6 + 8 = 21\)
3. \(865 - 2 = 863\)
4. \(62 - 58 = 4\)
5. a. By putting the greatest value in the hundreds place, and next greatest value in the tens place, the greatest sum will result. This can be compared to any other arrangements of the numbers and sums.
   
   b. By adding up each number as a ones number, using no 2-digit numbers, the least possible sum is created. (Note: If you were to create the least sum by creating two 2-digit numbers, this would be \(26 + 58 = 84\).)
   
   c. By creating the greatest value using three numbers and subtracting the least value from this, the greatest difference is created. This can be compared to any other arrangements of the numbers and differences.
   
   d. By creating the two 2-digit values that are closest together, beginning with the tens place, and then the ones place, the least difference is created. This can be compared to any other arrangements of the numbers and differences. (If students understand numbers less than zero, then the meaning of the least difference may need discussion to clarify: do we mean the least absolute value difference, or the least possible value of a difference? 4 is the least possible absolute value difference, while \(-863\) is the least possible value if you have \(2 - 863\).)
How Many Ways Can You Make a Sum of 18?

1. \(5 + 6 + 7, \ 4 + 6 + 8, \ 3 + 6 + 9, \ 2 + 7 + 9, \ 1 + 8 + 9, \ 3 + 7 + 8, \ 4 + 5 + 9\)

2. \(1 + 2 + 6 + 9, \ 1 + 2 + 7 + 8, \ 1 + 3 + 5 + 9, \ 1 + 3 + 6 + 8, \ 1 + 4 + 5 + 8, \ 1 + 4 + 6 + 7, \ 2 + 3 + 4 + 9, \ 2 + 3 + 5 + 8, \ 2 + 3 + 6 + 7, \ 2 + 4 + 5 + 7, \ 3 + 4 + 5 + 6\)

3. \(1 + 2 + 3 + 4 + 8, \ 1 + 2 + 3 + 5 + 7, \ 1 + 2 + 4 + 5 + 6\)

4. \(1 + 2 + 3 + 4 + 5 + 6 = 21\)

Since these are the least six numbers that can be used and the sum is still greater than 18, then it is not possible to create a sum of 18 with six different numbers from 1 to 9.
1–2.

a. \[
\begin{array}{ccc}
\frac{1}{2} & < & \frac{3}{4} & < & \frac{7}{8}
\end{array}
\]

A possible second solution: None

b. \[
\begin{array}{ccc}
\frac{1}{4} & < & \frac{1}{2} & < & \frac{7}{8}
\end{array}
\]

A possible second solution: \[
\begin{array}{ccc}
\frac{1}{4} & < & \frac{1}{2} & < & \frac{5}{8}
\end{array}
\]

c. \[
\begin{array}{ccc}
\frac{1}{8} & < & \frac{1}{4} & < & \frac{1}{2}
\end{array}
\]

A possible second solution: None

d. \[
\begin{array}{ccc}
\frac{3}{8} & < & \frac{1}{2} & < & \frac{3}{4}
\end{array}
\]

A possible second solution: \[
\begin{array}{ccc}
\frac{1}{8} & < & \frac{1}{2} & < & \frac{3}{4}
\end{array}
\]

e. \[
\begin{array}{ccc}
\frac{1}{4} & < & \frac{3}{8} & < & \frac{1}{2}
\end{array}
\]

A possible second solution: None

f. \[
\begin{array}{ccc}
\frac{1}{2} & < & \frac{5}{8} & < & \frac{3}{4}
\end{array}
\]

A possible second solution: None

3. Problem c is the only one in which the same number can be used for all three numerators. This means that each numerator represents the same number of parts; therefore, only the denominator, or size of the parts, makes a difference. So the denominators must be in order from least to greatest in terms of size of parts. Only c has denominators in this order.

4. Answers will vary.
Symbol and Cue Cards

1. Create a set of symbol cards and matching cue cards as shown on the following pages.
   - **Symbol cards**: Cards containing math terms, expressions, equations, etc.
   - **Cue cards**: Cards containing phrases that match one or more of the symbol cards. You may have more than one cue card for the same symbol card.

2. Play the game:
   a. Give each student (or pair of students) a set of symbol cards.
   b. Have students match symbol cards to cue cards by one of the following methods. Always check for understanding (correct matching). Students can also be required to record matches.
      i. Option 1: Someone (teacher) reads a cue card and students hold up the corresponding symbol card.
      ii. Option 2: Hand out a set of the cue cards to each student (or pair of students) and have students match cards.

**Symbol Cards**

<table>
<thead>
<tr>
<th>8 – n</th>
<th>n &lt; 8</th>
<th>8 + n</th>
</tr>
</thead>
<tbody>
<tr>
<td>8n</td>
<td>n &gt; 8</td>
<td>8(n)</td>
</tr>
<tr>
<td>n – 8</td>
<td>n = 8</td>
<td>n + 8</td>
</tr>
<tr>
<td>8 / n</td>
<td>n / 8</td>
<td>( \frac{n}{8} )</td>
</tr>
</tbody>
</table>
### Cue Cards

<table>
<thead>
<tr>
<th>The product of ( n ) and 8</th>
<th>8 is greater than ( n )</th>
<th>The sum of 8 and ( n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>The product of 8 and ( n )</td>
<td>8 is less than ( n )</td>
<td>The sum of ( n ) and 8</td>
</tr>
<tr>
<td>8 less than ( n )</td>
<td>( n ) is greater than 8</td>
<td>The difference between 8 and ( n )</td>
</tr>
<tr>
<td>( n ) less than 8</td>
<td>( n ) is less than 8</td>
<td>The difference between ( n ) and 8</td>
</tr>
<tr>
<td>8 more than ( n )</td>
<td>The quotient of 8 divided by ( n )</td>
<td>8 out of ( n ) equal parts</td>
</tr>
<tr>
<td>( n ) more than 8</td>
<td>The quotient of ( n ) divided by 8</td>
<td>( n ) out of 8 equal parts</td>
</tr>
<tr>
<td>8 divided by ( n )</td>
<td>8 ( n )'s</td>
<td>( n ) eighths</td>
</tr>
<tr>
<td>( n ) divided by 8</td>
<td>8 times ( n )</td>
<td>( n ) and 8 are the same</td>
</tr>
</tbody>
</table>